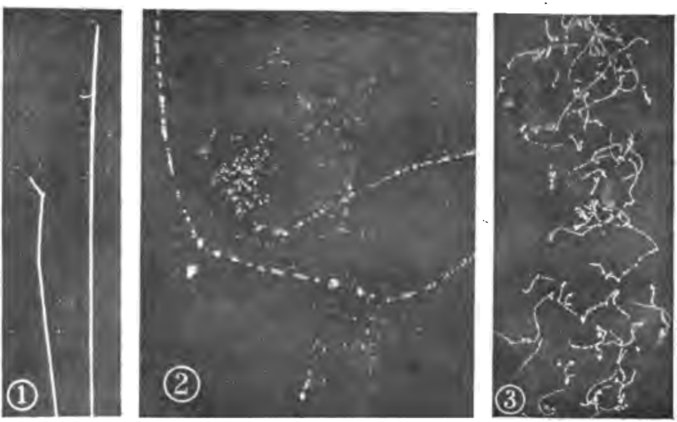
A RECENT ACHIEVEMENT OF APPLIED PHYSICS

André Beaumont speeding over the Marconi wireless station at Genoa



A RECENT ACHIEVEMENT OF PURE PHYSICS

Instantaneous photographs by C. T. R. Wilson. 1. Tracks of Alpha particles of radium through air. 2. Tracks of Beta particles through air. 3. Tracks of electrons ejected by X rays from air molecules (see p. 424)

A FIRST COURSE IN PHYSICS

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REVISED EDITION

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON

614.7

The Athenæum Press
GINN AND COMPANY · PRO-
PRIETORS · BOSTON · U.S.A.

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PREFACE

The chief aim of the first edition of this book was *to present elementary physics in such a way as to stimulate the pupil to do some thinking on his own account about the hows and whys of the physical world in which he lives.* With this end in view, we abandoned the formal, didactic method so largely in use during the preceding decade. In place of it we used a method which uniformly started with some simple experiment or some well-known phenomenon. The consideration of *how* a thing happened was then followed by a discussion of *why* it happened, definitions being in general inserted only after the need for them had been felt by the pupil. Finally, a carefully chosen set of questions and problems following each day's work, rather than each chapter, led the student to find for himself the connections between the phenomenon in hand and other familiar happenings. Such a method led inevitably to the final grouping of the apparently disconnected facts of physics about certain great underlying principles, such as the kinetic theory, the work principle, the electron theory, and the wave theory.

Since the appearance of the book we have been deeply gratified both by the fact that our method has met with continually increasing favor because of its demonstrated pedagogical efficiency, and by the fact that the developments of the past seven years in the science of physics itself have so amply justified our points of view. For example, within that time the fundamental conceptions of both the kinetic theory of matter and the atomic, or electronic, theory of electricity have become as well established as the theory of the rotation of the earth upon its axis, so that to fail to utilize them now in the interpretation of the phenomena of molecular physics

and of electricity would be precisely like teaching sunrise and sunset as merely observable facts without any reference to their interpretation in the light of the earth's rotation.

In the present revision our former method has been maintained. In addition, we have aimed to bring the subject matter thoroughly up to date, and to make certain changes which seemed desirable. The most important of these changes are as follows:

(1) The approach to the subject of physics has been made more simple and more interesting by postponing the chapter on force and motion until after the discussion of the fascinating phenomena of liquids and gases.

(2) The treatment of force and motion has been considerably simplified.

(3) The book has been shortened by about sixty pages, in order to give opportunity for an extended review at the end of the course.

(4) A carefully selected list of review questions and problems has been inserted at the end.

(5) The absolute units have been subordinated even more than in the first edition; for example, all electrical quantities are defined in this edition in terms of the practical, legal units.

(6) The presentation of the fundamental principles underlying the dynamo and motor has been notably simplified, and all of the operations involved in these machines have been reduced to one simple rule.

(7) The treatment of image formation has been greatly simplified through a more complete combination of the wave and ray methods than we used before.

(8) More than sixty new illustrations and a large number of new problems have been introduced.

(9) New half-tones have been inserted, illustrating some of the most notable achievements of modern physics, both in the field of application and of pure science.

(10) The portraits of some of the most eminent of modern physicists have been inserted, as well as those of the great pioneers of the science.

The frontispiece illustrates the combination of pure and applied physics, which is the guiding principle of the course.

For the sake of indicating in what directions omissions may be made if necessary, without interfering with continuity, paragraphs here and there have been thrown into fine print. These paragraphs will be easily distinguished from the classroom experiments, which are in the same type. They are for the most part descriptions of physical appliances.

Some teachers prefer to have the chapter on heat transference (X) follow immediately after the chapter on thermometry and expansion (VII). This order is as satisfactory to the authors as that given.

It is quite impossible for us to make suitable recognition of the assistance which has been derived from suggestions which have been sent to us from all over the United States. We owe an especial debt, however, to H. Clyde Krenerick, of Milwaukee; Willard R. Pyle, of New York; Willis E. Tower and Edwin S. Bishop, of Chicago.

THE UNIVERSITY OF CHICAGO

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A FIRST COURSE IN PHYSICS

CHAPTER I

MEASUREMENT

FUNDAMENTAL UNITS

1. Introductory. A certain amount of knowledge about familiar things comes to us all very early in life. We learn almost unconsciously, for example, that stones fall and balloons rise, that the teakettle stops boiling when removed from the fire, that telephone messages travel by electric currents, etc. The aim of physics is to set us to thinking about *how* and *why* such things happen, and to a less degree to acquaint us with other happenings which we may not have noticed or heard of previously. Most of our accurate knowledge about natural phenomena has been acquired by the making of careful measurements. To quote the words of Lord Kelvin, one of the greatest physicists of the last century: "When you can measure what you are speaking about, and express it in numbers, you know something about it; and when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of a science."

We can measure three fundamentally different kinds of quantities, — length, mass, and time, — and we shall find that all other measurements may be reduced to these three. Our

first problem in physics is then to learn something about the units in terms of which all our physical knowledge is expressed.

2. The historic standard of length. Nearly all civilized nations have at some time employed a unit of length the name of which bore the same significance as does *foot* in English. There can scarcely be any doubt, therefore, that in each country this unit has been derived from the length of the human foot. It is probable that in England, after the yard (a unit which is supposed to have represented the length of the arm of King Henry I) became established as a standard, the foot was arbitrarily chosen as one third of this standard yard. In view of such an origin it will be clear why no agreement existed among the units in use in different countries.

3. Relations between different units of length. It has also been true, in general, that in a given country the different units of length in common use, such, for example, as the inch, the hand, the foot, the fathom, the rod, the mile, etc., have been derived either from the lengths of different members of the human body or from equally unrelated magnitudes, and in consequence have been connected with one another by different, and often by very awkward, multipliers. Thus, there are 12 inches in a foot, 3 feet in a yard, $5\frac{1}{2}$ yards in a rod, 1760 yards in a mile, etc.

4. Relations between units of length, area, volume, and mass. A similar and even worse complexity exists in the relations of the units of length to those of area, capacity, and mass. Thus, there are $272\frac{1}{4}$ square feet in a square rod; $57\frac{3}{4}$ cubic inches in a quart, and $31\frac{1}{2}$ gallons in a barrel. Again the pound, instead of being the mass of a cubic inch or a cubic foot of water, or of some other common substance, is the mass of a cylinder of platinum, of inconvenient dimensions, which is preserved in London.

5. Origin of the metric system. At the time of the French Revolution the extreme inconvenience of existing weights and measures, together with the confusion arising from the use of different standards in different localities, led the National Assembly of France to appoint a commission to devise a more logical system. The result of the labors of this commission was the present metric system, which was introduced in France in 1793, and has since been adopted by the governments of most civilized nations except those of Great Britain and the United States; and even in these countries its use in scientific work is practically universal.

6. The standard meter. The standard *length* in the metric system is called the *meter*. It is the distance, at the freezing temperature, between two transverse parallel lines ruled on a bar of platinum-iridium (Fig. 1), which is kept at the International Bureau of Weights and Measures at Sèvres, near Paris.

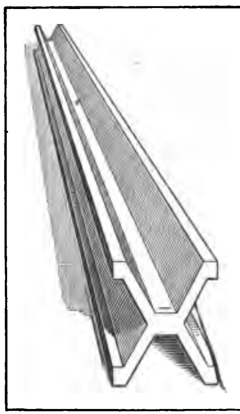


FIG. 1. The standard meter

In order that this standard length might be reproduced if lost, the commission attempted to make it one ten-millionth of the distance from the equator to the north pole, measured on the meridian of Paris. But since later measurements have thrown some doubt upon the exactness of the commission's determination of this distance, we now define the meter, not as any particular fraction of the earth's quadrant, but simply as the distance between the scratches on the bar mentioned above. This distance is 39.37 inches, or about 1.1 yards. On account of its more convenient size, the centimeter, one one-hundredth of a meter, is universally used for scientific purposes, as the fundamental unit of length.

7. Metric standard capacity. The standard unit of capacity is called the *liter*. It is the volume of a cube which is one tenth of a meter (about 4 inches) on a side, and is therefore equal to 1000 cubic centimeters (cc.). It is equivalent to 1.057 quarts. A liter and a quart are therefore roughly equivalent measures.

8. The metric standard of mass. In order to establish a connection between the unit of length and the unit of mass, the commission directed a committee of the French Academy to prepare a cylinder of platinum which should have the same weight as a liter of water at its temperature of greatest density, namely, 4° Centigrade (39° Fahrenheit). An exact equivalent of this cylinder made of platinum-iridium, and kept at Sèvres with the standard meter, now represents the standard of mass in the metric system. It is called the *standard kilogram*, and is equivalent to about 2.2 pounds. One one-thousandth of this mass was adopted as the fundamental unit of mass and was named the *gram*. For practical purposes, therefore, the *gram may be taken as equal to the mass of one cubic centimeter of water.*

9. The other metric units. The three standard units of the metric system — the meter, the liter, and the gram — have decimal multiples and submultiples, so that every unit of length, volume, or mass is connected with the unit of next higher denomination by an invariable multiplier, namely, ten.

The names of the multiples are obtained by adding the Greek prefixes, *deka* (ten), *hecto* (hundred), *kilo* (thousand); while the submultiples are formed by adding the Latin prefixes, *deci* (tenth), *centi* (hundredth), and *milli* (thousandth). Thus:

1 dekameter = 10 meters	1 decimeter = $\frac{1}{10}$ meter
1 hectometer = 100 meters	1 centimeter = $\frac{1}{100}$ meter
1 kilometer = 1000 meters	1 millimeter = $\frac{1}{1000}$ meter

The most common of these units, with the abbreviations which will henceforth be used for them, are the following:

meter (m.)	millimeter (mm.)	gram (g.)
kilometer (km.)	liter (l.)	kilogram (kg.)
centimeter (cm.)	cubic centimeter (cc.)	milligram (mg.)

1,000,000

10. Relations between the English and metric units. The following table gives the relation between the most common English and metric units.

1 inch (in.)	= 2.54 cm.	1 cm.	= .3937 in.
1 foot (ft.)	= 30.48 cm.	1 m.	= 1.094 yd. = 39.37 in.
1 mile (mi.)	= 1.609 km.	1 km.	= .6214 mi.
1 sq. in.	= 6.45 sq. cm.	1 sq. cm.	= .1550 sq. in.
1 sq. ft.	= 929.03 sq. cm.	1 sq. m.	= 1.196 sq. yd.
1 cu. in.	= 16.387 cc.	1 cc.	= .061 cu. in.
1 cu. ft.	= 28,317 cc.	1 cu. m.	= 1.308 cu. yd.
1 qt.	= .9463 l.	1 l.	= 1.057 qt.
1 grain	= 64.8 mg.	1 g.	= 15.44 grains
1 oz. av.	= 28.35 g.	1 g.	= .0353 oz.
1 lb. av.	= .4536 kg.	1 kg.	= 2.204 lb.

This table is inserted chiefly for reference; but the relations 1 in. = 2.54 cm., 1 m. = 39.37 in., 1 kilo (kg.) = 2.2 lb., 1 km. = .62 mi., should be memorized. Portions of a centimeter and of an inch scale are shown together in Fig. 2.

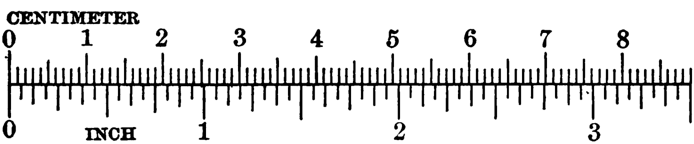


FIG. 2. Centimeter and inch scales

11. The standard unit of time. The *second* is taken among all civilized nations as the standard unit of time. It is $\frac{1}{86400}$ part of the time from noon to noon.

12. The three fundamental units. It is evident that measurements of both area and volume may be reduced simply to measurements of length; for an area is expressed as the product of two lengths, and a volume as the product of three lengths. For these reasons the units of area and volume are looked upon as *derived* units, depending on one *fundamental* unit, the unit of length.

Now it is found that just as measurements of area and of volume can be reduced to measurements of length, so the determination of any measurable quantities, such as the pressure in a steam boiler, the velocity of a moving train, the amount of electricity consumed by an electric lamp, the amount of magnetism in a magnet, etc., can be reduced simply to measurements of length, mass, and time. Hence *the centimeter, the gram, and the second are considered the three fundamental units.* Whenever any measurement has been reduced to its equivalent in terms of centimeters, grams, and seconds it is said, for short, to be expressed in C.G.S. (Centimeter-Gram-Second) units.

13. Measurement of length. Measuring the length of a body consists simply in comparing its length with that of the standard meter bar kept at the International Bureau. In order that this may be done conveniently, great numbers of rods of the same length as this standard meter bar have been made and scattered all over the world. They are our common meter sticks. They are divided into 10, 100, or 1000 equal parts, great care being taken to have all the parts of exactly the same length. The method of making a measurement with such a bar is more or less familiar to every one.

14. Measurement of mass. Similarly, measuring the mass of a body consists in comparing its mass with that of the standard kilogram. In order that this might be done conveniently, it was first necessary to construct bodies of the same mass as this kilogram and then to make a whole series of bodies whose masses were $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., of the mass of this kilogram; in other words, to construct a set of standard masses commonly called a *set of weights.*

With the aid of such a set of standard masses, the determination of the mass of any unknown body is made by first placing the body upon the pan *A* (Fig. 3) and counterpoising with shot, paper, etc., then replacing the unknown body by

as many of the standard masses as are required to bring the pointer back to O again. The mass of the body is equal to the sum of these standard masses. This is the rigorously correct method of making a weighing, and is called the *method of substitution*.

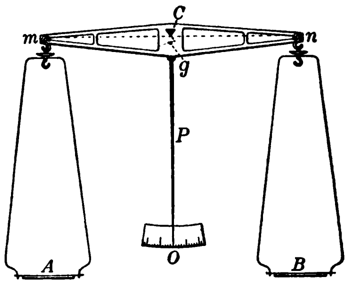


FIG. 3. The simple balance

If a balance is well constructed, however, a weighing may usually be made with sufficient accuracy by simply placing the unknown body upon one pan and finding the sum of the standard masses which must then be placed upon the other pan to bring the pointer again to O. This is the usual method of weighing. It gives correct results, however, only when the knife-edge C is exactly midway between the points of support m and n of the two pans. The method of substitution, on the other hand, is independent of the position of the knife-edge.

QUESTIONS AND PROBLEMS

1. The Twentieth Century Limited runs from New York to Chicago (967 mi.) in 18 hr. What is its average speed in miles per hour? in kilometers per hour?

2. Name as many advantages as you can which the metric system has over the English system. Can you think of any disadvantages?

3. What must you do to find the capacity in liters of a box when its length, breadth, and depth are given in meters? to find the capacity in quarts when its dimensions are given in feet?

4. Find the number of millimeters in 5 km. Find the number of inches in 3 mi. Which is the easier?

5. In the 1912 Gordon Bennett aviation cup race Jules Védrines in a Deperdussin monoplane made a world's record by flying 20 km. in 6 min. 55.9 sec. What was his speed in miles per hour?

6. A freely falling body starting from rest moves 490 cm. during the first second. Express this distance in feet.

†

DENSITY

15. Definition of density. When equal volumes of different substances, such as lead, wood, iron, etc., are weighed in the manner described above, they are found to have widely different masses. The term "density" is used to denote *the mass of unit volume* of a substance.

Thus, for example, in the English system the cubic foot is the unit of volume and the pound the unit of mass. Since 1 cubic foot of water is found to weigh 62.4 pounds, we say that in the English system *the density of water is* 62.4 *pounds per cubic foot.*

In the C.G.S. system the cubic centimeter is taken as the unit of volume and the gram as the unit of mass. Hence we say that in this system the density of water is 1 gram per cubic centimeter, for it will be remembered that the gram was taken as the mass of 1 cubic centimeter of water. Unless otherwise expressly stated, density is now universally understood to mean density in C.G.S. units, that is, *the density of a substance is the mass in grams of* 1 *cubic centimeter of that substance.* For example, if a block of cast iron 3 cm. wide, 8 cm. long, and 1 cm. thick weighs 177.6 g., then, since there are 24 cc. in the block, the mass of 1 cc., that is, the density, is equal to $\frac{177.6}{24}$, or 7.4 g. per cc.

The density of some of the most common substances is given in the following table:

DENSITIES OF SOLIDS

(In grams per cubic centimeter)

Aluminium	2.58	Lead	11.3
Brass	8.5	Nickel	8.9
Copper	8.9	Oak	.8
Cork	.24	Pine	.5
Glass	2.6	Platinum	21.5
Gold	19.3	Silver	10.53
Iron (cast)	7.4	Tin	7.29
Iron (wrought)	7.86	Zinc	7.15

Densities of Liquids

(In grams per cubic centimeter)

Alcohol	.79	Hydrochloric acid	1.27
Carbon bisulphide	1.29	Mercury	13.6
Glycerin	1.26	Olive oil	.91

16. Relation between mass, volume, and density. Since the volume of a body is equal to the number of cubic centimeters which it contains, and since its density is by definition the number of grams in 1 cubic centimeter, its mass, that is, the total number of grams which it contains, must evidently be equal to its volume times its density. Thus, if the density of iron is 7.4 and if the volume of an iron body is 100 cc., the mass of this body in grams must equal $7.4 \times 100 = 740$. To express this relation in the form of an equation, let M represent the mass of a body, that is, its total number of grams; V its volume, that is, its total number of cubic centimeters; and D its density, that is, the number of grams in 1 cubic centimeter; then

$$D = \frac{M}{V}. \tag{1}$$

This equation merely states the definition of density in algebraic form.

17. Distinction between density and specific gravity. The term "specific gravity" is used to denote *the ratio between the weight of a body and the weight of an equal volume of water.* Thus, if a cubic centimeter of iron weighs 7.4 times as much as a cubic centimeter of water, its specific gravity is 7.4. But the density of iron in C.G.S. units is 7.4 grams per cubic centimeter, for by definition density in that system is the mass per cubic centimeter. It is clear, then, that *density in C.G.S. units is numerically the same as specific gravity.*

Specific gravity is the same in all systems, since it simply expresses how many times heavier a body is than an equal volume of water. Density, however, which we have defined as the mass per unit volume, is different in different systems. Thus,

in the English system the density of iron is 461 pounds per cubic foot (7.4×62.4), since we have found that water weighs 62.4 pounds per cubic foot and iron weighs 7.4 times as much as an equal volume of water.

Since we shall henceforth use the term "density" to signify exclusively density in the C.G.S. system of units, we shall have little further use in this book for the term "specific gravity." *

QUESTIONS AND PROBLEMS

1. If a wooden beam is $30 \times 20 \times 500$ cm. and has a mass of 150 kg., what is the density of wood?

2. Would you attempt to carry home a block of gold the size of a peck measure? (Consider a peck equal to 8 l. See table, p. 8.)

3. What is the mass of a liter of alcohol?

4. How many cubic centimeters in a block of brass weighing 34 g.?

5. What is the weight in metric tons of a cube of lead 2 m. on an edge? (A metric ton is 1000 kilos, or about 2200 lb.)

6. Find the volume in liters of a block of platinum weighing 45.5 kilos.

7. Find the density of a steel sphere of radius 1 cm. and weight 32.7 g.

8. One kilogram of alcohol is poured into a cylindrical vessel and fills it to a depth of 8 cm. Find the cross section of the cylinder.

9. A capillary glass tube weighs .2 g. A thread of mercury 10 cm. long is drawn into the tube, when it is found to weigh .6 g. Find the cross section of the capillary tube.

10. Find the length of a lead rod 1 cm. in diameter and weighing 1 kg.

* Laboratory exercises on length, mass, and density measurements should accompany or follow this chapter. See, for example, Experiments 1, 2, and 3 of the authors' manual.

CHAPTER II

PRESSURE IN LIQUIDS

Liquid Pressure beneath a Free Surface

18. Force beneath the surface of a liquid. We are all conscious of the fact that in order to lift a kilogram of mass we must exert an upward pull. Experience has taught us that the greater the mass, the greater the force which we must exert. In fact, the force is commonly taken as numerically equal to the mass lifted. This is called the *weight measure* of a force. *A push or pull which is equal to that required to support a gram of mass is called a gram of force.*

To investigate the nature of the forces beneath the free surface of a liquid, we shall use a pressure gauge of the form shown in Fig. 4. If the rubber diaphragm which is stretched across the mouth of a thistle tube A is pressed in lightly with the finger, the drop of ink B will be observed to move forward in the tube T, but it will return again to its first position as soon as the finger is removed. If the pressure of the finger is increased, the drop will move forward a greater distance than before. We may therefore take the amount of motion of the drop as a measure of the amount of force acting on the diaphragm.

Now let A be pushed down first 2, then 4, then 8 cm. below the surface of the water (Fig. 4). The motion of the index B will show that the upward force continually increases as the depth increases.

Careful measurements made in the laboratory will show that *the force is directly proportional to the depth.**

*It is recommended that quantitative laboratory work on the law of depths and on the use of manometers accompany this discussion. See, for example, Experiments 4 and 5 of the authors' manual.

Let the diaphragm A (Fig. 4) be pushed down to some convenient depth, for example, 10 centimeters, and the position of the index noted. Then let it be turned sidewise so that its plane is vertical (see a, Fig. 4), and adjusted in position until its center is exactly 10 centimeters beneath the surface, that is, until the *average* depth of the diaphragm is the same as before. The position of the index will show that the force is also exactly the same as before.

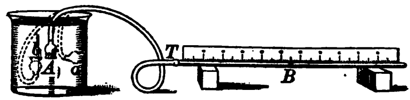


FIG. 4. Gauge for measuring liquid pressure

Let the diaphragm then be turned to the position b, so that the gauge measures the *downward* force at a depth of 10 centimeters. The index will show that this force is again the same.

We conclude, therefore, that *at a given depth a liquid presses up and down and sidewise on a given surface with exactly the same force.*

19. Magnitude of the force. If a vessel like that shown in Fig. 5 is filled with a liquid, the force against the bottom is obviously equal to the weight of the column of liquid resting upon the bottom. Thus, if F represents this force in grams, A the area in square centimeters, h the depth in centimeters, and d the density in grams per cubic centimeter, we shall have

$$F = Ahd. \tag{1}$$

FIG. 5

Since, as was shown by the experiment of the preceding section, the force is the same in all directions at a given depth, we have the following general rule:

The force which a liquid exerts against any surface is equal to the area of the surface times its average depth times the density of the liquid.

It is important to remember that "average depth" means the vertical distance from the level of the free surface to the center of the area in question.

20. Pressure in liquids. Thus far attention has been confined to the total force exerted by a liquid against the *whole* of a given surface. It is often more convenient to consider the surface divided into square centimeters or square inches, and consider the force on one of these units of area. In physics the word "pressure" is used exclusively to denote the *force per unit area.* Pressure is thus a measure of the *intensity* of the force acting on a surface, and does not depend at all on the area of the surface. Since, by § 19, $F = Ahd$, and since by definition the pressure p is equal to the force per unit area, we have

$$p = \frac{F}{A} = hd. \qquad (2)$$

Therefore *the pressure at a depth of h centimeters below the surface of a liquid of density d is hd grams per square centimeter.*

If the height is given in feet and the density in pounds per cubic foot, namely, 62.4, then the product hd gives pressure in pounds per square foot. Dividing by 144 gives the result in pounds per square inch.

21. Levels of liquids in connecting vessels. It is a perfectly familiar fact that when water is poured into a teapot it stands at exactly the same level in the spout as in the body of the teapot; or if it is poured into a number of connected tubes, like those shown in Fig. 6, the surfaces of the liquid in the various tubes lie in the same horizontal plane. Now the pressure at c, Fig. 7, was shown by the experiment of § 18 to be equal to the density of the liquid times the depth cg. The pressure at o in the opposite direction must be equal to that at c, since the liquid does not tend to move in either direction. Hence the pressure at o must be ks times the density.

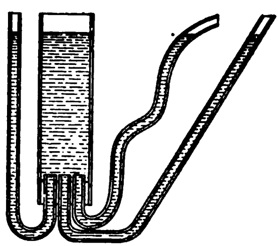


FIG. 6. Water level in communicating vessels

If water is poured in at *s* so that the height *ks* is increased, the pressure to the left at *o* becomes greater than the pressure to the right at *c*, and a flow of water takes place to the left until the heights are again equal.

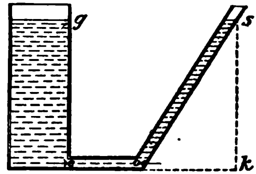


FIG. 7. Why water seeks its level

It follows from these observations on the level of water in connected vessels that the *pressure beneath the surface of a liquid depends simply on the vertical depth beneath the free surface, and not at all on the size or shape of the vessel.*

QUESTIONS AND PROBLEMS

1. If the areas of the surfaces AB in Fig. 8, (1) and (2) are the same, and if water is poured into each vessel at D till it stands at the same height above AB, how will the downward force on AB in Fig. 8, (2) compare with that in Fig. 8, (1)? Test your answer, if possible, by making AB a piece of cardboard and pouring water in at D in each case until the cardboard is forced off.

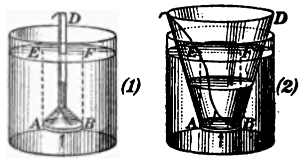


FIG. 8. Illustrating hydrostatic paradox

2. Soundings at sea are made by lowering some kind of a pressure gauge. When this gauge reads 1.3 kg. per square centimeter, what is the depth? (Density of sea water = 1.026.)

3. If the pressure at a tap on the first floor reads 80 lb. per square inch, and at a tap two floors above 68 lb., what is the difference in feet between the levels of the two taps?

4. If the vessel shown in Fig. 10, (1), p. 15, has a base of 200 sq. cm. and if the water stands 100 cm. deep, what is the total force on the bottom?

5. If the weight of the empty vessel in Fig. 10, (1) is small compared with the weight of the contained water, will the force required to lift the vessel and water be greater or less than the force exerted by the water against the bottom? Explain.

6. Find the total force against the gate of a lock if its width is 60 ft. and the depth of the water 20 ft. Will it have to be made stronger if it holds back a lake than if it holds back a small pond?

7. A whale when struck with a harpoon will often dive straight down as much as 400 fathoms (2400 ft.). If the body is 60 ft. long and has an average circumference of 15 ft., what is the total force to which it is subjected?

8. A hole 5 cm. square is made in a ship's bottom 7 m. below the water line. What force in kilograms is required to hold a board above the hole?

9. Thirty years ago standpipes were generally straight cylinders. To-day they are more commonly of the form shown in Fig. 9. What are the advantages of each form?

FIG. 9. A water reservoir

PASCAL'S LAW

22. Transmission of pressure by liquids. From the fact that pressure within a free liquid depends simply upon the depth and density of the liquid, it is possible to deduce a very surprising conclusion, which was first stated by the famous French scientist, mathematician, and philosopher, Pascal (1623–1662).

Let us imagine a vessel of the shape shown in Fig. 10, (1), to be filled with water up to the level *ab*. For simplicity let the upper portion be assumed to be 1 square centimeter in cross section. Since the density of water is 1, the force with which it presses against any square centimeter of the interior surface which is h centimeters beneath the level *ab* is h grams. Now let 1 gram of water (that is, 1 cubic centimeter) be poured into the tube. Since each square centimeter of surface which was before h centimeters beneath the level of the water in the tube is now $h+1$ centimeters beneath this level, the new pressure which the water exerts

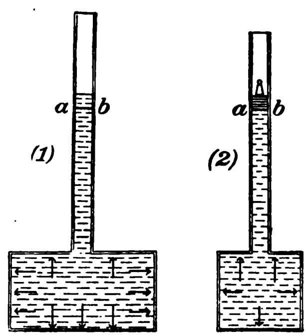


FIG. 10. Proof of Pascal's law

against it is $h+1$ grams; that is, applying 1 gram of force to the square centimeter of surface *ab* has added 1 gram to the force exerted by the liquid against each square centimeter of the interior of the vessel. Obviously it can make no difference whether the pressure which was applied to the surface *ab* was due to a weight of water or to a piston carrying a load, as in Fig. 10, (2), or to any other cause whatever. We thus arrive at Pascal's conclusion that *pressure applied anywhere to a body of confined liquid is transmitted undiminished to every portion of the surface of the containing vessel.*

23. Multiplication of force by the transmission of pressure by liquids. Pascal himself pointed out that with the aid of the principle stated above we ought to be able to transform a very small force into one of unlimited magnitude. Thus, if the area of the cylinder *ab* (Fig. 11) is 1 sq. cm., while that of the cylinder *AB* is 1000 sq. cm., a force of 1 kg. applied to *ab* would be transmitted by the liquid so as to act with a force of 1 kg. on each square centimeter of the surface *AB*. Hence the total upward force exerted against the piston *AB* by the 1 kg. applied at *ab* would be 1000 kg. Pascal's own words are as follows: "A vessel full of water is a new principle in mechanics, and a new machine for the multiplication of force to any required extent, since one man will by this means be able to move any given weight."

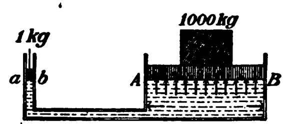


FIG. 11. Multiplication of force by transmission of pressure

24. **The hydraulic press.** The experimental proof of the correctness of the conclusions of the preceding paragraph is furnished by the hydraulic press, an instrument now in common use for subjecting to enormous pressures paper, cotton, etc., and for punching holes through iron plates, testing the strength of iron beams, extracting oil from seeds, making dies, embossing metal, etc.

Such a press is represented in section in Fig. 12. As the small piston p is raised, water from the cistern C enters the piston chamber through the valve v. As soon as the down stroke begins, the valve v closes, the valve v' opens, and the pressure applied on the piston p is transmitted through

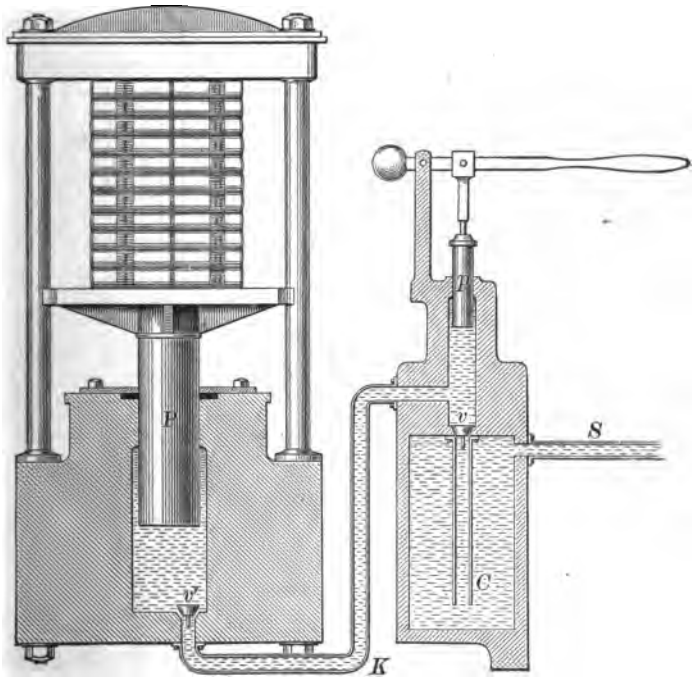


FIG. 12. Diagram of a hydraulic press

the tube K to the large reservoir, where it acts on the large cylinder P with a force which is as many times that applied to p as the area of P is times the area of p.

Hand presses similar to that shown in Fig. 12 are often made which are capable of exerting a compressing force of from 500 to 1000 tons.

25. No gain in the product of force times distance. It should be noticed that, while the force acting on AB (Fig. 11) is 1000 times as great as the force acting on ab, the distance

through which the piston AB is pushed up in a given time is but $\frac{1}{1000}$ of the distance through which the piston ab moves down. For, forcing ab down a distance of 1 centimeter crowds but 1 cubic centimeter of water over into the large cylinder, and this additional cubic centimeter can raise the level of the water there but $\frac{1}{1000}$ centimeter. We see, therefore, that the product of the force acting by the distance moved is precisely the same at both ends of the machine. This important conclusion will be found in our future study to apply to all machines.

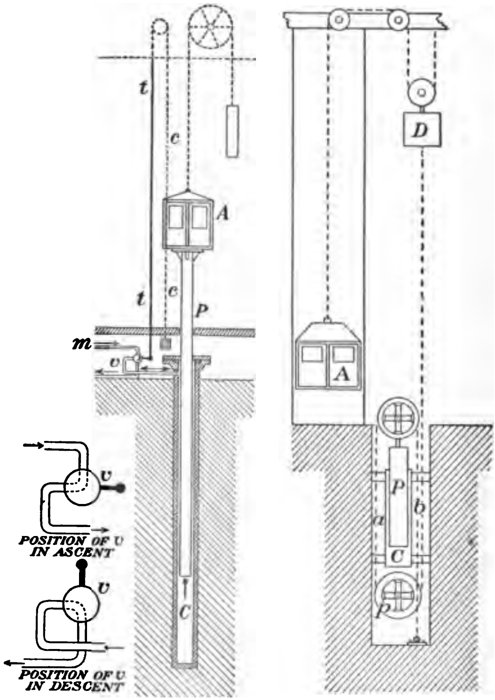


FIG. 13 FIG. 14

Diagrams of hydraulic elevators

26. The hydraulic elevator. Another very common application of the principle of transformation of pressure by liquids is found in the hydraulic elevator. The simplest form of such an elevator is shown in Fig. 13. The cage A is borne on the top of a long piston P which runs in a cylindrical pit C of the same depth as the height to which the carriage must ascend. Water enters the pit either directly from the water mains m of the city's supply, or, if this does not furnish sufficient pressure, from a special reservoir on top of the building. When the elevator boy pulls *up* on the cord cc, the valve v opens so as to make connection from m into C. The

elevator then ascends. When *cc* is pulled *down*, *v* turns so as to permit the water in *C* to escape into the sewer. The elevator then descends.

Where speed is required the motion of the cylinder is communicated indirectly to the cage by a system of pulleys like that shown in Fig. 14. With this arrangement a foot of upward motion of the cylinder *P* causes the counterpoise *D* of the cage to descend 2 feet, for it is clear from the figure that when the cylinder goes up 1 foot enough rope must be pulled over the fixed pulley *p* to lengthen each of the two strands *a* and *b* 1 foot. Similarly, when the counterpoise descends 2 feet the cage ascends 4 feet. Hence the cage moves four times as fast and four times as far as the cylinder. The elevators in the Eiffel Tower in Paris are of this sort. They have a total travel of 420 feet and are capable of lifting 50 people 400 feet per minute.

27. City water supply. Fig. 15 illustrates the method by which a city is often supplied with water from a distant source. The aqueduct from the lake *a* passes under a road *r*, a brook *b*, a hill *H*, and into a reservoir *e*, from which it is forced by the pump *p* into the standpipe *P*, whence it is distributed to the houses of the city. If a static condition prevailed in the whole system, then the water level in *e* would of necessity be

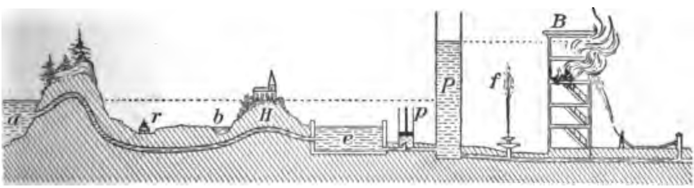


FIG. 15. City water supply from lake

the same as that in *a*, and the level in the pipes of the building *B* would be the same as that in the standpipe *P*. But when the water is flowing, the friction of the mains causes the level in *e* to be somewhat less than that in *a*, and that in *B* less than that in *P*. It is on account of the friction both of the air and of the pipes that the fountain *f* does not actually rise nearly as high as the ideal limit shown in the figure (see dotted line).

28. Artesian wells. It is in the principle of transmission of pressure by liquids that artesian wells find their explanation. Fig. 16 is an ideal section of what geologists call an artesian basin. The stratum *A* is composed of some porous material such as sand, open-textured sandstone, or broken rock, through which the water can percolate easily. Above and below it are strata *C* and *B* of clay, slate, or some other material impervious to water. The porous layer is filled with water which finds entrance at the outcropping margins. As soon as a boring

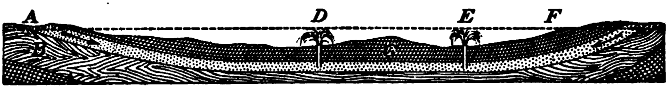


Fig. 16. Artesian wells

is made through the layer *C* the water gushes forth because of the transmission of pressure from the higher levels. A well of this sort exists near Leipzig, Germany, which is 5735 feet deep. Many artesian wells have been bored in the desert of Sahara and an abundant water supply found at a depth of 200 feet. Great numbers of these wells exist in the United States, notable ones being located at Chicago, Louisville (Kentucky), and Charleston (South Carolina). The artesian basins in which the wells are found are often a hundred miles or more in width.

QUESTIONS AND PROBLEMS

1. How does your city get its water? How is the pressure in the pipes maintained?

2. A jug full of water may often be burst by striking a blow on the cork. If the surface of the jug is 200 sq. in. and the cross section of the cork 1 sq. in., what total force acts on the interior of the jug when a 10-lb. blow is struck on the cork?

3. If the water pressure in the city mains is 70 lb. to the square inch, how high above the town is the top of the water in the standpipe?

4. A cubical box 10 cm. on a side is half filled with mercury and half with water. Find the total force in grams on the bottom; on each side.

5. The water pressure in the city mains is 80 lb. to the square inch. The diameter of the piston of a hydraulic elevator of the type shown in Fig. 13 is 10 in. If friction could be disregarded, how heavy a load could the elevator lift? If 30 % of the ideal value must be allowed for frictional loss, what load will the elevator lift?

6. Fig. 17 represents an instrument commonly known as the hydrostatic bellows. If the base *C* is 20 in. square and the tube is filled with water to a depth of 5 ft. above the top of *C*, what is the value of the weight which the bellows can support?

7. A hydraulic press having a piston 1 in. in diameter exerts a force of 10,000 lb. when 10 lb. are applied to this piston. What is the diameter of the large piston?

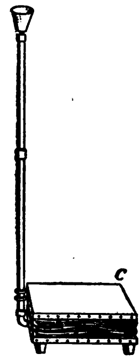


FIG. 17
Hydrostatic bellows

THE PRINCIPLE OF ARCHIMEDES *

29. Loss of weight of a body in a liquid. The preceding experiments have shown that an upward force acts against the bottom of any body immersed in a liquid. If the body is a boat, cork, piece of wood, or any body which floats, it is clear that, since it is in equilibrium, this upward force must be equal to the weight of the body. Even if the body does not float, everyday observation shows that it still loses a portion of its natural weight, for it is well known that it is easier to lift a stone under water than in air; or, again, that a man in a bath tub can support his whole weight by pressing lightly against the bottom with his fingers. It was indeed this very observation which first led the old Greek philosopher Archimedes (287–212 B.C.) to the discovery of the exact law which governs the loss of weight of a body in a liquid.

Hiero, the tyrant of Syracuse, had ordered a gold crown made, but suspected that the artisan had fraudulently used silver as well as gold in its construction. He ordered Archimedes to discover whether or not this were true. How to do so without destroying the crown was at first a puzzle to the old philosopher. While in his daily bath, noticing the loss of weight of his own body, it suddenly occurred to him that *any*

* A laboratory exercise on the experimental proof of Archimedes' principle should precede or accompany this discussion. See, for example, Experiment 6 of the authors' manual.

body immersed in a liquid must lose a weight equal to the weight of the displaced liquid. He is said to have jumped at once to his feet and rushed through the streets of Syracuse crying, "Eureka, eureka!" (I have found it, I have found it!)

30. Theoretical proof of Archimedes' principle. It is probable that Archimedes, with that faculty which is so common among men of great genius, saw the truth of his conclusion without going through any logical process of proof. Such a proof, however, can easily be given. Thus, since the upward force on the bottom of the block *abcd* (Fig. 18) is equal to the weight of the column of liquid *obce*, and since the downward force on the top of this block is equal to the weight of the column of liquid *oade*, it is clear that the upward force must exceed the downward force by the weight of the column of liquid *abcd*; that is, *the buoyant force exerted by the liquid is exactly equal to the weight of the displaced liquid.*

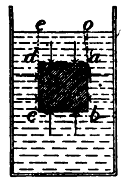


FIG. 18. Proof that an immersed body is buoyed up by a force equal to the weight of the displaced liquid

The reasoning is exactly the same, no matter what may be the nature of the liquid in which the body is immersed, nor how far the body may be beneath the surface. Further, if the body weighs more than the liquid which it displaces, it must sink, for it is urged down with the force of its own weight, and up with the lesser force of the weight of the displaced liquid. But if it weighs less than the displaced liquid, then the upward force due to the displaced liquid is greater than its own weight, and consequently it must rise to the surface. When it reaches the surface the downward force on the top of the block, due to the liquid, becomes zero. The body must, however, continue

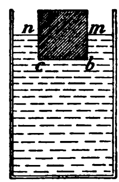
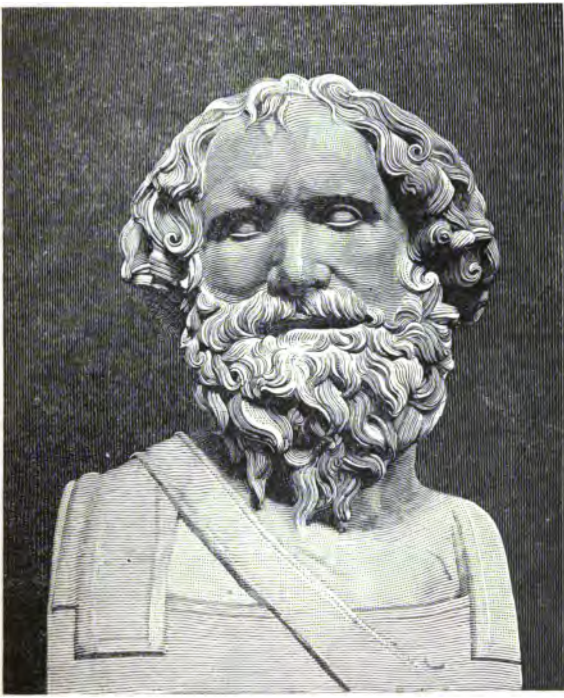


FIG. 19. Proof that a floating body is buoyed up by a force equal to the weight of the displaced liquid



Archimedes (287–212 B.C.)

(Bust in Naples Museum)

The celebrated geometrician of antiquity; lived at Syracuse, Sicily; first made a determination of π and computed the area of the circle; discovered the laws of the lever and was author of the famous saying, "Give me where I may stand and I will move the world"; discovered the laws of flotation; invented various devices for repelling the attacks of the Romans in the siege of Syracuse; on the capture of the city, while in the act of drawing geometrical figures in a dish of sand, he was killed by a Roman soldier to whom he cried out, "Don't spoil my circle."

to rise until the upward force on its bottom is equal to its own weight. But this upward force is always equal to the weight of the displaced liquid, that is, to the weight of the column of liquid *mbcn* (Fig. 19).

Hence *a floating body must displace its own weight of the liquid in which it floats.* This statement is embraced in the original statement of Archimedes' principle, for a body which floats has lost its whole weight.

31. Density of a heavy solid. The density of a body is, by definition, its mass divided by its volume. It is always possible to obtain the mass of a body by weighing it, but it is not, in general, possible to obtain the volume of an irregular body from measurements of its dimensions. Archimedes' principle, however, furnishes an accurate and easy method for obtaining the volume of any solid, however irregular, for by the preceding paragraph this volume is *numerically* equal to the loss of weight in water. Hence the equation which defines density, namely,

FIG. 20. Method of weighing a body under water

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

becomes in this case

$$\text{Density} = \frac{\text{Mass}}{\text{Loss of weight in water}}. \qquad (3)$$

Fig. 20 shows the usual arrangement for finding the weight in water.

32. Density of a solid lighter than water. If the body is too light to sink of itself, we may still obtain its volume by forcing it beneath the surface with a sinker. Thus, suppose w_1 represents the weight on the right pan of the balance when the body is in air and the sinker in water, as in Fig. 21; while

w_2 is the weight on the right pan when both body and sinker are under water. Then $w_1 - w_2$ is obviously the buoyant effect of the water on the body alone, and is therefore equal to the weight of the displaced water, which is numerically equal to the volume of the body.

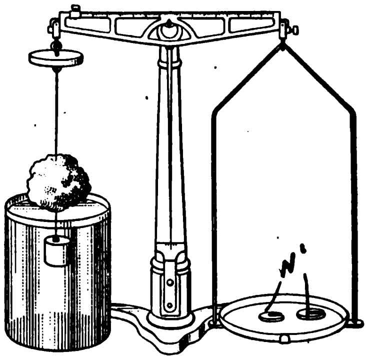


FIG. 21. Method of finding density of a light solid

33. Density of liquids by the hydrometer method. The commercial hydrometer such as is now in common use for testing the density of alcohol, milk, acids, sugar solutions, etc. is of the form shown in Fig. 22. The stem is calibrated by trial so that the density of any liquid may be read upon it directly. The principle involved is that a floating body sinks until it displaces its own weight. By making the stem very slender the sensitiveness of the instrument may be made very great. Why?

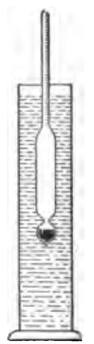


FIG. 22. Constant-weight hydrometer

34. Density of liquids by "loss of weight" method. If any suitable solid be weighed, first in air, then in water, and then in some liquid of some unknown density, by the principle of Archimedes, the loss of weight in the liquid is equal to the weight of the liquid displaced, and the loss in water is equal to the weight of the water displaced. Obviously the volume of liquid displaced is the same, since in each case it is just the volume of the body. If we divide the loss of weight in the liquid by the loss of weight in water, we are dividing the weight of a given volume of liquid by

the weight of an equal volume of water. By § 17 this gives us at once the specific gravity of the liquid, which is the same as its density in the C.G.S. system. Therefore, *to find the density of a liquid, divide the loss of weight of some solid in it by the loss of weight of the same body in water.**

QUESTIONS AND PROBLEMS

1. What fraction of the volume of a block of wood will float above water if its density is .5? if its density is .6? if its density is .9? State in general what fraction of the volume of a floating body is under water.

2. If an iceberg rises 100 ft. above water, how far does it extend below water? (Assume the density of the ice to be .9 that of sea water.)

3. If a barge 30 ft. by 15 ft. sank 4 in. when an elephant was taken aboard, what was the elephant's weight?

4. The hull of a modern battleship is made almost entirely of steel, its walls being of steel plates from 6 to 18 in. thick. Explain how it can float.

5. Will the water line of a boat rise or fall as it passes from fresh into salt water?

6. If a 150-lb. man can just float, what is his volume?

7. A hollow steel body weighing 1 kg. just floats. What is its volume?

8. What is the volume of a whale which weighs 30 tons?

9. If each boat of a pontoon bridge is 100 ft. long and 75 ft. wide at the water line, how much will it sink when a locomotive weighing 100 tons passes over it?

10. A block of wood 10 in. high sinks 6 in. in water. Find the density of the wood.

11. If this block sank 7 in. in oil, what would be the density of the oil?

12. To what depth would it sink in turpentine of density .87?

13. A graduated glass cylinder contains 190 cc. of water. An egg weighing 40 g. is dropped into the glass; it sinks to the bottom and raises the water to the 225-cc. mark. Find the density of the egg.

14. A cube of iron 10 cm. on a side weighs 7500 g. What will it weigh in alcohol of density .82?

* Laboratory experiments on the determination of the densities of solids and liquids should follow or accompany the discussion of this chapter. See, for example, Experiments 7 and 8 of the authors' manual.

CHAPTER III

PRESSURE IN AIR

Barometric Phenomena

35. The weight of air. To ordinary observation air is scarcely perceptible. It appears to have no weight and to offer no resistance to bodies passing through it. But if a bulb be balanced as in Fig. 23, then removed and filled with air under pressure by a few strokes of a bicycle pump, it will be found, when placed on the balance again, to be heavier than it was before. On the other hand, if the bulb be connected with an air pump and exhausted, it will be found to have lost weight. Evidently, then, air can be put into and taken out of a vessel, weighed, and handled, just like a liquid or a solid.

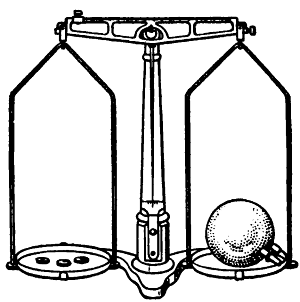


Fig. 23. Proof that air has weight

We are accustomed to say that bodies are "as light as air," yet careful measurement shows that it takes but 12 cubic feet of air to weigh a pound, so that a single large room contains more air than an ordinary man can lift. Thus the air in a room 60 feet by 30 feet by 15 feet weighs more than a ton. The exact weight of air at the freezing temperature and under normal atmospheric conditions is .001293 gram per cubic centimeter, that is, 1.293 grams per liter. A given volume of air therefore weighs $\frac{1}{773}$ as much as an equal volume of water.

36. Proof that air exerts pressure. Since air has weight, it is to be inferred that air, like a liquid, exerts force against any surface immersed in it. The following experiments prove this.

Let a rubber membrane be stretched over a glass vessel, as in Fig. 24. As the air is exhausted from beneath the membrane the latter will be observed to be more and more depressed until it will finally burst under the pressure of the air above.

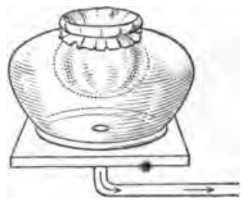


FIG. 24. Rubber membrane stretched by weight of air

Again, let a tin can be partly filled with water and the water boiled. The air will be expelled by the escaping steam. While the boiling is still going on, let the can be tightly corked, then placed in a sink or tray and cold water poured over it. The steam will be condensed and the weight of the air outside will crush the can.

37. Cause of the rise of liquids in exhausted tubes. If the lower end of a long tube be dipped into water and the air exhausted from the upper end, water will rise in the tube. We prove the truth of this statement every time we draw lemonade through a straw. The old Greeks and Romans explained such phenomena by saying that "nature abhors a vacuum," and this explanation was still in vogue in Galileo's time. But in 1640 the Duke of Tuscany had a deep well dug near Florence, and found to his surprise that no water pump which could be obtained would raise the water higher than about 32 feet above the level in the well. When he applied to the aged Galileo for an explanation, the latter replied that evidently "nature's horror of a vacuum did not extend beyond 32 feet." It is quite likely that Galileo suspected that the pressure of the air was responsible for the phenomenon, for he had himself proved before that air had weight, and, furthermore, he at once devised another experiment to test, as he said, the "power of a vacuum." He died in 1642 before the experiment

was performed, but suggested to his pupil, Torricelli, that he continue the investigation.

38. Torricelli's experiment. Torricelli argued that if water would rise 32 feet, then mercury, which is about 13 times as heavy as water, ought to rise but $\frac{1}{13}$ as high. To test this inference he performed in 1643 the following famous experiment:

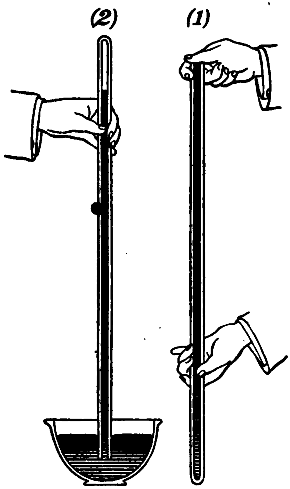


FIG. 25. Torricelli's experiment

Let a tube about 4 feet long, which is sealed at one end, be completely filled with mercury, as in Fig. 25, (1), then closed with the thumb and inverted, and the bottom then immersed in a dish of mercury, as in Fig. 25, (2). When the thumb is removed from the bottom of the tube, the mercury will fall away from the upper end of the tube in spite of the fact that in so doing it will leave a vacuum above it, and its upper surface will, in fact, stand about $\frac{1}{13}$ of 32 feet, that is, between 29 and 30 inches above the mercury in the dish.

Torricelli concluded from this experiment that the rise of liquids in exhausted tubes is due to an outside pressure exerted by the atmosphere on the surface of the liquid, and not to any mysterious sucking power created by the vacuum.

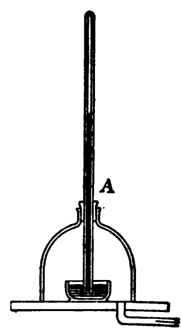


FIG. 26. Barometer falls when air pressure on the mercury surface is reduced

39. Further decisive tests. An unanswerable argument in favor of this conclusion will be furnished if the mercury in the tube falls as soon as the air is removed from above the surface of the mercury in the dish.

To test this point, let the dish and tube be placed on the table of an air pump, as in Fig. 26, the tube passing through a tightly fitting rubber

stopper A in the bell jar. As soon as the pump is started, the mercury in the tube will, in fact, be seen to fall. As the pumping is continued it will fall nearer and nearer to the level in the dish, although it will not usually reach it for the reason that an ordinary vacuum pump is not capable of producing as good a vacuum as that which exists in the top of the tube. As the air is allowed to return to the bell jar the mercury will rise in the tube to its former level.

40. Amount of the atmospheric pressure. Torricelli's experiment shows exactly how great the atmospheric pressure is, since this pressure is able to balance a column of mercury of definite length. In accordance with Pascal's law the downward pressure exerted by the atmosphere on the surface of the mercury in the dish (Fig. 27) is transmitted as an exactly equal upward pressure on the layer of mercury inside the tube at the same level as the mercury outside. But the downward pressure at this point within the tube is equal to hd, where d is the density of mercury and h is the depth below the surface b. Since the average height of this column at sea level is found to be 76 centimeters, and since the density of mercury is 13.6, the downward pressure inside the tube at a is equal to 76 times 13.6, or 1033.6 grams per square centimeter. Hence the atmospheric pressure acting on the surface of the mercury in the dish is 1033.6 grams, or roughly 1 kilogram, per square centimeter. This amounts to about 15 pounds per square inch.

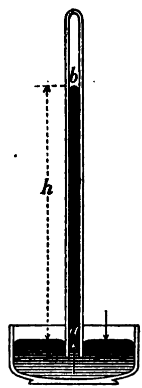


FIG. 27. Air column to top of atmosphere balances the mercury column ab

41. Pascal's experiment. Pascal thought of another way of testing whether or not it were indeed the weight of the outside air which sustains the column of mercury in an exhausted tube. He reasoned that, since the pressure in a liquid diminishes on

ascending toward the surface, atmospheric pressure ought also to diminish on passing from sea level to a mountain top. As no mountain existed near Paris, he carried Torricelli's apparatus to the top of a high tower and found, indeed, a slight fall in the height of the column of mercury. He then wrote to his brother-in-law, Perrier, who lived near Puy de Dome, a mountain in the south of France, and asked him to try the experiment on a larger scale. Perrier wrote back that he was "ravished with admiration and astonishment" when he found that on ascending 1000 meters the mercury sank about 8 centimeters in the tube. This was in 1648, five years after Torricelli's discovery.

At the present day geological parties actually ascertain differences in altitude by observing the change in the barometric pressure as they ascend or descend. A fall of 1 millimeter in the barometric height corresponds to an ascent of about 12 meters.

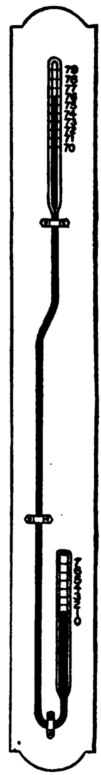


Fig. 28. The barometer

42. The barometer. The modern barometer (Fig. 28) is essentially nothing more nor less than Torricelli's tube. Taking a barometer reading consists simply in accurately measuring the height of the mercury column. This height varies from 73 to 76.5 centimeters in localities which are not far above sea level, the reason being that disturbances in the atmosphere affect the pressure at the earth's surface in the same way in which eddies and high waves in a tank of water would affect the liquid pressure at the bottom of the tank.

The barometer does not directly foretell the weather, but it has been found that a low or rapidly falling pressure is usually accompanied, or soon followed, by stormy conditions.

Hence the barometer, although not an infallible weather prophet, is nevertheless of considerable assistance in forecasting weather conditions some hours ahead. Further, by comparing at a central station the telegraphic reports of barometer readings made every few hours at stations all over the country, it is possible to determine in what direction the atmospheric eddies which cause barometer changes and stormy conditions are traveling, and hence to "forecast" the weather even a day or two in advance.

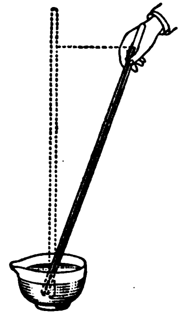


FIG. 29. Effect of inclining a barometer tube

43. The first barometers. Torricelli actually constructed a barometer not essentially different from that shown in Fig. 28, and used it for observing changes in the atmospheric pressure; but perhaps the most interesting of the early barometers was that set up about 1650 by the famous old German physicist Otto von Guericke of Magdeburg (1602–1686). He used for his barometer a water column the top of which passed through the roof of his house. A wooden image which floated on the upper surface of the water appeared above the housetop in fair weather but retired from sight in foul, a circumstance which led his neighbors to charge him with being in league with Satan.

44. Effect of inclining a barometer. If a barometer tube is inclined in the manner shown in Fig. 29, the top of the mercury will be found to remain always in the same horizontal plane. Explain, remembering that pressure equals height times density (Fig. 5).

FIG. 30. An aneroid barometer

45. The aneroid barometer. Since the mercurial barometer is somewhat long and inconvenient to carry, geological and surveying parties commonly use an instrument called the *aneroid barometer* (Fig. 30). It consists essentially of an air-tight cylindrical box D, the top of which is a metallic diaphragm

which bends slightly under the influence of change in the atmospheric pressure. This motion of the top of the box is multiplied by a delicate system of levers and communicated to a hand *B*, which moves over a dial whose readings are made to correspond to the readings of a mercury barometer. These instruments are made so sensitive as to indicate a change in pressure when they are moved no farther than from a table to the floor.

QUESTIONS AND PROBLEMS

1. What is your explanation of why "nature abhors a vacuum"?

2. Find the weight of the air contained in a room $18 \times 12 \times 4.5$ m.

3. If a barometer were sunk in water so that the lower mercury surface stood 1 m. below the surface of the water, what would be the reading of the barometer, the barometric height at the surface being 75.42 cm.?

4. If a circular piece of wet leather, having a string attached to the middle, is pressed down on a flat smooth stone, as in Fig. 31, the latter may often be lifted by pulling on the string. Is it pulled up or pushed up? Explain.

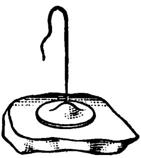


FIG. 31

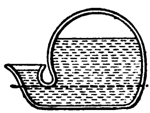


FIG. 32

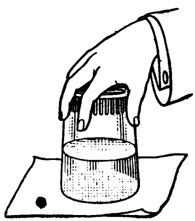


FIG. 33

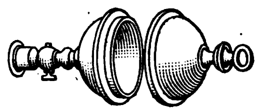


FIG. 34. Magdeburg hemispheres

5. Why does not the ink run out of a pneumatic inkstand like that shown in Fig. 32?

6. If the variation of the height of a mercury barometer is 2 in., how far did the image rise and fall in Otto von Guericke's water barometer? (See § 43.)

7. If a tumbler is filled with water, and a piece of writing paper is placed over the top, it may be inverted, as in Fig. 33, without spilling the water. Explain. What is the function of the paper?

8. Magdeburg hemispheres (Fig. 34) are so called because they were invented by Otto von Guericke, who was mayor of Magdeburg. When the lips of the hemispheres are placed in contact and the air exhausted from between them, it is found very difficult to pull them apart. Why?

9. Von Guericke's original hemispheres are still preserved in the museum at Berlin. Their interior diameter is 22 in. On the cover of

OTTO VON GUERICKE (1602–1686)

German physicist, astronomer, and man of affairs; mayor of Magdeburg; invented the air pump in 1650, and performed many new experiments with liquids and gases; discovered electrostatic repulsion; constructed the famous Magdeburg hemispheres which four teams of horses could not pull apart (see p. 32)

the book which describes his experiments is a picture which represents 4 teams of horses on each side of the hemispheres trying to separate them. The experiment was actually performed in this way before the German emperor Ferdinand III. If the air was all removed from the interior of the hemispheres, what force in pounds was in fact required to pull them apart? (Find the atmospheric force on a circle of 11 in. radius.)

Compressibility and Expansibility of Air

46. Incompressibility of liquids. Thus far we have found very striking resemblances between the conditions which exist at the bottom of a body of liquid and those which exist at the bottom of the great ocean of air in which we live. We now come to a most important difference. It is well known that if 2 liters of water be poured into a tall cylindrical vessel, the water will stand exactly twice as high as if the vessel contain but 1 liter; or if 10 liters be poured in, the water will stand 10 times as high as if there be but 1 liter. This means that the lowest liter in the vessel is not measurably compressed by the weight of the water above it.

It has been found by carefully devised experiments that compressing weights enormously greater than these may be used without producing a marked effect; namely, when a cubic centimeter of water is subjected to the stupendous pressure of 3,000,000 grams, its volume is reduced to but .90 cubic centimeter. Hence we say that water, and liquids generally, are practically incompressible. Had it not been for this fact we should not have been justified in taking the pressure at any depth below the surface of the sea as the simple product of the depth by the density at the surface.

47. Compressibility of air. When we study the effects of pressure on air we find a wholly different behavior from that described above for water. It is very easy to compress a body of air to one half, one fifth, or one tenth of its normal volume, as we prove every time we inflate a pneumatic tire or cushion of any sort. Further, the *expansibility* of air, that is, its tendency

to spring back to a larger volume as soon as the pressure is relieved, is proved every time a tennis ball or a football bounds, or the cork is driven from a popgun.

But it is not only air which has been crowded into a pneumatic cushion by some sort of a pressure pump which is in this state of readiness to expand as soon as the pressure is diminished; the ordinary air of the room will expand in the same way if the pressure to which it is subjected is relieved.

Thus, let a liter beaker with a sheet of rubber dam tied tightly over the top be placed under the receiver of an air pump. As soon as the pump is set into operation the inside air will expand with sufficient force to burst the rubber or greatly to distend it, as shown in Fig. 35.

Again, let two bottles be arranged as in Fig. 36, one being stoppered air-tight, while the other is uncorked. As soon as the two are placed under the receiver of an air pump and the air exhausted, the water in *A* will pass over into *B*. When the air is readmitted to the receiver the water will flow back. Explain.

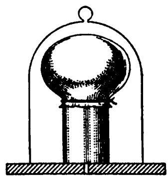


FIG. 35

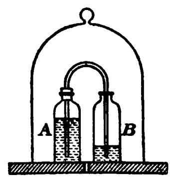


FIG. 36

Illustrations of the expansibility of air

48. Why hollow bodies are not crushed by atmospheric pressure. The preceding experiments show why the walls of hollow bodies are not crushed in by the enormous forces which the weight of the atmosphere exerts against them. For the air inside such bodies presses their walls out with as much force as the outside air presses them in. In the experiment of § 36 the inside air was removed by the escaping steam. When this steam was condensed by the cold water, the inside pressure became very small and the outside pressure then crushed the can. In the experiment shown in Fig. 35 it was the outside pressure which was removed by the air pump, and the pressure of the inside air then burst the rubber.

49. Boyle's law. The first man to investigate the exact relation between the change in the pressure exerted by a confined body of air and its change in volume was an Irishman, Robert Boyle (1626–1691). We shall repeat a modified form of his experiment much more carefully in the laboratory; but the following will illustrate the method by which he discovered one of the most important laws of physics.

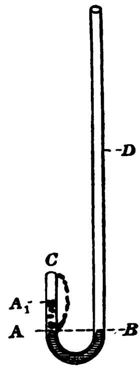


Fig. 37. Method of demonstrating Boyle's law

Let mercury be poured into a bent glass tube until it stands at the same level in the closed arm AC as in the open arm BD (Fig. 37). There is now confined in AC a certain volume of air under the pressure of one atmosphere. Call this pressure P_1. Let the length AC be measured and called V_1. Then let mercury be poured into the long arm until the level in this arm is as many centimeters above the level in the short arm as there are centimeters in the barometer height. The confined air is now under a pressure of two atmospheres. Call it P_2. Let the new volume $A_1C (= V_2)$ be measured. It will be found to be just half its former value.

Hence we learn that doubling the pressure exerted upon a body of air halves its volume. If we had tripled the pressure, we should have found the volume reduced to one third its initial value, etc. That is, *the pressure which a given quantity of air at constant temperature exerts against the walls of the containing vessel is inversely proportional to the volume occupied.* This is algebraically stated thus:

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}, \quad \text{or} \quad P_1V_1 = P_2V_2. \tag{1}$$

This is Boyle's law. It may also be stated in slightly different form. Doubling, tripling, or quadrupling the pressure must double, triple, or quadruple the *density*, since the volume is made only one half, one third, or one fourth as much, while

the mass remains unchanged. Hence *the pressure which air exerts is directly proportional to its density, or, algebraically,*

$$\frac{P_1}{P_2} = \frac{D_1}{D_2}.^* \qquad (2)$$

50. Extent and character of the earth's atmosphere. From the facts of compressibility and expansibility of air we may know that the air, unlike the sea, must become less and less dense as we ascend from the bottom toward the top. Thus at the top of Mont Blanc, where the barometer height is but 38 centimeters, or one half of its value at sea level, the density also must, by Boyle's law, be just one half as much as at sea level.

No one has ever ascended higher than 7 miles, which was approximately the height attained in 1862 by the two daring English aëronauts, Glasier and Coxwell. At this altitude the barometric height is but about 7 inches, and the temperature about $-60°$ F. Both aëronauts lost the use of their limbs and Mr. Glasier became unconscious. Mr. Coxwell barely succeeded in grasping with his teeth the rope which opened a valve and caused the balloon to descend. Again, on July 31, 1901, the French aëronaut M. Berson rose without injury to a height of about 7 miles (35,420 feet), his success being due to the artificial inhalation of oxygen.

By sending up self-registering thermometers and barometers in balloons which burst at great altitudes, the instruments being protected by parachutes from the dangers of rapid fall, the atmosphere has been explored to a height of 30,500 meters (18.95 miles), this being the height attained on September 1, 1910, by a little balloon holding 6.2 cubic meters of hydrogen at the surface, which was sent up at Huron, South Dakota, by William R. Blair of the United States Government

* A laboratory experiment on Boyle's law should follow this discussion. See, for example, Experiment 9 of the authors' manual.

Observatory, Mount Weather, Virginia. These extreme heights are calculated from the indications of the self-registering barometers.

At a height of 35 miles the density of the atmosphere is estimated to be but $\frac{1}{30000}$ of its value at sea level. By calculating how far below the horizon the sun must be when the

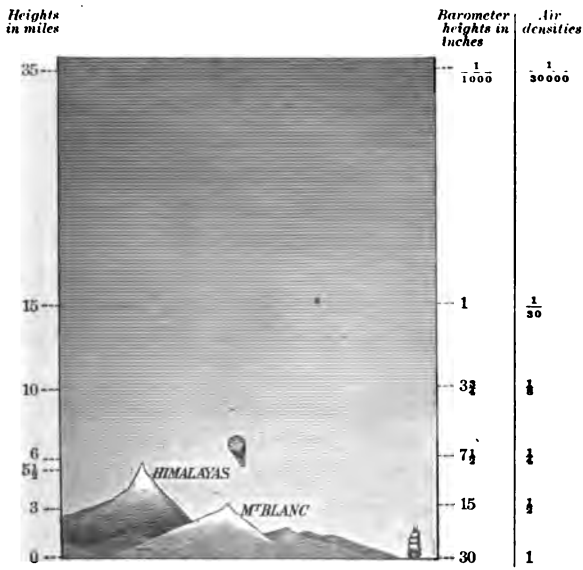


FIG. 38. Extent and character of atmosphere

last traces of color disappear from the sky, we find that at a height as great as 45 miles there must be air enough to reflect some light. How far beyond this an extremely rarified atmosphere may extend, no one knows. It has been estimated at all the way from 100 to 500 miles. These estimates are based on observations of the height at which meteors first become visible, on the height of the aurora borealis, and on the darkening of the surface of the moon just before it is eclipsed by the shadow of the solid earth.

51. Height of the "homogeneous atmosphere." Although, then, we cannot tell to what height the atmosphere extends, we do know with certainty that the weight of a column of air 1 square centimeter in cross section and reaching from the earth's surface to the extreme limits of the atmosphere will just balance a column of mercury 76 centimeters high, for this was shown by Torricelli's experiment. Since 1 cubic centimeter of air at the earth's surface weighs about 1.2 milligrams, that is, since the density of air is about .0012, or $\frac{1}{800}$ that of water, and since mercury is about 13.6 times as heavy as water, it follows that if the air had the same density at all altitudes which it has at the earth's surface, its height would be $76 \times 13.6 \times 800$ centimeters, that is, 8.2 kilometers, or about 5 miles. The tops of the Himalayas would therefore rise above it. This height of 5 miles, which is the height to which the air would extend if it, like the ocean, had the same density throughout, is called the *height of the homogeneous atmosphere.*

52. Density of air below sea level. The same cause which makes air diminish rapidly in density as we ascend above sea level must produce a rapid increase in its density as we descend below this level. It has been calculated that if a boring could be made in the earth 35 miles deep, the air at the bottom would be one thousand times as dense as at the earth's surface. Therefore wood and even water would float in it.

QUESTIONS AND PROBLEMS

1. Why is the reading of a barometer incorrect if the barometer tube is not strictly vertical?

2. Under ordinary conditions a gram of air occupies about 800 cc. Find what volume a gram will occupy at the top of Mont Blanc (altitude 15,810 ft.), where the barometer indicates that the pressure is only about one half what it is at sea level.

3. The mean density of the air at sea level is about .0012. What is its density at the top of Mont Blanc? What fractional part of the

earth's atmosphere has one left beneath him when he ascends to the top of this mountain?

4. If Glasier and Coxwell rose in their balloon until the barometric height was only 18 cm., how many inhalations were they obliged to make in order to obtain the same amount of air which they could obtain at the surface in one inhalation?

5. With the aid of the experiment in which the rubber dam was burst under the exhausted receiver of an air pump, explain why high mountain climbing often causes pain and bleeding in the ears and nose. Why does deep diving produce similar effects?

6. Blow as hard as possible into the tube of the bottle shown in Fig. 39. Then withdraw the mouth and explain all of the effects observed.

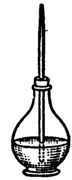


FIG. 39

7. If a bottle or cylinder is filled with water and inverted in a dish of water, with its mouth beneath the surface (see Fig. 40), the water will not run out. Why?

8. If a bent rubber tube is inserted beneath the cylinder and air blown in at *o* (Fig. 40), it will rise to the top and displace the water. This is the method regularly used in collecting gases. Explain what forces the gas up into it, and what causes the water to descend in the tube as the gas rises.

9. Why must the bung be removed from a cider barrel in order to secure a proper flow from the faucet?

10. When a bottle full of water is inverted, the water will gurgle out instead of issuing in a steady stream. Why?

11. There is a pressure of 80 cm. of mercury on 1000 cc. of gas. What pressure must be applied to reduce the volume to 600 cc. if the temperature is kept constant?

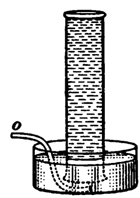


FIG. 40

12. What sort of a change in volume do the bubbles of air which escape from a diver's suit experience as they ascend to the surface?

PNEUMATIC APPLIANCES

53. The siphon. Let a rubber or glass tube be filled with water and then placed in the position shown in Fig. 41. Water will be found to flow through the tube from vessel *A* into vessel *B*. If, then, *B* be raised until the water in it is at a higher level than that in *A*, the direction of flow will be reversed. This instrument, which is called the *siphon*, is very useful for removing liquids from vessels which cannot be overturned, or for drawing off the upper layers of a liquid without disturbing the lower layers.

The explanation of the siphon's action is readily seen from Fig. 41. Since the tube *acb* is full of water, water must evidently flow through it if the force which pushes it one way is greater than that which pushes it the other way. Now the upward pressure at *a* is equal to atmospheric pressure minus the downward pressure due to the water column *ad*; while the upward pressure at *b* is the atmospheric pressure minus the downward pressure due to the water column *be*. Hence the pressure at *a* exceeds the pressure at *b* by the pressure due to the water column *fb*. The siphon will evidently cease to act when the water is at the same level in the two vessels, since then $fb = 0$, and the forces acting at the two ends of the tube are therefore equal and opposite. It will also cease to act when the bend *c* is more than 34 feet above the surface of the water in *A*, since then a vacuum will form at the top, atmospheric pressure being unable to raise water to a height greater than this in either tube.

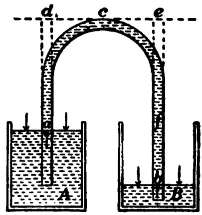


FIG. 41. The siphon

Would a siphon flow in a vacuum?

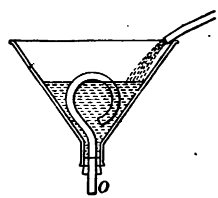


FIG. 42. Intermittent siphon

54. The intermittent siphon. Fig. 42 represents an intermittent siphon. If the vessel is at first empty, to what level must it be filled before the water will flow out at *o*? To what level will the water then fall before the flow will cease?

55. The air pump. The air pump was invented in 1650 by Otto von Guericke, mayor of Magdeburg, Germany, who deserves the greater credit, since he was apparently wholly without knowledge of the discoveries which Galileo, Torricelli, and Pascal had made a few years earlier regarding the character of the earth's atmosphere. A simple form of such a pump is shown in Fig. 43. When the piston is raised the air

from the receiver R expands into the cylinder B through the valve A. When the piston descends it compresses this air, and thus closes the valve A and opens the exhaust valve C. Thus with each double stroke a certain fraction of the air in the receiver is transferred from R through the cylinder to the outside.

In many pumps the valve C is in the piston itself.

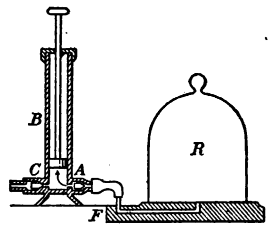


FIG. 43. A simple air pump

56. The compression pump. A compression pump is nothing but an exhaust pump with the valves reversed, so that A closes and C opens on the upstroke, and A opens and C closes on the downstroke. In its cheaper forms, for example, the common bicycle pump, the valve C is often replaced by a very simple device called a cup valve. This valve consists of a disk of leather a little larger than the barrel of the pump, attached to a loosely fitting metal piston. When the piston is raised the air passes in around the leather, but when it is lowered the leather is crowded closely against the walls, so that there is no escape for the air (Fig. 44).

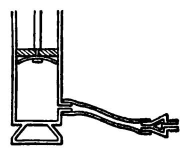


FIG. 44. The cup valve

Compressed air finds so many applications in such machines as air drills (used in mining), air brakes, air motors, etc., that the compression pump must be looked upon as of much greater importance industrially than the exhaust pump.

57. The lift pump. The common water pump, shown in Fig. 45, has been in use at least since the time of Aristotle (fourth century B.C.). It will be seen from the figure that it is nothing more nor less than a simplified form of air pump. In fact, in the earlier strokes we are simply exhausting air from the pipe below the valve b. Water could never be obtained at S, even with a perfect pump, if the valve b were

not within 34 feet of the surface of the water in *W*. Why? On account of mechanical imperfections this limit is usually about 28 feet instead of 34. Let the student analyze, stroke by stroke, the operation of pumping water from a well with the pump of Fig. 45. Why will pouring in a little water at the top, that is, "priming," often assist greatly in starting such a pump?

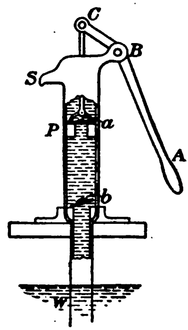


FIG. 45. The lift pump

58. The force pump. Fig. 46 illustrates the construction of the force pump, a device commonly used when it is desired to deliver water at a point higher than the position at which it is convenient to place the pump itself. Let the student analyze the action of the pump from a study of the diagram.

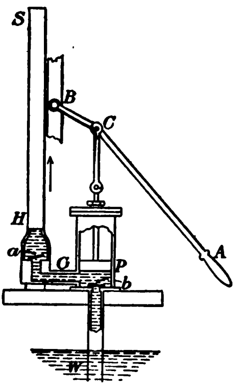


FIG. 46. The force pump

It will be seen that the discharge from such an arrangement as that shown in Fig. 46 must be intermittent, since no water can flow up the pipe *HS* when the piston *P* is ascending. In order to make the flow continue during the upstroke, an air chamber, such as that shown in Fig. 47, is always inserted between the valve *a* (Fig. 46) and the discharge point. As the water is forced violently into this chamber by the downward motion of the piston it compresses the confined air. It is, then, the reaction of this compressed air which is immediately responsible for the flow in the discharge tube, and as this reaction is continuous the flow is also continuous.

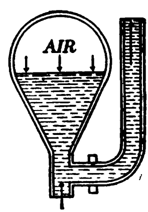


FIG. 47. The air dome of a force pump

Fig. 48 represents one of the most familiar types of force pump, the double-acting steam fire engine. Let the student analyze the action of the pump from a study of the diagram.

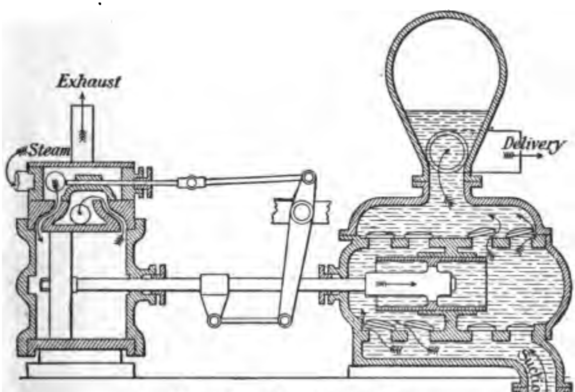


FIG. 48. The fire engine

59. The Cartesian diver. Descartes (1596–1650), the great French philosopher, invented an odd device which illustrates at the same time the principle of the transmission of pressure by liquids, the principle of Archimedes, and the compressibility of gases. A hollow glass image in human shape [Fig. 49, (1)] has an opening in the lower end. It is partly filled with water and partly with air, so that it will just float. By pressing on the rubber diaphragm at the top of the vessel it may be made to sink or rise at will. Explain. If the diver is not available, a small bottle or test tube [see Fig. 49, (2)] may be used instead. It works equally well, and brings out the principle even better. The modern submarine is essentially nothing but a huge Cartesian diver. The volume of the air in its chambers is changed by forcing water in or out.

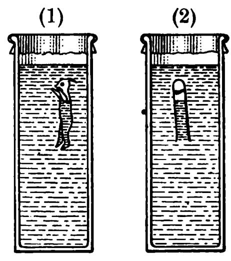


FIG. 49. The Cartesian diver

60. The balloon. A reference to the proof of Archimedes' principle (§ 30, p. 22) will show that it must apply as well to gases as to liquids. *Hence any body immersed in air is buoyed up by a force which is equal to the weight of the displaced air.* The body will therefore rise if its own weight is less than the weight of the air which it displaces.

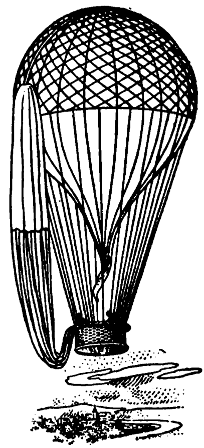


FIG. 50. The balloon

A balloon is a large silk bag (Fig. 50) varnished so as to be air-tight, and filled either with hydrogen or with common illuminating gas. The former gas weighs about .09 kilogram per cubic meter and common illuminating gas weighs about .75 kilogram per cubic meter. It will be remembered that ordinary air weighs about 1.20 kilograms per cubic meter. It will be seen, therefore, that the lifting power of hydrogen per cubic meter, namely, $1.20 - .09 = 1.11$, is more than twice the lifting power of illuminating gas, $1.20 - .75 = .45$. Nevertheless, on account of the comparative cheapness of the latter gas its use is very much more common.

Ordinarily a balloon is not completely filled at the start, for if it were, since the outside pressure is continually diminishing as it ascends, the pressure of the inside gas would subject the bag to enormous strain, and would surely burst it before it reached any considerable altitude. But if it is but partially inflated at the start, it can increase in volume as it ascends by simply inflating to a greater extent. Thus a balloon which ascends until the pressure is but 7 centimeters of mercury should be only about one fourth inflated when it is at the surface of the earth.

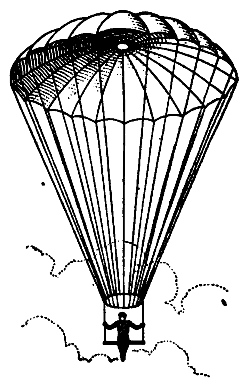


FIG. 51. The parachute

The parachute seen hanging from the side of the balloon in Fig. 50 is a huge umbrella-like affair with which the aëronaut may descend in safety to the earth. After opening, as in Fig. 51, it descends very slowly on account of the enormous surface exposed to the air. The hole in the top allows air to escape slowly and thus keeps the parachute upright.

Monster Zeppelin Dirigible Airship "Hansa" sailing over Hamburg, September, 1912

400 ft. long; 48 ft. wide; total gas capacity, in 17 separate compartments, 25,000 cubic yards; carries 24 passengers and a crew of 8; speed 40 miles per hour; horse power of each of her 3 motors, 145; carrying capacity 20 tons, 15 of which represent weight of ship, motors, and supplies; sails on regular schedule between Frankfort and Düsseldorf

61. The diving bell. The diving bell (Fig. 52) is a heavy bell-shaped body with rigid walls, which sinks of its own weight. Formerly the workmen who went down in the bell had at their disposal only the amount of air confined within it, and the water rose to a certain height within the bell on account of the compression of the air. But in modern practice the air is forced in from the surface through a connecting tube *a* (Fig. 53) by means of a force pump *h*. This arrangement, in addition to furnishing a continual supply of fresh air, makes it possible to force the water down to the level of the bottom of the bell. In practice a continual stream of bubbles is kept flowing out from the lower edge of the bell, as shown in Fig. 53.

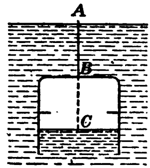


FIG. 52. The diving bell

The pressure of the air within the bell must, of course, be the pressure existing within the water at the depth of the level of the water inside the bell, that is, in Fig. 52 at the depth AC. Thus at a depth of 34 feet the pressure is 2 atmospheres. Diving bells are used for putting in the foundations of bridge piers, doing subaqueous excavating, etc. The so-called *caisson*, much used in bridge building, is simply a huge stationary diving bell, which the workmen enter through compartments provided with air-tight doors. Air is pumped into it precisely as in Fig. 53.

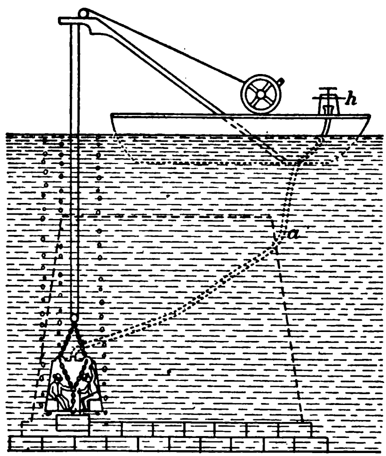


FIG. 53. Laying foundations of piers with the diving bell

62. The diving suit. For most purposes, except those of heavy engineering, the diving suit has now replaced the diving bell. This suit is made of rubber with a metal helmet. The diver is sometimes connected with the surface by a tube (Fig. 54) through which air is forced down to him. It passes out into the water through the valve v in his suit. But more commonly the diver is entirely independent of the surface, carrying air under a pressure of about 40 atmospheres in a tank on his back. This air is allowed to escape gradually through the suit and out into the water through the valve v as fast as the diver needs it. When he wishes to rise to the surface he simply admits enough air to his suit to make him float.

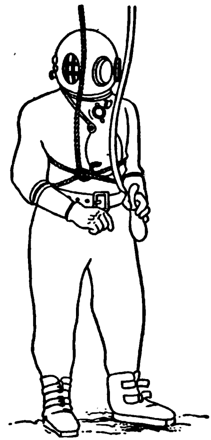


FIG. 54. The diving suit

In all cases the diver is subjected to the pressure existing at the depth at which the suit or bell communicates with the outside water. Divers seldom work at depths greater than 60 feet, and 80 feet is usually considered the limit of safety. But in building the bridge over the Mississippi at St. Louis, Missouri, the bells with their divers were sunk to a depth of 110 feet, while a case is on record of a diver who, in investigating a wreck off the coast of South America, sank to a depth of 201 feet.

The diver experiences pain in the ears and above the eyes when he is ascending or descending, but not when at rest. This is because it requires some time for the air to penetrate into the interior cavities of the body and establish equal pressure in both directions.

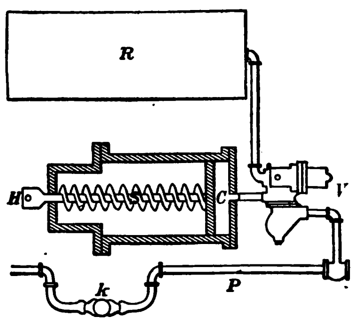


FIG. 55. The Westinghouse air brake

63. The air brake. Fig. 55 is a diagram which shows the essential features of the Westinghouse air brake. P is an air pipe leading to the engine, where a compression pump maintains air in the main cylinder under a pressure of about 70 pounds to the square inch. R is an auxiliary reservoir which is placed under each car, and which connects with P

through the triple valve V. So long as the pressure from the engine is on in P, the valve V is open in such a way that there is direct communication between P and R. But as soon as the pressure in P is diminished, either by the engineer or by the accidental breaking of the hose coupling k, which connects P from car to car, the compressed air in R operates the valve in V so as to shut off connection between R and P and to open connection between R and the cylinder C. The piston H is thus driven powerfully to the left and sets the brake shoes against the wheels through the operation of levers attached to H. When it is desired to take off the brakes, pressure is again turned on in P. This operation opens V in such a way as to permit the compressed air in C to escape, and the spring S then pulls back the brake shoes from the wheels.

64. The bellows. Fig. 56 shows the construction of the ordinary blacksmith's bellows. When the handle a rises and the point b in consequence falls, the valve v opens and air from the outside enters the lower compartment C_1. When a is pulled down and b thus made to ascend, v at once closes, and as soon as the pressure within C_1 has risen to the same value as that maintained in C_2 by the weights W, the valve v' opens and air passes from C_1 to C_2. With this arrangement it will be seen that the current of air which issues from C_2 through the nozzle is continuous rather than intermittent, as it would be if there were but one compartment and one valve.

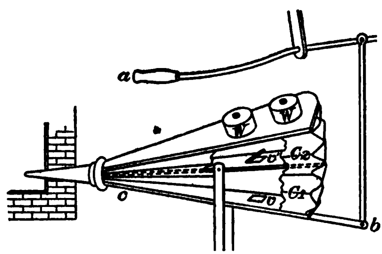


FIG. 56. A blacksmith's bellows

65. The gas meter. The gas meter is a device which differs little in principle from the blacksmith's bellows. Gas from the city supply enters the meter through P (Fig. 57) and passes through the openings o and o_1 into the compartments B and B_1 of the meter. Here its pressure forces in the diaphragms d and d_1. The gas already contained in A and A_1 is therefore pushed out to the burners through the openings o' and o_1' and

the pipe P_1. As soon as the diaphragm d has moved as far as it can to the right, a lever which is worked by the movement of d causes the slide valve u to move to the left, thus closing o and shutting off connection between P and B, but at the same time opening o' and allowing the gas from P to enter compartment A through o'. A quarter of a cycle later u_1 moves to the right and connects A_1 with P and B_1 with P_1. If u and u_1 were set so as to work exactly together, there would be slight fluctuations in the gas pressure at P_1. The movement of the diaphragms is recorded by a clockwork device, the dials of which indicate the number of cubic feet of gas which have passed through the meter.

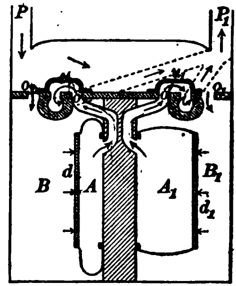


FIG. 57. The gas meter

QUESTIONS AND PROBLEMS

1. How many of the laws of liquids and gases do you find illustrated in the experiment of the Cartesian diver?

2. Let a siphon of the form shown in Fig. 58 be made by filling a flask one third full of water, closing it with a cork through which pass two pieces of glass tubing, as in the figure, and then inverting so that the lower end of the straight tube is in a dish of water. If the bent arm is of considerable length, the fountain will play forcibly and continuously until the dish is emptied. Explain.

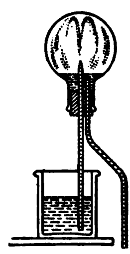


FIG. 58

3. Pneumatic dispatch tubes are now used in many large stores for the transmission of small packages. An exhaust pump is attached to one end of the tube in which a tightly fitting carriage moves, and a compression pump to the other. If the air is half exhausted on one side of the carriage, and has twice its normal density on the other, find the propelling force acting on the carriage when the area of its cross section is 50 sq. cm.

4. Pascal proved by an experiment that a siphon would not run if the bend in the arm were more than 34 ft. above the upper water level. He made it run, however, by inclining it sidewise until the bend was less than 34 ft. above this level. Explain.

5. If the cylinder of an air pump is of the same size as the receiver, what fractional part of the air is removed by one complete stroke? What fractional part is left after 3 strokes? after 10?

6. If the cylinder of an air pump is one third the size of the receiver, what fractional part of the original air will be left after 5 strokes?

What will a barometer within the receiver read, the outside pressure being 76?

7. Theoretically, can a vessel ever be completely exhausted by an air pump, even if mechanically perfect?

8. Why, in pumping water, is more and more force required at each succeeding stroke until the water begins to flow?

9. If the air in the air dome of a fire engine is reduced to one tenth of its normal volume, under what pressure is the water at the mouth of the nozzle?

10. What is the lifting power of a balloon which is filled with hydrogen and has a volume of 1000 cu. m.? (Take the weight of air as 1.2 g. per liter and that of hydrogen as one fourteenth that of air.)

11. Pressure tests for boilers or steel tanks of any kind are always made by filling them with water rather than with air. Why?

12. If the cylinder C (Fig. 55) has a diameter of 8 in., what is the force applied to the brakes at H? (Take the pressure in C as 70 lb. per sq. in.)

13. When will a balloon cease to rise?

14. If a diving bell (Fig. 52) is sunk until the level of the water within it is 1033 cm. beneath the surface, to what fraction of its initial volume has the inclosed air been reduced? (1033 g. per sq. cm. = 1 atmosphere.)

15. If a diver's tank has a volume of 2 cu. ft. and contains air under a pressure of 40 atmospheres, to what volume will the air expand when it is released at a depth of 34 ft. under water?

16. If the water within a diving bell is at a depth of 1033 cm. beneath the surface of a lake, what is the density of the air inside, if at the surface the density of air is .0012 and its pressure 76 cm.? What would be the reading of a barometer within the bell?

CHAPTER IV

MOLECULAR MOTIONS

KINETIC THEORY OF GASES

66. Molecular constitution of matter. In order to account for some of the simplest facts in nature, — for example, the fact that two substances often apparently occupy the same space at the same time, as when two gases are crowded together in the same vessel, or when sugar is dissolved in water, — it is now universally assumed that all substances are composed of very minute particles called *molecules*. Spaces are supposed to exist between these molecules, so that when one gas enters a vessel which is already full of another gas, the molecules of the one scatter themselves about among the molecules of the other. Since molecules cannot be seen with the most powerful microscopes, it is evident that they must be very minute. The number of them contained in a cubic centimeter of air is 27 billion billion (27×10^{18}). It would take as many as a thousand molecules laid side by side to make a speck long enough to be seen with the best microscopes.

67. Evidence for molecular motions in gases. Certain very simple observations lead us to the conclusion that the molecules of gases, even in a still room, must be in continual and quite rapid motion. Thus, if a little chlorine, or ammonia, or any gas of powerful odor is introduced into a room, in a very short time it will have become perceptible in all parts of the room. This shows clearly that enough of the molecules of the gas to effect the olfactory nerves must have found their way across the room.

Again, chemists tell us that if two globes, one containing hydrogen and the other carbon dioxide gas, be connected as in Fig. 59 and the stopcock between them opened, after a few hours chemical analysis will show that each of the globes contains the two gases in exactly the same proportions — a result which is at first sight very surprising, since carbon dioxide gas is about twenty-two times as heavy as hydrogen. This mixing of gases in apparent violation of the laws of weight is called *diffusion.*

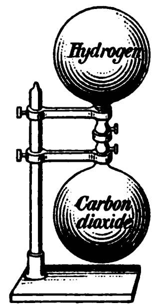


FIG. 59. Illustrating the diffusion of gases

We see then that such simple facts as the transference of odors and the diffusion of gases furnish very convincing evidence that the molecules of a gas are not at rest, but are continually moving about.

68. Molecular motions and the indefinite expansibility of a gas. Perhaps the most striking property which we have found gases to possess is the property of indefinite or unlimited expansibility. The existence of this property was demonstrated by the fact that we were able to obtain a high degree of exhaustion by means of an air pump. No matter how much air was removed from the bell jar, the remainder at once expanded and filled the entire vessel. In fact, it was only because of this property that the air pump was able to perform its functions at all.

In order to explain these facts it used to be assumed that the molecules of gases exert mutual repulsion upon one another. This theory has now, however, been completely abandoned, for it has been conclusively shown that no such repulsions exist. The motions of the molecules alone furnish a thoroughly satisfactory explanation of the phenomenon. As soon as the piston of the air pump is drawn up, some of the molecules follow it because they were already moving in that

direction, and not on account of any repulsion exerted upon them by the molecules below. The phenomenon is precisely the same as that illustrated in Fig. 59, where the carbon dioxide molecules moved up into the globe containing hydrogen; only in the latter case the operation took much more time because the upward motion of the carbonic acid molecules was hindered by collisions with the hydrogen molecules.

The fact that, however rapidly the piston of the air pump is drawn up, gas always appears to follow it instantly, leads us to the conclusion that the natural velocity possessed by the molecules of gas must be very considerable.

69. Molecular motions and gas pressures. If the molecules of gases do not repel one another, how are we to account for the fact that gases exert such pressures as they do against the walls of the vessels which contain them? We have found that in an ordinary room the air presses against the walls with a force of 15 pounds to the square inch. Within an automobile tire this pressure may amount to as much as 100 pounds, and the steam pressure within the boiler of an engine is often as high as 240 pounds per square inch. Yet in all these cases we may be certain that the molecules of the gas are separated from each other by distances which are large in comparison with the diameters of the molecules; for when we reduce steam to water, it shrinks to $\frac{1}{1600}$ of its original volume, and when we reduce air to the liquid form, it shrinks to about $\frac{1}{800}$ of its ordinary volume.

The explanation is at once apparent when we reflect upon the *motions* of the molecules. For just as a stream of water particles from a hose exerts a continuous force against a wall on which it strikes, so the blows which the innumerable molecules of a gas strike against the walls of the containing vessel must constitute a continuous force tending to push out these walls. Indeed, when we give up the wholly untenable notion of molecular repulsions, there is no other way in which we

can account for the fact that vessels containing only gas — balloons, for example — do not collapse under the enormous external pressures to which we know them to be subjected.

70. Explanation of Boyle's law. It will be remembered that it was discovered in the last chapter that when the density of a gas is doubled, the temperature remaining constant, the pressure is found to double also. When the density was trebled, the pressure was trebled, etc. This, in fact, was the assertion of Boyle's law. Now this is exactly what would be expected if the pressure which a gas exerts against a given surface is due to blows struck by an enormous number of swiftly moving molecules; for doubling the number of molecules in the given space — that is, doubling the density — would simply double the number of blows struck per second against that surface, and hence would double the pressure. While the kinetic theory of gases which is here presented accounts in this simple way for Boyle's law, the theory of molecular repulsions cannot be reconciled with it.

71. Brownian movements and molecular motions. It has recently been found possible to demonstrate the existence of molecular motions in gases in a very direct and very striking way. It is found that very minute oil drops suspended in perfectly stagnant air, instead of being themselves at rest, are ceaselessly dancing about just as though they were endowed with life. It has now been definitely proved (1913) that these motions, which are known as the *Brownian movements*, are the direct result of the bombardment which the droplets receive from the flying molecules of the gas with which they are surrounded; for at a given instant this bombardment is not the same on all sides, and hence the suspended particle, if it is minute enough, is pushed hither and thither as the resultant force is first in one direction, then in another. Diminishing either the size of the drops or the density of the surrounding gas increases the violence of the motions, the first because a

light body is more easily moved than a heavy one, the second because a body once set in motion is more quickly stopped by a dense gas than by a very rare one. At a pressure of $\frac{1}{100}$ atmosphere the dancing motions of small drops is exceedingly striking. There can be no doubt, then, that what the oil drops are here seen to be doing, the molecules themselves are also doing, only in a much more lively way.

72. Molecular velocities. From the known weight of a cubic centimeter of air under normal conditions, and the known force which it exerts per square centimeter, — namely, 1033 grams, — it is possible to calculate the velocity which its molecules must possess in order that they may produce by their collisions against the walls this amount of force. Further, since a cubic centimeter of hydrogen which is in condition to exert the same pressure as a cubic centimeter of air weighs only one fourteenth as much as the air, it is evident that the hydrogen molecules must be moving much more rapidly than the air molecules, or else they could not exert the same pressure. The result of the calculation gives to the air molecules under normal conditions a velocity of about 445 meters per second, while it assigns to the hydrogen molecules the enormous speed of 1700 meters (a mile) per second. The speed of a cannon ball is seldom greater than 800 meters (2500 feet) per second. It is easy to see then, since the molecules of gases are endowed with such speeds, why air, for example, expands instantly into the space left behind by the rising piston of the air pump, and why any gas always fills completely the vessel which contains it.

73. Diffusion of gases through porous walls. Strong evidence for the correctness of the above views is furnished by the following experiment:

Let a porous cup of unglazed earthenware be closed with a rubber stopper through which a glass tube passes, as in Fig. 60. Let the tube be dipped into a dish of colored water, and a jar containing hydrogen

James Clerk-Maxwell
(1831–1879)

One of the greatest of mathematical physicists; born in Edinburgh, Scotland; professor of natural philosophy at Marischal College, Aberdeen, in 1856, of physics and astronomy in Kings College, London, in 1860, and of experimental physics in Cambridge University from 1871 to 1879; one of the most prominent figures in the development of the kinetic theory of gases and the mechanical theory of heat; author of the electromagnetic theory of light — a theory which has become the basis of nearly all modern theoretical work in electricity and optics (see p. 416)

Heinrich Rudolph Hertz
(1857–1894)

One of the most brilliant of German physicists, who, in spite of his early death at the age of thirty-seven, made notable contributions to theoretical physics, and left behind the epoch-making experimental discovery of the electromagnetic waves predicted by Maxwell. Wireless telegraphy is but an application of this discovery of so-called "Hertzian" waves (see p. 413). The capital discovery that ultra-violet light discharges negatively electrified bodies is also due to Hertz

placed over the porous cup, or let the jar simply be held in the position shown in the figure, and illuminating gas passed into it by means of a rubber tube connected with a gas jet. The rapid passage of bubbles out through the water will show that the gaseous pressure inside the cup is rapidly increasing. Now let the bell jar be lifted, so that the hydrogen is removed from the outside. Water will at once begin to rise in the tube, showing that the inside pressure is now rapidly decreasing.

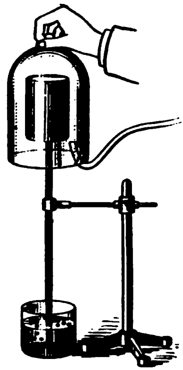


FIG. 60. Diffusion of hydrogen through porous cup

The explanation is as follows: We have learned that the molecules of hydrogen have about four times the velocity of the molecules of air. Hence, if there are as many hydrogen molecules per cubic centimeter outside the cup as there are air molecules per cubic centimeter inside, the hydrogen molecules will strike the outside of the wall four times as frequently as the air molecules will strike the inside. Hence, in a given time, the number of hydrogen molecules which pass into the interior of the cup through the little holes in the porous material will be four times as great as the number of air particles which pass out. Since the inside is thus gaining molecules faster than it is losing them, and since the pressure of a gas at a given temperature is determined solely by the number of molecules which are bombarding the wall, the inside pressure must increase until the number per cubic centimeter inside is so much larger than the number outside that molecules pass out as fast as they pass in. When the bell jar is removed the hydrogen which has passed inside now begins to pass out faster than the outside air passes in, and hence the inside pressure is diminished.

74. Temperature and molecular velocity. The effects which are observed when a gas is heated furnish further evidence that its molecules are in motion.

Let a bulb of air *B* be connected with a water manometer *m*, as in Fig. 61. If the bulb is warmed by holding a Bunsen burner beneath it, or even by placing the hand upon it, the water at *m* will at once begin to descend, showing that the pressure exerted by the air contained in the bulb has been increased by the increase in its temperature. If *B* is cooled with ice or ether, the water will rise at *m*.

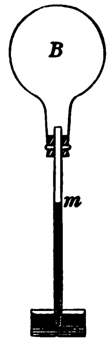


Fig. 61. Expansion of air by heat

Now if gas pressure is due to the bombardment of the walls by the molecules of the gas, since the number of molecules in the bulb can scarcely have been changed by slightly heating it, we are forced to conclude that the increase in pressure is due to an increase in the *velocity* of the molecules which are already there. The temperature of a given gas, then, from the standpoint of the kinetic theory, is determined simply by the mean velocity of the gas molecules. To increase the temperature is to increase the average velocity of the molecules, and to diminish the temperature is to diminish this average molecular velocity. The theory thus furnishes a very simple and natural explanation of the fact of the expansion of gases with a rise in temperature.

QUESTIONS AND PROBLEMS

1. Automobile tires are pumped up to a pressure of 80 lb. per sq. in. What is the density of the contained air? (1 atmosphere = 14.7 lb.)

2. If a vessel containing a small leak is filled with hydrogen at a pressure of 2 atmospheres, the pressure falls to 1 atmosphere about 4 times as fast as when the same experiment is tried with air. Can you see a reason for this?

3. A liter of air at a pressure of 76 cm. is compressed so as to occupy 400 cc. What is the pressure against the walls of the containing vessel?

4. If an open vessel contains 250 g. of air when the barometric height is 750 mm., what weight will the same vessel contain at the same temperature when the barometric height is 740 mm.?

5. The density of air is .001293 when the temperature is 0° C. and the pressure 76 cm. How large must a vessel be to contain a kilogram of air when the temperature is 0° C. and the pressure 75 cm.?

6. On a day on which the barometric height is 76 cm. the volume of the space above the mercury in a Torricelli tube is 10 cc., and on account of air in this space the mercury in the tube stands only 74 cm. high. How high will the mercury stand above the cistern if the tube is pulled up out of the dish so that the space above is 20 cc.?

7. Find the pressure to which the diver was subjected who descended to a depth of 201 ft. Find the density of the air in his suit, the density at the surface being .00118 and the temperature being assumed to remain constant. Take the pressure at the surface as 75 cm.

8. A bubble of air which escaped from this diver's suit would increase to how many times its volume on reaching the surface?

Molecular Motions in Liquids

75. Molecular motions in liquids and evaporation. Evidence that the molecules of liquids as well as those of gases are in a state of perpetual motion is found, first, in the familiar facts of evaporation.

We know that the molecules of a liquid in an open vessel are continually passing off into the space above; for it is only a matter of time when the liquid completely disappears and the vessel becomes dry. Now it is hard to imagine a way in which the molecules of a liquid thus pass out of the liquid into the space above, unless these molecules, while in the liquid condition, are in motion. As soon, however, as such a motion is assumed, the facts of evaporation become perfectly intelligible. For it is to be expected that in the jostlings and collisions of rapidly moving liquid molecules an occasional molecule will acquire a velocity much greater than the average. This molecule may then, because of the unusual speed of its motion, break away from the attraction of its neighbors and fly off into the space above. This is indeed the mechanism by which we now believe that the process of evaporation goes on from the surface of any liquid.

76. Molecular motions and the diffusion of liquids. One of the most convincing arguments for the motions of molecules in gases was found in the fact of diffusion. But precisely the same sort of phenomena are observable in liquids.

Let a few lumps of blue litmus be pulverized and dissolved in water. Let a tall glass cylinder be half filled with this water and a few drops of ammonia added. Let the remainder of the litmus solution be turned red by the addition of one or two cubic centimeters of nitric acid. Then let this acidulated water be introduced into the bottom of the jar through a thistle tube (Fig. 62). In a few minutes the line of separation between the acidulated water and the blue solution will be fairly sharp; but in the course of a few hours, even though the jar is kept perfectly quiet, the red color will be found to have spread considerably toward the top of the jar, showing that the acid molecules have gradually found their way toward the top.


FIG. 62. Diffusion of liquids

Certainly, then, the molecules of a liquid must be endowed with the power of independent motion. Indeed, every one of the arguments for molecular motions in gases applies with equal force to liquids. Even the Brownian movements can be seen in liquids, though they are here so small that high power microscopes must be used to make them apparent.

77. Molecular motions and the expansion of liquids. The fact of the expansion of gases with a rise of temperature was looked upon as evidence that the molecules of gases are in motion, the velocity of this motion increasing with an increase in temperature. But precisely the same property belongs to liquids also.

Thus, let the bulb (Fig. 63) be heated with a Bunsen burner. The contained liquid will be found to expand and rise in the tube.

It is natural to infer that the cause of this increase in volume is the same as before; that is, the velocity of the molecules of the liquid has been increased by the rise in temperature,

and they have therefore jostled one another farther apart, and thus caused the whole volume to be enlarged. According to this view, then, an increase in temperature in a liquid, as in a gas, means an increase in the mean velocity of the molecules, and conversely a decrease in temperature means a decrease in this average velocity.

FIG. 63. Expansion of a liquid

78. Evaporation and temperature. If it is true that increase in temperature means increase in the mean velocity of molecular motion, then the number of molecules which chance in a given time to acquire the velocity necessary to carry them into the space above the liquid ought to increase as the temperature increases; that is, evaporation ought to take place more rapidly at high temperatures than at low. Common observation teaches that this is true. Damp clothes become dry under a hot flatiron but not under a cold one; the sidewalk dries more readily in the sun than in the shade; we put wet objects near a hot stove or radiator when we wish them to dry quickly.

PROPERTIES OF VAPORS

79. Saturated vapor. If a liquid is placed in an open vessel, there ought to be no limit to the number of molecules which can be lost by evaporation, for as fast as the molecules emerge from the liquid they are carried away by air currents. As a matter of fact, experience teaches that water left in an open dish does waste away until the dish is completely dry.

But suppose that the liquid is evaporating into a closed space, such as that shown in Fig. 64. Since the molecules which leave the liquid cannot escape from the space S, it is clear that as time goes on the number of molecules which have passed off from the liquid into this space must continually

increase; in other words, the density of the vapor in S must grow greater and greater. But there is an absolutely definite limit to the density which the vapor can attain. For, as soon as it reaches a certain value, depending on the temperature and on the nature of the liquid, the number of molecules returning per second to the liquid surface will be exactly equal to the number escaping. The vapor is then said to be *saturated.*

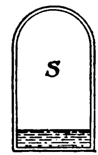


FIG. 64. A saturated vapor

If the density of the vapor is lessened temporarily by increasing the size of the vessel S, more molecules will escape from the liquid per second than return to it until the density of the vapor has regained its original value.

If, on the other hand, the density of the vapor has been increased by compressing it, more molecules return to the liquid per second than escape, and the density of the vapor falls quickly to its "saturated" value. We learn, then, that *the density of the saturated vapor of a liquid depends on the temperature alone, and cannot be affected by changes in volume.*

80. Pressure of a saturated vapor. Just as a gas exerts a pressure against the walls of the containing vessel by the blows of its moving molecules, so also does a confined vapor. But at any given temperature the density of a saturated vapor can have only a definite value, that is, there can be only a definite number of molecules per cubic centimeter. It follows, therefore, that just as at any temperature the saturated vapor can have only one density, so also it can have only one pressure. This pressure is called *the pressure of the saturated vapor* corresponding to the given temperature.

Let four Torricelli tubes be set up as in Fig. 65, and with the aid of a curved pipette (Fig. 65) let a drop of ether be introduced into the bottom of tube *1*. This drop will at once rise to the top and a portion of it will evaporate into the vacuum which exists above the mercury. The pressure of this vapor will push down the mercury column, and the

number of centimeters of this depression will of course be a measure of the pressure of the vapor. It will be observed that the mercury will fall almost instantly to the lowest level which it will ever reach — a fact which indicates that it takes but a very short time for the condition of saturation to be attained. In the same way let alcohol and water be introduced into tubes *2* and *3* respectively.

While the pressure of the saturated ether vapor at the temperature of the room will be found to be as much as 40 centimeters, that of alcohol will be found to be but 4 or 5 centimeters, and that of water only 1 or 2 centimeters.

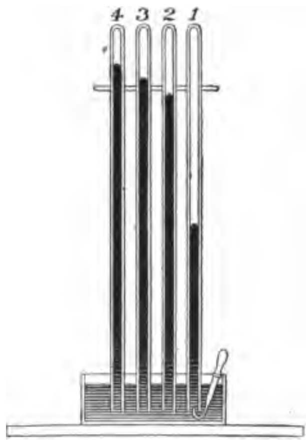


Fig. 65. Vapor pressures of saturated vapors

Let a Bunsen flame be passed quickly across the tubes of Fig. 65 near the upper level of the mercury. The vapor pressure will increase rapidly in all of the tubes, as shown by the fall of the mercury columns. This will be especially noticeable in the case of the ether.

The experiment proves that both the pressure and the density of a saturated vapor increase rapidly with the temperature. This was to have been expected from our theory; for increasing the temperature of the liquid increases the mean velocity of its molecules and hence increases the number which attain each second the velocity necessary for escape.

Let air be introduced into tube *4* until the mercury stands at about the same height as in tube *1*. Let pieces of ice be held against tubes *1* and *4* near the top of the mercury. The mercury will rise in both, but much more rapidly in the ether tube than in the air tube, thus showing that the ether vapor is condensing.

The experiment shows that if the temperature of a saturated vapor is diminished, it condenses until its density is reduced

to that corresponding to saturation at the lower temperature. How rapidly the density and pressure of saturation increase with temperature may be seen from the following table:

TABLE OF CONSTANTS OF SATURATED WATER VAPOR

The table shows the pressure P, in millimeters of mercury, and the density D of aqueous vapor saturated at temperatures t° C.

t.	P.	D.	t.	P.	D.	t.	P.	D.
– 10°	2.2	.0000023	4°	6.1	.0000064	18°	15.3	.0000152
– 9°	2.3	.0000025	5°	6.5	.0000068	19°	16.3	.0000162
– 8°	2.5	.0000027	6°	7.0	.0000073	20°	17.4	.0000172
– 7°	2.7	.0000029	7°	7.5	.0000077	21°	18.5	.0000182
– 6°	2.9	.0000032	8°	8.0	.0000082	22°	19.6	.0000193
– 5°	3.2	.0000034	9°	8.5	.0000087	23°	20.9	.0000204
– 4°	3.4	.0000037	10°	9.1	.0000093	24°	22.2	.0000216
– 3°	3.7	.0000040	11°	9.8	.0000100	25°	23.5	.0000229
– 2°	3.9	.0000042	12°	10.4	.0000106	26°	25.0	.0000242
– 1°	4.2	.0000045	13°	11.1	.0000112	27°	26.5	.0000256
0°	4.6	.0000049	14°	11.9	.0000120	28°	28.1	.0000270
1°	4.9	.0000052	15°	12.7	.0000128	30°	31.5	.0000301
2°	5.3	.0000056	16°	13.5	.0000135	35°	41.8	.0000393
3°	5.7	.0000060	17°	14.4	.0000144	40°	54.9	.0000509

81. The influence of air on evaporation. We observed that when a drop of ether was inserted into a Torricelli tube the mercury fell *very suddenly* to its final position, showing that in a vacuum the condition of saturation is reached almost instantly. This was to have been expected from the great velocities which we found the molecules of gases and vapors to possess.

In order to see what effect the presence of air has upon evaporation, let a drop of ether be introduced into a Torricelli tube which is partly filled with air. The mercury will not now be found to sink instantly to its final level as it did before, but although it will fall rapidly at first, it will continue to fall slowly for several hours. At the end of a day, if the temperature has remained constant, it will show a depression which indicates a vapor pressure of the ether just as great as that existing in a tube which contains no air.

The experiment leads, then, to the rather remarkable conclusion that *just as much liquid will evaporate into a space which is already full of air as into a vacuum.* The air has no effect except to retard greatly the *rate of evaporation.*

82. Explanation of the retarding influence of air on evaporation. This retarding influence of air on evaporation is easily explained by the kinetic theory; for while in a vacuum the molecules which emerge from the surface fly at once to the top of the vessel, when air is present the escaping molecules collide with the air molecules before they have gone any appreciable distance away from the surface (probably less than .00001 centimeter), and only work their way up to the top after an almost infinite number of collisions. Thus, while the space immediately above the liquid may become saturated very quickly, it requires a long time for this condition of saturation to reach the top of the vessel.

It must not be forgotten, however, that at a given temperature the *pressure* existing within a vessel containing gases is simply due to the total number of molecules per cubic centimeter which are striking blows against each square centimeter of the wall. Therefore, when a liquid evaporates into a closed vessel already containing air, the pressure gradually increases, and *is ultimately equal to the air pressure plus the pressure of the saturated vapor.* When a liquid evaporates in an open vessel, — that is, under constant pressure, — its molecules crowd out an equal number of molecules of air.

QUESTIONS AND PROBLEMS

1. Salt is heavier than water. Why does not all the salt in a mixture of salt and water settle to the bottom?

2. The space above the mercury in a Torricelli is filled with saturated ether vapor. Its volume is 20 cc. and the height of the mercury is 36 cm. The tube is pushed down into the mercury cistern until the volume occupied by the vapor is 10 cc. What is now the height of the mercury?

3. If the inside of a barometer tube is wet when it is filled with mercury, will the height of the mercury be the same as in a dry tube?

4. At a temperature of 15° C. what will be the error in the barometric height indicated by a barometer which contains moisture? (See Table of Constants of Saturated Water Vapor, p. 62.)

5. Why do clothes dry more quickly on a windy day than on a quiet day?

6. If dry air were placed in a closed vessel when the barometer was 76 cm., and a dish of water then introduced within the closed space, what pressure would finally be attained within the vessel if the temperature were kept at 18° C.?

7. How many grams of water will evaporate at 20° C. into a closed room 18 × 20 × 4 m.? (See table, p. 62, for density of saturated water vapor at 20° C.)

Hygrometry, or the Study of Moisture Conditions in the Atmosphere *

83. Condensation of water vapor from the air. Were it not for the retarding influence of air upon evaporation we should be obliged to live in an atmosphere which would be always completely saturated with water vapor; for the evaporation from oceans, lakes, and rivers would almost instantly saturate all the regions of the earth. This condition — one in which moist clothes would never dry, and in which all objects would be perpetually soaked in moisture — would be exceedingly uncomfortable, if not altogether unendurable.

But on account of the slowness with which, as the last experiment showed, evaporation takes place into air, the water vapor which always exists in the atmosphere is usually far from saturated, even in the immediate neighborhood of lakes and rivers. Since, however, the amount of vapor which is necessary to produce saturation rapidly decreases with a fall in temperature, if the temperature decreases continually in some unsaturated locality, it is clear that a point must soon

* It is recommended that this subject be preceded by a laboratory determination of dew point, humidity, etc. See, for example, Experiment 10 of the authors' manual.

be reached at which the amount of vapor already existing in a cubic centimeter of the atmosphere is the amount corresponding to saturation. Then, in accordance with the facts discovered in § 80, if the temperature still continues to fall, the vapor must begin to condense. Whether it condenses as dew, or cloud, or fog, or rain will depend upon how and where the cooling takes place.

84. The formation of dew. If the cooling is due to the natural radiation of heat from the earth at night after the sun's warmth is withdrawn, the atmosphere itself does not fall in temperature nearly as rapidly as do solid objects on the earth, such as blades of grass, trees, stones, etc. The layers of air which come into immediate contact with these cooled bodies are themselves cooled, and as they thus reach a temperature at which the amount of moisture which they already contain is in a saturated condition, they begin to deposit this moisture, in the form of dew, upon the cold objects. The drops of moisture which collect on an ice pitcher in summer illustrate perfectly the whole process.

85. The formation of fog. If the cooling at night is so great as not only to bring the grass and trees below the temperature at which the vapor in the air in contact with them is in a state of saturation, but also to lower the whole body of air near the earth below this temperature, then the condensation takes place not only on the solid objects but also on dust particles suspended in the atmosphere. This constitutes a fog.

86. The formation of clouds, rain, hail, and snow. When the cooling of the atmosphere takes place at some distance *above* the earth's surface, as when a warm current of air enters a cold region, if the resultant temperature is below that at which the amount of moisture already in the air is sufficient to produce saturation, this excessive moisture immediately condenses about floating dust particles and forms a *cloud*. If the cooling is sufficient to free a considerable amount of

moisture, the drops become large and fall as *rain.* If this falling rain passes through cold regions, it freezes into *hail.* If the temperature at which condensation begins is below freezing, the condensing moisture forms into *snowflakes.*

87. The dew point. The temperature to which the atmosphere must be cooled in order that condensation may begin is called the *dew point.* This temperature may be found by partly filling with water a brightly polished vessel of 200 or 300 cubic centimeters capacity and dropping into it little pieces of ice, stirring thoroughly at the same time with a thermometer. The dew point is the temperature indicated by the thermometer at the instant a film of moisture appears upon the polished surface. In winter the dew point is usually below freezing, and it will therefore be necessary to add salt to the ice and water in order to make the film appear. The experiment may be performed equally well by bubbling a current of air through ether contained in a polished tube (Fig. 66).

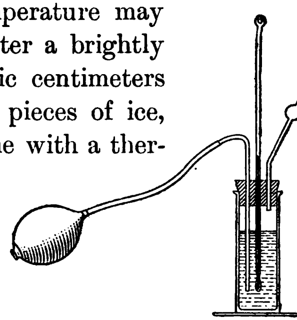


FIG. 66. Apparatus for determining dew point

88. Humidity of the atmosphere. From the dew point and table given in § 80, p. 62, we can easily find what is commonly known as the *relative humidity*, or the *degree of saturation* of the atmosphere. This quantity is defined as *the ratio between the amount of moisture actually present in the air per cubic centimeter and the amount which would be present if the air were completely saturated.* This is precisely the same as the ratio between the pressure which the water vapor present in the air exerts and the pressure which it would exert if it were present in sufficient quantity to be in the saturated condition. An example will make clear the method of finding the relative humidity.

Suppose that the dew point were found to be 15° C. on a day on which the temperature of the room was 25° C. The amount of moisture actually present in the air then saturates it at 15° C. We see from the *P* column in the table that the pressure of saturated vapor at 15° C. is 12.7 millimeters. This is, then, the pressure exerted by the vapor in the air at the time of our experiment. Running down the table, we see that the amount of moisture required to produce saturation at the temperature of the room, that is, at 25°, would exert a pressure of 23.5 millimeters. Hence at the time of the experiment the air contains 12.7/23.5, or .54, as much water vapor as it might hold. We say, therefore, that the air is 54% saturated, or that the relative humidity is 54%.

89. Practical value of humidity determinations. From humidity determinations it is possible to obtain much information regarding the likelihood of rain or frost. Such observations are continually made for this purpose at all meteorological stations. Further, they are made in greenhouses to see that the air does not become too dry for the welfare of the plants, and also in hospitals and public buildings, and even in private dwellings, in order to insure the maintenance of hygienic living conditions. For the most healthful conditions the relative humidity should be kept at from 50% to 60%.

90. Cooling effect of evaporation. Let three shallow dishes be partly filled, the first with water, the second with alcohol, and the third with ether, the bottles from which these liquids are obtained having stood in the room long enough to acquire its temperature. Let three students carefully read as many thermometers, first before their bulbs have been immersed in the respective liquids and then after. In every case the temperature of the liquid in the shallow vessel will be found to be somewhat lower than the temperature of the air, the difference being greatest in the case of ether and least in the case of water.

It appears from this experiment that an evaporating liquid assumes a temperature somewhat lower than its surroundings, and that the substances which evaporate the most readily, that is, those which have the greatest vapor pressures at a given temperature (see § 80), assume the lowest temperatures.

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Another way of establishing the same truth is to place a few drops of each of the above liquids in succession on the bulb of the arrangement shown in Fig. 61, and observe the rise of water in the stem; or, more simply still, to place a few drops of each liquid on the back of the hand, and notice that the order in which they evaporate — namely, ether, alcohol, water — is the order of greatest cooling.

91. Explanation of the cooling effect of evaporation. The kinetic theory furnishes a simple explanation of the cooling effects of evaporation. We saw that in accordance with this theory evaporation means an escape from the surface of those molecules which have acquired velocities considerably above the average. But such a continual loss from a liquid of its most rapidly moving molecules involves, of course, a continual diminution of the average velocity of the molecules left behind in the liquid state, and this means a decrease in the temperature of the liquid (see §§ 74 and 77).

Again, we should expect the amount of cooling to be proportional to the rate at which the liquid is losing molecules. Hence, of the three liquids studied, ether should cool most rapidly, since it shows the highest vapor pressure at a given temperature and therefore the highest rate of emission of molecules. The alcohol should come next in order, and the water last, as was in fact observed.

92. Freezing by evaporation. In § 81 it was shown that a liquid will evaporate much more quickly into a vacuum than into a space containing air. Hence if we place a liquid under the receiver of an air pump and exhaust rapidly, we ought to expect a much greater fall in temperature than when the liquid evaporates into air. This conclusion may be strikingly verified as follows:

Let a thin watch glass be filled with ether and placed upon a drop of cold water, preferably ice water, which rests upon a thin glass plate. Let the whole arrangement be placed underneath the receiver of an air pump and the air rapidly exhausted. After a few minutes of pumping the watch glass will be found frozen to the plate.

By evaporating liquid helium in this way Professor Kamerlingh Onnes of Leyden, in 1911, attained the lowest temperature which has ever been reached, namely, – 271.3° C. or – 456.3° F.

93. Effect of air currents upon evaporation. Let four thermometer bulbs, the first of which is dry, the second wetted with water, the third with alcohol, and the fourth with ether, be rapidly fanned and their respective temperatures observed. The results will show that in all of the wetted thermometers the fanning will considerably augment the cooling, but the dry thermometer will be wholly unaffected.

The reason that fanning thus facilitates evaporation, and therefore cooling, is that it removes the saturated layers of vapor which are in immediate contact with the liquid and replaces them by unsaturated layers into which new evaporation may at once take place. From the behavior of the dry-bulb thermometer, however, it will be seen that fanning produces cooling only when it can thus hasten evaporation. A dry body at the temperature of the room is not cooled in the slightest degree by blowing a current of air across it.

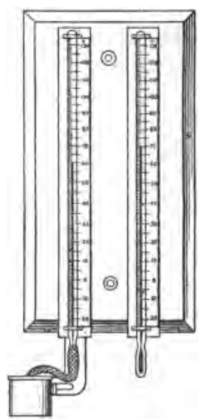


Fig. 67. Wet- and dry-bulb hygrometer

94. The wet- and dry-bulb hygrometer. In the wet- and dry-bulb hygrometer (Fig. 67) the principle of cooling by evaporation finds a very useful application. This instrument consists of two thermometers, the bulb of one of which is dry, while that of the other is kept continually moist by a wick dipping into a vessel of water. Unless the air is saturated the wet bulb indicates a lower temperature than the dry one, for the reason that evaporation is continually taking place from its surface. How much lower will depend on how

rapidly the evaporation proceeds; and this in turn will depend upon the relative humidity of the atmosphere. Thus in a completely saturated atmosphere no evaporation whatever takes place at the wet bulb, and it consequently indicates the same temperature as the dry one. By comparing the indications of this instrument with those of the dew-point hygrometer (Fig. 66), tables have been constructed which enable one to determine at once from the readings of the two thermometers both the relative humidity and the dew point. On account of their convenience instruments of this sort are used almost exclusively in practical work. They are not very reliable unless the air is made to circulate about the wet bulb before the reading is taken. In scientific work this is always done.

95. Effect of increased surface upon evaporation. Let a small test tube containing a few drops of water be dipped into a larger tube, or a small glass, containing ether, as in Fig. 68, and let a current of air be forced rapidly through the ether with an aspirator, in the manner shown. The water within the tube will be frozen in a few minutes.

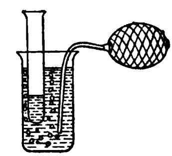


FIG. 68. Freezing water by the evaporation of ether

The effect of passing bubbles through the ether is simply to increase enormously the evaporating surface, for the ether molecules which could before escape only at the upper surface can now escape into the air bubbles as well.

96. Factors affecting evaporation. The above results may be summarized as follows: The rate of evaporation depends (1) on the nature of the evaporating liquid; (2) on the temperature of the evaporating liquid; (3) on the degree of saturation of the space into which the evaporation takes place; (4) on the density of the air or other gas above the evaporating surface; (5) on the rapidity of the circulation of the air above the evaporating surface; (6) on the extent of the exposed surface of the liquid.

MOLECULAR MOTIONS IN SOLIDS

97. Evidence for molecular motions in solids. We have inferred that the molecules of liquids are in motion, in part at least, from the fact that liquids increase in volume when the temperature is raised, and from the fact that molecules of the liquid can usually be detected in a gaseous condition above the surface. Both of these reasons apply just as well in the case of solids.

Thus the facts of expansion of solids with an increase in temperature may be seen on every side. Railroad rails are laid with spaces between their ends so that they may expand during the heat of summer without crowding each other out of place. Wagon tires are made smaller than the wheels which they are to fit. They are then heated until they become large enough to be driven on, and in cooling they shrink again and thus grip the wheels with immense force. A common lecture-room demonstration of expansion is the following:

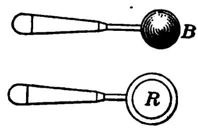


FIG. 69. Expansion of solids

Let the ball *B*, which when cool just slips through the ring *R*, be heated in a Bunsen flame. It will now be found too large to pass through the ring; but if the ring is heated, or if the ball is again cooled, it will pass through easily (see Fig. 69).

98. Evaporation of solids, — sublimation. That the molecules of a solid substance are found in a vaporous condition above the surface of the solid, as well as above that of a liquid, is proved by the often observed fact that ice and snow evaporate even though they are kept constantly below the freezing point. Thus wet clothes dry in winter after freezing. An even better proof is the fact that the odor of camphor can be detected many feet away from the camphor crystals. The

evaporation of solids may be rendered visible by the following striking experiment:

Let a few crystals of iodine be placed on a watch glass and heated gently with a Bunsen flame. The visible vapor of iodine will appear above the crystals, though none of the liquid is formed. A great many substances at high temperatures pass thus from the solid to the gaseous condition without passing through the liquid stage at all. This process is called *sublimation*.

99. Diffusion of solids. It has recently been demonstrated that if a layer of lead is placed upon a layer of gold, molecules of gold may in time be detected throughout the whole mass of the lead. This diffusion of solids into one another at ordinary temperature has been shown only for these two metals, but at higher temperatures, for example 500° C., all of the metals show the same characteristics to quite a surprising degree.

The evidence for the existence of molecular motions in solids is then no less strong than in the case of liquids.

100. The three states of matter. Although it has been shown that in accordance with current belief the molecules of all substances are in very rapid motion, and that the temperature of a given substance, whether in the solid, liquid, or gaseous condition, is determined by the average velocity of its molecules, yet differences exist in the kind of motion which the molecules in the three states possess. Thus in the solid state it is probable that the molecules oscillate with great rapidity about certain fixed points, always being held by the attractions of their neighbors, that is, by the *cohesive forces* (see § 139), in practically the same positions with reference to other molecules in the body. In rare instances, however, as the facts of diffusion show, a molecule breaks away from its constraints. In liquids, on the other hand, while the molecules are, in general, as close together as in solids, they slip about with perfect ease over one another and thus have no

fixed positions. This assumption is necessitated by the fact that liquids adjust themselves readily to the shape of the containing vessel. In gases the molecules are comparatively far apart, as is evident from the fact that a cubic centimeter of water occupies about 1600 cubic centimeters when it is transformed into steam; and furthermore, they exert practically no cohesive force upon one another, as is shown by the indefinite expansibility of gases.

QUESTIONS AND PROBLEMS

1. Does dew "fall"?

2. Why does sprinkling the street on a hot day make the air cooler?

3. Why is the heat so oppressive on a very damp day in summer?

4. Would fanning produce any feeling of coolness if there were no moisture on the face?

5. If there were moisture on the face, would fanning produce any feeling of coolness in a saturated atmosphere?

6. If a glass beaker and a porous earthenware vessel are filled with equal amounts of water at the same temperature, in the course of a few minutes a noticeable difference of temperature will exist between the two vessels. Which will be the cooler, and why? Will the difference in temperature between the two vessels be greater in a dry or in a moist atmosphere?

7. Why are icebergs frequently surrounded with fog?

8. What weight of water is contained in a room 5 × 5 × 3 m. if the relative humidity is 60% and the temperature 20° C.? (See table, p. 62.)

9. Why will an open, narrow-necked bottle containing ether not show as low a temperature as an open shallow dish containing the same amount of ether?

10. A morning fog generally disappears before noon. Explain the reason for its disappearance.

11. What becomes of the cloud which you see about a blowing locomotive whistle? Is it steam?

12. Dew will not usually collect on a pitcher of ice water standing in a warm room on a cold winter day. Explain.

CHAPTER V

FORCE AND MOTION

DEFINITION AND MEASUREMENT OF FORCE

101. Distinction between a gram of mass and a gram of force. If a gram of mass is held in the outstretched hand, a downward pull upon the hand is felt. If the mass is 50,000 g. instead of 1, this pull is so great that the hand cannot be held in place. The cause of this pull we assume to be an attractive force which the earth exerts on the matter held in the hand, and *we define the gram of force as the amount of the earth's pull at its surface upon one gram of mass.*

Unfortunately, in ordinary conversation we often fail altogether to distinguish between the idea of mass and the idea of force, and use the same word "gram" to mean sometimes a certain *amount of matter*, and at other times the *pull of the earth upon this amount of matter.* That the two ideas are, however, wholly distinct is evident from the consideration that the amount of matter in a body is always the same, no matter where the body is in the universe, while the pull of the earth upon that amount of matter decreases as we recede from the earth's surface. It will help to avoid confusion if we reserve the simple term "gram" to denote exclusively an amount of matter, that is, a mass, and use the full expression "gram of force" wherever we have in mind the pull of the earth upon this mass.

102. Method of measuring forces. When we wish to compare accurately the pulls exerted by the earth upon different masses, we find such sensations as those described in the

preceding paragraph very untrustworthy guides. An accurate method, however, of comparing these pulls is that furnished by the stretch produced in a spiral spring. Thus the pull of the earth upon a gram of mass at its surface will stretch a given spring a given distance *ab* (Fig. 70). The pull of the earth upon 2 grams of mass is found to stretch the spring a larger distance *ac*, upon 3 grams a still larger distance *ad*, etc. We have only to place a fixed surface behind the pointer and make lines upon it corresponding to the points to which it is stretched by the pull of the earth upon different masses in order to graduate a spring balance (Fig. 71), so that it will thenceforth measure the values of any pulls exerted upon it, no matter how these pulls may arise. Thus, if a man stretch the spring so that the pointer is opposite the mark corresponding to the pull of the earth upon 2 grams of mass, we say that he exerts 2 grams of force. If he stretch it the distance corresponding to the pull of the earth upon 3 grams of mass, he exerts 3 grams of force, etc. The spring balance thus becomes an instrument for measuring forces.

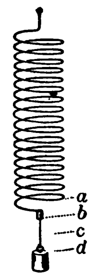


FIG. 70. Method of measuring forces

103. The gram of force varies slightly in different localities. With the spring balance it is easy to verify the statement made above, that the force of the earth's pull decreases as we recede from the earth's surface; for upon a high mountain the stretch produced by a given mass is indeed found to be slightly less than at the sea level. Furthermore, if the balance is simply carried from point to point over the earth's surface, the stretch is still found to vary slightly. For example, at Chicago it is about one part in 1000 less than it is at Paris, and near the equator it is five parts in 1000 less

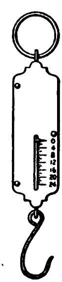


FIG. 71. The spring balance

than it is near the pole. This is due in part to the earth's rotation, and in part to the fact that the earth is not a perfect sphere, and in going from the equator toward the pole we are coming closer and closer to the center of the earth. We see, therefore, that *the gram of force is not an absolutely invariable unit of force.*

Composition and Resolution of Forces

104. Graphic representation of force. A force is completely defined when its *magnitude*, its *direction*, and the *point at which it is applied* are given. Since the three characteristics of a straight line are its *length*, its *direction*, and the *point at which it starts*, it is obviously possible to represent forces by means of straight lines. Thus, if we wish to represent the fact that a force of 8 pounds, acting in an easterly direction, is applied at the point A (Fig. 72), we draw a line 8 units long, beginning at the point A and extending to the right. The length of this line then represents the magnitude of the force; the direction of the line, the direction of the force; and the starting point of the line, the point at which the force is applied.

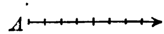


Fig. 72. Graphic representation of a single force

105. Resultant of two forces acting in the same line. *The resultant of two forces is defined as that single force which will produce the same effect upon a body as is produced by the joint action of the two forces.*

If two spring balances are attached to a small ring and pulled in the same direction until one registers 10 g. of force and the other 5, it will be found that a third spring balance attached to the same point and pulled in the opposite direction will register exactly 15 g. when there is equilibrium; that is, *resultant of two similarly directed forces is equal to the sum of the two forces.*

Similarly, *the resultant of two oppositely directed forces applied at the same point is equal to the difference between them, and its direction is that of the greater force.*

106. Equilibrant. In the last experiment the pull in the spring balance which registered 15 g. was not the resultant of the 5 g. and 10 g. forces; it was rather a force equal and opposite to that resultant. Such a force is called an *equilibrant. The equilibrant of a force or forces is that single force which will just prevent the motion which the given forces tend to produce.* It is equal and opposite to the resultant.

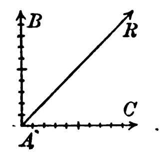


FIG. 73. Direction of resultant of two equal forces at right angles

107. The resultant of forces acting at an angle. If a body at A is pulled toward the east with a force of 10 lb. (represented in Fig. 73 by the line AC) and toward the north with a force of 10 lb. (represented in the figure by the line AB), the effect upon the motion of the body must, of course, be the same as though some single force acted somewhere between AC and AB. If the body *moves* under the action of the two equal forces, it may be seen from symmetry that it must move along a line midway between AC and AB, that is, along the line AR. This line therefore indicates the *direction* as well as the point of application of the resultant of the forces AC and AB.

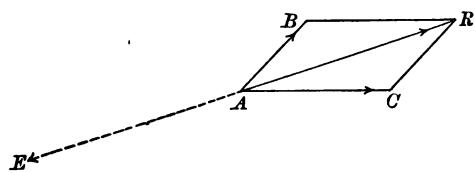


FIG. 74. Resultant of two forces at an angle is represented by the diagonal of the parallelogram of which the forces are sides

If the two forces are not equal, then the resultant will lie nearer the larger force. As a matter of fact, the following experiment will show that *if the two given forces are represented in direction and in magnitude by the lines AB and AC* (Fig. 74),

then their resultant will be exactly represented both in direction and in magnitude by the diagonal AR of the parallelogram of which AB and AC are sides.

Let the rings of two spring balances be hung over nails B and C in the rail at the top of the blackboard (Fig. 75), and let a weight W be tied near the middle of the string joining the hooks of the two balances. The force of the earth's attraction for the weight W is then exactly equal and opposite to the resultant of the two forces exerted by the spring balances; that is, OW is the *equilibrant* of the forces exerted by the balances. Let the lines OA and OD be drawn upon the blackboard behind the string, and upon these lines let distances Oa and Ob be laid off which contain as many units of length as there are units of force indicated by the balances E and F respectively. Then let a parallelogram be constructed upon Oa and Ob as sides. The diagonal of this parallelogram will be found in the first place to be exactly vertical, that is, in the *direction* of the resultant, since it is exactly opposite to OW; and in the second place, the *length* of the diagonal will be found to contain as many units of length as there are units of force in the earth's attraction for W (W must, of course, be expressed in the same units as the balance readings). Therefore the diagonal OR represents in direction, in magnitude, and in point of application the resultant of the two forces represented by Oa and Ob.

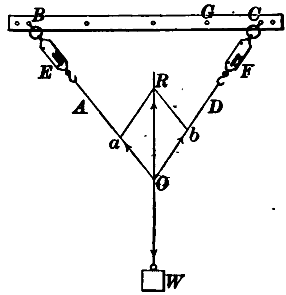


FIG. 75. Experimental proof of parallelogram law

108. Component of a force. Whenever a force acts upon a body in some direction other than that in which the body is free to move, it is clear that the full effect of the force cannot be spent in producing motion. For example, suppose that a force is applied in the direction OR (Fig. 76) to a car on an elevated track. Evidently OR produces two distinct effects upon the car: on the one hand, it moves the car along the track; and on the other, it presses it down against the rails. These two effects might be produced just as well by two

separate forces acting in the directions *OA* and *OB* respectively. The value of the single force which, acting in the direction *OA*, will produce the same motion of the car on the track as is produced by *OR*, is called the *component* of *OR* in the direction *OA*. Similarly, the value of the single force which, acting in the direction *OB*, will produce the same pressure against the rails as is produced by the force *OR*, is called the component of *OR* in the direction *OB*. In a word, *the component of a force in a given direction is the effective value of the force in that direction.*

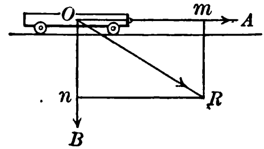


FIG. 76. Component of a force

109. Magnitude of the component of a force in a given direction. Since, from the definition of component just given, the two forces, one to be applied in the direction *OA* and the other in the direction *OB*, are together to be exactly equivalent to *OR* in their effect on the car, their magnitudes must be represented by the sides of a parallelogram of which *OR* is the diagonal. For in § 107 it was shown that if any one force is to have the same effect upon a body as two forces acting simultaneously, it must be represented by the diagonal of a parallelogram the sides of which represent the two forces. Hence, conversely, if two forces are to be equivalent in their joint effect to a single force, they must be sides of the parallelogram of which the single force is the diagonal. Hence the following rule: *To find the component of a force in any given direction, construct upon the given force as a diagonal a rectangle the sides of which are respectively parallel and perpendicular to the direction of the required component. The length of the side which is parallel to the given direction represents the magnitude of the component which is sought.* Thus, in the above illustration, the line *Om* completely represents the component of *OR* in the direction *OA*, and the line *On* represents the component of *OR* in the direction *OB*.

Again, when a boy pulls on a sled with a force of 10 lb. in the direction OR (Fig. 77), the force with which the sled is urged forward is represented by the length of Om, which is seen to be but 9.3 lb. instead of 10 lb.

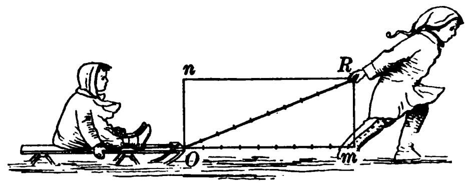


Fig. 77. Horizontal component of pull on a sled

To apply the test of experiment to the conclusions of the preceding paragraph, let a wagon be placed upon an inclined plane (Fig. 78), the height of which, bc, is equal to one half its length ab. In this case the force acting on the wagon is the weight of the wagon, and its direction is downward. Let this force be represented by the line OR. Then by the construction of the preceding paragraph, the line Om will represent the value of the force which is pulling the carriage down the plane, and the line On the value of the force which is producing pressure against the plane. Now since the triangle ROm is similar to the triangle abc (for $\angle mOR = \angle abc$, $\angle RmO = \angle acb$, and $\angle ORm = \angle bac$), we have

$$\frac{Om}{OR} = \frac{bc}{ab},$$

that is, in this case, since bc is equal to one half of ab, Om is one half of OR. Therefore the force which is necessary to prevent the wagon from sliding down the plane should be equal to one half its weight. To test this conclusion let the wagon be weighed on the spring balance and then placed on the plane in the manner shown in the figure. The pull indicated by the balance will, indeed, be found to be one half of the weight of the wagon.

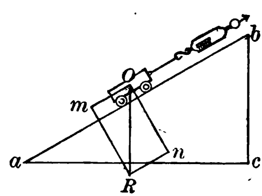


Fig. 78. Component of weight parallel to an inclined plane

The equation $Om / OR = bc/ab$ gives us the following rule for finding the force necessary to prevent a body from moving down an inclined plane, namely, *the force which must be applied to a body to hold it in place upon an inclined plane bears the same ratio to the weight of the body that the height of the plane bears to its length.*

110. Component of gravity effective in producing the motion of the pendulum. When a pendulum is drawn aside from its position of rest (Fig. 79), the force acting on the bob is its weight, and the direction of this force is vertical. Let it be represented by the line OR. The component of this force in the direction in which the bob is free to move is On, and the component at right angles to this direction is Om. The second component Om simply produces stretch in the string and pressure upon the point of suspension. The first component On is alone responsible for the motion of the bob. A consideration of the figure shows that this component becomes larger and larger the greater the displacement of the bob. When the bob is directly beneath the point of support the component producing motion is zero. Hence a pendulum can be permanently at rest only when its bob is directly beneath the point of suspension.*

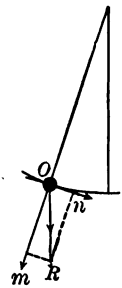


FIG. 79. Force acting on displaced pendulum

QUESTIONS AND PROBLEMS

1. In Fig. 80 the line on represents the pull of gravity on a kite, and the line om represents the pull of the boy on the string. What is the name given to the force represented by the line oR?

2. If the force of the wind against the kite is represented by the line AB, and it is considered to be applied at o, what must be the relation between the force oR and the component of AB parallel to oR when the kite is in equilibrium under the action of the existing forces.

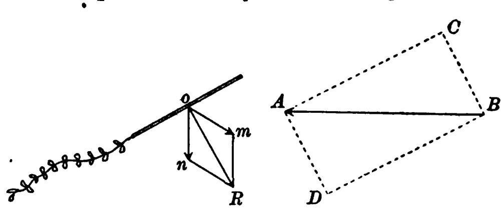


FIG. 80. Forces acting on a kite

3. If the wind increases, why does the kite rise higher?

* It is recommended that the study of the laws of the pendulum be introduced into the laboratory work at about this point (see Experiment 12, authors' manual).

4. Represent graphically a force of 30 lb. acting southeast and a force of 40 lb. acting southwest at the same point. What will be the magnitude of the resultant, and what will be its approximate direction?

5. The engines of a steamer can drive it 12 mi. an hour. How fast can it go up a stream in which the current is 5 ft. per second? How fast can it come down the same stream?

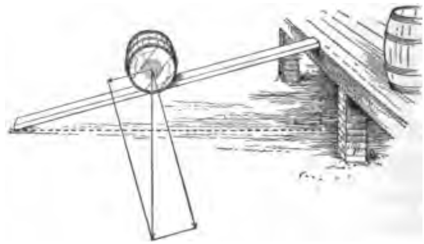


FIG. 81. Force necessary to prevent a barrel from rolling down an inclined plane

6. The wind drives a steamer east with a force which would carry it 12 mi. per hour, and its propeller is driving it south with a force which would carry it 15 mi. per hour. What distance will it actually travel in an hour? Draw a diagram to represent the exact path.

7. A boy pulls a loaded sled weighing 200 lb. up a hill which rises 1 ft. in 5 measured along the slope. Neglecting friction, how much force must he exert?

8. If the barrel of Fig. 81 weighs 200 lb., with what force must a man push parallel to the skid to keep the barrel in place if the skid is 9 ft. long and the platform 3 ft. high?

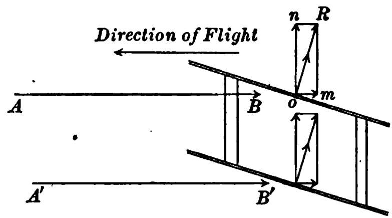


FIG. 82. Forces acting on an aëroplane in flight

9. Why does a workman lower the handles of a wheelbarrow when he wishes to push the load over an obstacle?

10. Could a kite be flown from an automobile when there is no wind? Explain.

11. Show from Fig. 82 what force supports an aëroplane in flight. (Remember that oR, the component of the wind pressure AB perpendicular to the plane, is the only force acting out of which a support for the aëroplane can be derived.)

12. What force will be required to support a 50-lb. ball on an inclined plane of which the length is 10 times the height?

13. A boy is able to exert a force of 75 lb. Neglecting friction, how long an inclined plane must he have in order to push a truck weighing 350 lb. up to a doorway 3 ft. above the ground?

GRAVITATION

111. Newton's law of universal gravitation. In order to account for the fact that the earth pulls bodies toward itself, and at the same time to account for the fact that the moon and planets are held in their respective orbits about the earth and the sun, Sir Isaac Newton (1642–1727) first announced the law which is now known as the law of universal gravitation. This law asserts first that *every body in the universe attracts every other body with a force which varies inversely as the square of the distance between the two bodies.* This means that if the distance between the two bodies considered is doubled, the force will become only one fourth as great; if the distance is made three, four, or five times as great, the force will be reduced to one ninth, one sixteenth, or one twenty-fifth of its original value, etc.

The law further asserts that if the distance between two bodies remains the same, *the force with which one body attracts the other is proportional to the product of the masses of the two bodies.* Thus we know that the earth attracts 3 cubic centimeters of water with three times as much force as it attracts 1, that is, with a force of 3 grams. We know also, from the facts of astronomy, that if the mass of the earth were doubled, its diameter remaining the same, it would attract 3 cubic centimeters of water with twice as much force as it does at present, that is, with a force of 6 grams (multiplying the mass of one of the attracting bodies by 3 and that of the other by 2 multiplies the forces of attraction by 3 × 2, or 6). In brief, then, Newton's law of universal gravitation is as follows: *Any two bodies in the universe attract each other with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.*

112. Variation of the force of gravity with distance above the earth's surface. If a body is spherical in shape and of uniform density, it attracts external bodies with the same force as

though its mass were concentrated at its center. Since, therefore, the distance from the surface to the center of the earth is about 4000 miles, we learn from Newton's law that the earth's pull upon a body 4000 miles above its surface is but one fourth as much as it would be at the surface.

It will be seen, then, that if a body be raised but a few feet or even a few miles above the earth's surface, the decrease in its weight must be a very small quantity, for the reason that a few feet or a few miles is a small distance compared with 4000 miles. As a matter of fact, at the top of a mountain 4 miles high 1000 grams of mass is attracted by the earth with 998 grams instead of 1000 grams of force.

113. Center of gravity. From the law of universal gravitation it follows that every particle of a body upon the earth's surface is pulled toward the earth. It is evident that the sum of all these little pulls on the particles of which the body is composed must be equal to the total pull of the earth upon the body. Now it is always possible to find one single point in a body at which a single force equal in magnitude to the weight of the body and directed upward can be applied so that the body will remain at rest in whatever position it is placed.

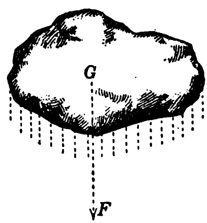


FIG. 83. Center of gravity of an irregular body

This point is called the *center of gravity* of the body. Since this force counteracts entirely the earth's pull upon the body, it must be equal and opposite to the resultant of all the small forces which gravity is exerting upon the different particles of the body. Hence the center of gravity may be defined as the point of application of the resultant of all the little downward forces; that is, it is *the point at which the entire weight of the body may be considered as concentrated.* The earth's attraction for a body is therefore always considered not as a multitude of little forces but as one single force F (Fig. 83) equal to the

pull of gravity upon the body and applied at its center of gravity G. It is evident, then, that *under the influence of the earth's pull, every body tends to assume the position in which its center of gravity is as low as possible.*

114. Method of finding center of gravity experimentally. From the above definition it will be seen that the most direct way of finding the center of gravity of any flat body, like that shown in Fig. 84, is to find the point upon which it will balance.

Let an irregular sheet of zinc be thus balanced on the point of a pencil or the head of a pin. Let a small hole be punched through the zinc at the point of balance, and let a needle be thrust through this hole. When the needle is held horizontally the zinc will be found to remain at rest, no matter in what position it is turned.

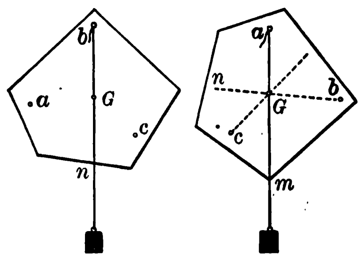


FIG. 84. Locating center of gravity

To illustrate another method for finding the center of gravity of the zinc, let it be supported from a pin stuck through a hole near its edge, that is, b (Fig. 84). Let a plumb line be hung from the pin, and let a line bn be drawn through b on the surface of the zinc parallel to and directly behind the plumb line. Let the zinc be hung from another point a, and another line am be drawn in a similar way.

Since the earth's attraction may be considered as a single force applied at the center of gravity, the zinc can remain at rest only when the center of gravity is directly beneath the point of support (see § 113). It must therefore lie somewhere on the line am. For the same reason it must lie on the line bn. But the only point which lies on both of these lines is their point of intersection G. *The point of intersection, then, of any two vertical lines dropped through two different points of suspension locates the center of gravity of a body.*

115. Stable equilibrium. A body is said to be in *stable equilibrium* if it tends to return to its original position when very

slightly tipped out of that position. A pendulum, a chair, a cube resting on its side, a cone resting on its base, are all illustrations.

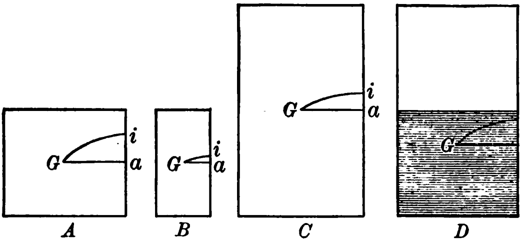


FIG. 85. Illustration of varying degrees of stability

In general, a body is in stable equilibrium whenever tipping it slightly tends to raise its center of gravity. Thus in Fig. 85 all of the bodies *A*, *B*, *C*, *D*, are in stable equilibrium, for in order to overturn any one of them, its center of gravity *G* must be raised through the height *ai*. If the weights are all alike, that one will be most stable for which *ai* is greatest.

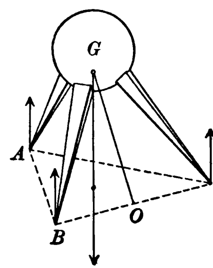


FIG. 86. Body in stable equilibrium

The condition of stable equilibrium for bodies which rest upon a horizontal plane is that a vertical line through the center of gravity shall fall within the base, the base being defined as the polygon formed by connecting the points at which the body touches the plane, as *ABC* (Fig. 86); for it is clear that in such a case a slight displacement must raise the center of gravity along the arc of which *OG* is the radius. If the vertical line drawn through the center of gravity fall outside the base, as in Fig. 87, the body must always fall.

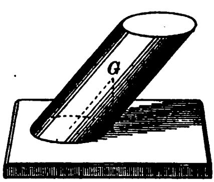


FIG. 87. Body not in equilibrium

The condition of stable equilibrium for bodies supported from a single point is that the point of support be above the center of gravity. For example, the beam of a balance cannot be in stable equilibrium so that it will return to the horizontal position when slightly displaced, unless its center of gravity *g* (Fig. 3, p. 7) is below the knife-edge *C*. (The pans are not to be considered, since they are not rigidly connected to the beam.)

116. Neutral equilibrium. A body is said to be in *neutral equilibrium* when, after a slight displacement, it tends neither to return to its original position nor to move farther from it. Examples of neutral equilibrium are a spherical ball lying on a smooth plane, a cone lying on its side, a wheel free to rotate about a fixed axis through its center, or any body supported at its center of gravity. In general, a body is in neutral equilibrium when a slight displacement neither raises nor lowers its center of gravity.

117. Unstable equilibrium. A body is in *unstable equilibrium* when, after a slight tipping, it tends to move farther from its original position. A cone balanced on its point or an egg on its end are examples. In all such cases a slight tipping lowers the center of gravity, and the motion then continues until the center of gravity is as low as circumstances will permit. The condition for unstable equilibrium in the case of a body supported by a point is that the center of gravity shall be above the point of support. Fig. 88 illustrates the three kinds of equilibrium.

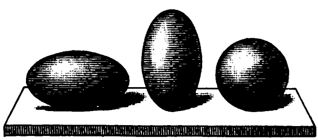


FIG. 88. Stable, unstable, and neutral equilibrium

QUESTIONS AND PROBLEMS

1. What is the most stable position of a brick? What the least? Why?

2. Where is the center of gravity of a hoop? of a cubical box? Is the latter more stable when empty or full? Why?

3. Why is it unsafe to stand up in a canoe?

4. Where must the center of gravity of the beam of a balance be with reference to the supporting knife-edge C (Fig. 3, p. 7)? Why? Could you make a weighing if C and g coincided? Why?

5. The pull of the earth on a body at its surface is 100 kg. Find the pull on the same body 4000 mi. above the surface; 1000 mi. above the surface; 3 mi. above the surface. (Take the earth's radius as 4000 mi.)

6. What is the object of ballast in a ship?

7. Explain why the toy shown in Fig. 89 will not lie upon its side, but instead rises to the vertical position. Does the center of gravity actually rise?

8. What purpose is served by the tail of a kite?

9. If a lead pencil is balanced on its point on the finger, it will be in unstable equilibrium, but if two knives are stuck into it, as in Fig. 90, it will be in stable equilibrium. Why?

10. Why does a man lean forward when he climbs a hill?

11. Do you get more sugar to the pound in Calcutta than in Aberdeen? Explain.

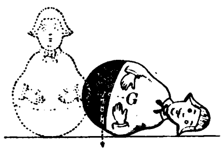


FIG. 89

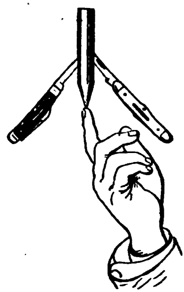


FIG. 90

FALLING BODIES

118. Galileo's early experiments. Many of the familiar and important experiences of our lives have to do with falling bodies. Yet when we ask ourselves the simplest question which involves quantitative knowledge about gravity, such as, for example, Would a stone and a piece of lead dropped from the same point reach the ground at the same or at different times? most of us are uncertain as to the answer. In fact, it was the asking and the answering of this very question by Galileo about 1590 which may be considered as the starting point of modern science.

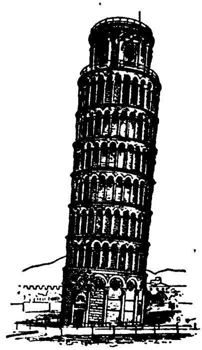


FIG. 91. Leaning tower of Pisa, from which were performed some of Galileo's famous experiments on falling bodies

Ordinary observation teaches that light bodies like feathers fall slowly and heavy bodies like stones fall rapidly, and up to Galileo's time it was taught in the schools that bodies fall with "velocities proportional to their weights." Not

GALILEO (1564–1642)

Great Italian physicist, astronomer, and mathematician; "founder of experimental science"; was son of an impoverished nobleman of Pisa; studied medicine in early youth, but forsook it for mathematics and science; was professor of mathematics at Pisa and at Padua; discovered the laws of falling bodies and the laws of the pendulum; was the creator of the science of dynamics; constructed the first thermometer; first used the telescope for astronomical observations; discovered Jupiter's satellites and the spots on the sun. Modern physics begins with Galileo

content with book knowledge, however, Galileo tried it himself. In the presence of the professors and students of the University of Pisa he dropped balls of different sizes and materials from the top of the tower of Pisa (Fig. 91), 180 feet high, and found that they fell in practically the same time. He showed that even very light bodies like paper fell with velocities which approached more and more nearly those of heavy bodies the more compactly they were wadded together. From these experiments he inferred that *all bodies, even the lightest, would fall at the same rate were it not for the resistance of the air.*

That the air resistance is indeed the chief factor in the slowness of fall of feathers and other light objects can be shown by pumping the air out of a tube containing a feather and a coin (Fig. 92). The more complete the exhaustion the more nearly do the feather and coin fall side by side when the tube is inverted. The air pump, however, was not invented until sixty years after Galileo's time.

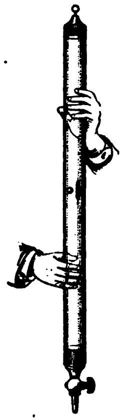


FIG. 92. Feather and coin fall together in a vacuum

119. Exact proof of Galileo's conclusion. We can demonstrate the correctness of Galileo's conclusion in still another way, one which he himself used.

Let balls of iron and wood, for example, be started together down the inclined plane of Fig. 93. They will be found to keep together all the way down. (If they roll in a groove, they should have the same diameter; otherwise, size is immaterial.) The experiment differs from that of the freely falling bodies only in that the resistance of the air is here more nearly negligible because the balls are moving more slowly. In order to make them move still more slowly and at the same time to eliminate completely all possible effects due to the friction of the plane, let us follow Galileo and suspend the different balls as the bobs of pendulums of exactly the same length, two meters long at least, and start them swinging through equal arcs. Since now the bobs, as they pass through any given position, are merely moving

very slowly down identical inclined planes (Fig. 79), it is clear that this is only a refinement of the last experiment. We shall find that the times of fall, that is, *the periods*, of the pendulums are exactly the same.

We conclude from the above experiment, with Galileo and with Newton who performed it with the utmost care a hundred years later, that *in a vacuum the velocity acquired per second by a freely falling body is exactly the same for all bodies.*

120. Relation between distance and time of fall. Having found that, barring air resistance, all bodies fall in exactly the same way, we shall next try to find what relation exists between distance and time of fall; and since a freely falling body falls so rapidly as to make direct measurements upon it difficult, we shall adopt Galileo's plan of studying the laws of falling bodies through observing the motions of a ball rolling down an inclined plane.

Let a grooved board 17 or 18 ft. long be supported as in Fig. 93, one end being about a foot above the other. Let the side of the board be divided into feet, and let the block *B* be set just 16 ft. from the starting point of the ball *A*. Let a metronome or a clock beating seconds be started, and the marble released at the instant of one click of the metronome. If the marble does not hit the block so that the click produced by the impact of the ball coincides exactly with the fifth click of the metronome, alter the inclination until this is the case. (This adjustment may well be made by the teacher before class.) Now start the marble again at some click of the metronome and note that it crosses the 1-ft. mark exactly at the end of the first second, the 4-ft. mark at the end of the second second, the 9-ft. mark at the end of the third second, and hits *B* at the 16-ft. mark at the end of the fourth second. This can be tested more accurately by placing *B* successively at the 9-ft., the 4-ft., and the 1-ft. mark, and noting that the click produced by the impact coincides exactly with the proper click of the metronome.

We conclude then, with Galileo, that *the distance traversed by a falling body in any number of seconds is the distance traversed the first second times the square of the number of*

seconds; that is, if D represents the distance traversed the first second, S the total space, and t the number of seconds, $S = Dt^2$.

121. Relation between velocity and time of fall. In the last paragraph we investigated the distances traversed in one, two, three, etc. seconds. Let us now investigate the *velocities* acquired on the same inclined plane in one, two, three, etc., seconds.

Let a second grooved board M be placed at the bottom of the incline, in the manner shown in Fig. 93. To eliminate friction it should be given a slight slant, just sufficient to cause the ball to roll along it with

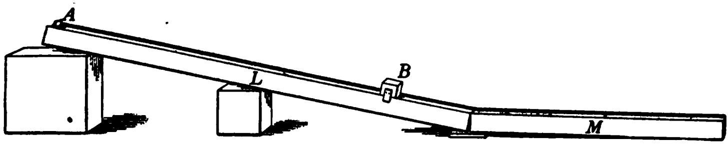


FIG. 93. Spaces traversed and velocities acquired by falling bodies in one, two, three, etc. seconds

uniform velocity. Let the ball be started at a distance D up the incline, D being the distance which in the last experiment it was found to roll during the first second. It will then just reach the bottom of the incline at the instant of the second click. Here it will be freed from the influence of gravity, and will therefore move along the lower board with the velocity which it had at the end of the first second. It will be found that when the block is placed at a distance exactly equal to $2D$ from the bottom of the incline, the ball will hit it at the exact instant of the third click of the metronome, that is, exactly two seconds after starting; hence the velocity acquired in one second is $2D$. If the ball is started at a distance $4D$ up the incline, it will take it two seconds to reach the bottom, and it will roll a distance $4D$ in the next second; that is, in two seconds it acquires a velocity $4D$. In three seconds it will be found to acquire a velocity $6D$, etc.

The experiment shows, first, that the gain in velocity each second is the same; and second, that the amount of this gain is numerically equal to twice the distance traversed the first second. *Motion,* like the above, *in which velocity is gained at a constant rate, is called uniformly accelerated motion.*

In uniformly accelerated motion the gain each second in the velocity is called the acceleration. It is numerically equal to twice the distance traversed the first second. It is usually denoted by the letter a.

122. Formal statement of the laws of falling bodies. Putting together the results of the last two paragraphs, we obtain the following table in which D represents the distance traversed the first second in any uniformly accelerated motion.

Number of Seconds (t)	Velocity at the End of Each Second (v)	Gain in Velocity Each Second (a)	Total Distance Traversed (S)
1	$2D$	$2D$	$1D$
2	$4D$	$2D$	$4D$
3	$6D$	$2D$	$9D$
4	$8D$	$2D$	$16D$
...	...	...	...
t	$2tD$	$2D$	t^2D

Since D was shown in § 121 to be equal to one half of the acceleration a, we have at once, by substituting $\frac{1}{2}a$ for D in the last line of the table,

$$v = at, \tag{1}$$

$$S = \tfrac{1}{2}at^2. \tag{2}$$

These formulas are simply the algebraic statement of the facts brought out by our experiments, but the reasons for these facts may be seen as follows:

Since in uniformly accelerated motion the acceleration a is the velocity in centimeters per second gained each second, it follows at once that when a body starts from rest, the velocity which it has at the end of t seconds is given by $v = at$. This is formula (1).

To obtain formula (2) we have only to reflect that distance traversed is always equal to the average velocity multiplied by the time. When the initial velocity is zero, as in this case, and the final velocity is at, average velocity $= (0 + at) \div 2 = \frac{1}{2}at$. Hence

$$S = \tfrac{1}{2}at^2.$$

This is formula (2).

These are the fundamental formulas of uniformly accelerated motion, but it is sometimes convenient to obtain the final velocity v directly from

the total distance of fall S, or vice versa. This may of course be done by simply substituting in (2) the value of t obtained from (1), namely $\frac{v}{a}$. This gives

$$v = \sqrt{2\,aS}. \tag{3}$$

123. Acceleration of a freely falling body. If in the above experiment the slope of the plane be made steeper, the results will obviously be precisely the same, except that the acceleration has a larger value. If the board is tilted until it becomes vertical, the body becomes a freely falling body. In this case the distance traversed the first second is found to be 490 centimeters, or 16.08 feet. Hence the acceleration expressed in centimeters is 980, in feet 32.16. This acceleration of free fall, called the *acceleration of gravity*, is usually denoted by the letter g. For freely falling bodies, then, the three formulas of the last paragraph become

$$v = gt, \tag{4}$$

$$S = \tfrac{1}{2}\,gt^2, \tag{5}$$

$$v = \sqrt{2\,gS}. \tag{6}$$

To illustrate the use of these formulas, suppose we wish to know with what velocity a body will hit the earth if it falls from a height of 200 meters or 20,000 centimeters. From (6) we get

$$v = \sqrt{2 \times 980 \times 20{,}000} = 6261 \text{ cm. per sec.}$$

124. Height of ascent. If we wish to find the height S to which a body projected vertically upward will rise, we reflect that the time of ascent must be the initial velocity divided by the upward velocity which the body loses per second, that is, $t = \frac{v}{g}$; and the height reached must be this multiplied by the average velocity $\frac{v + 0}{2}$; that is,

$$S = \frac{v^2}{2\,g}, \quad \text{or} \quad v = \sqrt{2\,gS}. \tag{7}$$

Since (7) is the same as (6), we learn that in a vacuum the speed with which a body must be projected upward to rise to a given height is the same as the speed which it acquires in falling from the same height.

125. The laws of the pendulum. The first law of the pendulum was found in § 119, namely,

1. *The periods of pendulums of equal lengths swinging through short arcs are independent of the weight and material of the bobs.*

Let the two pendulums of § 119 be set swinging through arcs of lengths 5 centimeters and 25 centimeters respectively. We shall find thus the second law of the pendulum, namely,

2. *The period of a pendulum swinging through a short arc is independent of the amplitude of the arc.*

Let pendulums $\frac{1}{4}$ and $\frac{1}{9}$ as long as the above be swung with it. The long pendulum will be found to make only one vibration, while the others are making two and three respectively. The third law of the pendulum is therefore

3. *The periods of pendulums are directly proportional to the square roots of their lengths.*

The accurate determination of g is never made by direct measurement, for the laws of the pendulum just established make this instrument by far the most accurate one obtainable for this determination. It is only necessary to measure the length of a long pendulum and the time t between two successive passages of the bob across the mid-point and then to substitute in the formula $t = \pi \sqrt{\frac{l}{g}}$ in order to obtain g with a high degree of precision. The deduction of this formula is not suitable for an elementary text, but the formula itself may well be used for checking the above value of g.

$v = gt$

QUESTIONS AND PROBLEMS

1. A boy dropped a stone from a bridge and noticed that it struck the water in just 3 sec. How fast was it going when it struck? How high was the bridge above the water?

2. How high is a balloon from which a stone falls to earth in 10 sec.?

3. With what speed does a bullet strike the earth, if it is dropped from the Eiffel Tower, 335 m. high?

4. If the acceleration of a marble rolling down an inclined plane is 20 cm. per second, what velocity will it have at the bottom, the plane being 7 m. long?

5. If a man can jump 3 ft. high on the earth, how high could he jump on the moon, where g is $\frac{1}{6}$ as much?

6. If a body sliding without friction down an inclined plane moves 40 cm. during the first second of its descent, and if the plane is 500 cm. long and 40.8 cm. high, what is the value of g? (Remember that the acceleration down the incline is simply the component (§ 108) of g parallel to the incline.)

7. How far will a body fall in half a second?

8. Fig. 94 represents the pendulum and "escapement" of a clock. The escapement wheel D is urged in the direction of the arrow by the clock weights or spring. The slight pushes communicated by the teeth of the wheel keep the pendulum from dying down. Show how the length of the pendulum controls the rate of the clock.

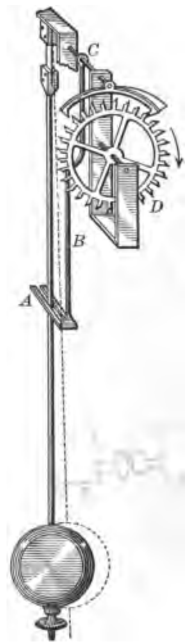


FIG. 94

NEWTON'S LAWS OF MOTION

126. First law — inertia. It is a matter of everyday observation that bodies in a moving train tend to move toward the forward end when the train stops and toward the rear end when the train starts; that is, bodies in motion seem to want to keep on moving, and bodies at rest to remain at rest.

Again, a block will go farther when driven with a given blow along a surface of glare ice than when knocked along an asphalt pavement. The reason which every one will assign for this is that there is more friction between the block and the asphalt than between the block and the ice. But when would the body stop if there were no friction at all?

Astronomical observations furnish the most convincing answer to this question, for we cannot detect any retardation at all in the motions of the planets as they swing around the sun through empty space.

Furthermore, since mud flies off *tangentially* from a rotating carriage wheel, or water from a whirling grindstone, and since, too, we have to lean inward to prevent ourselves from falling

outward in going around a curve, it appears that bodies in motion tend to maintain not only the *amount* but also the *direction* of their motion.

In view of observations of this sort Sir Isaac Newton in 1686 formulated the following statement and called it the first law of motion.

Every body continues in its state of rest or uniform motion in a straight line unless impelled by external force to change that state.

This property, which all matter possesses, of resisting any attempt to start it if at rest, to stop it if in motion, or in any way to change either the direction or amount of its motion, is called inertia.

127. Centrifugal force. It is inertia alone which prevents the planets from falling into the sun, which causes a rotating sling to pull hard on the hand until the stone is released, and which then causes the stone to fly off tangentially. It is inertia which makes rotating liquids move out as far as possible from the axis of rotation (Fig. 95), which makes flywheels sometimes burst, which makes the equatorial diameter of the earth greater than the polar, which makes the heavier milk move out farther than the lighter cream in the dairy separator, etc. *Inertia manifesting itself in this tendency of the parts of rotating systems to move away from the center of rotation is called centrifugal force.*

Fig. 95. Illustrating centrifugal force

128. Momentum. The quantity of motion possessed by a moving body is defined as the product of the mass and the velocity of the body. It is commonly called *momentum.* Thus a 10-gram bullet moving 50,000 centimeters per second has 500,000 units of momentum. A 1000-kg. pile driver moving

SIR ISAAC NEWTON (1642–1727)

English mathematician and physicist, "prince of philosophers"; professor of mathematics at Cambridge University; formulated the law of gravitation; discovered the binomial theorem; invented the method of the calculus; announced the three laws of motion which have become the basis of the science of mechanics; made important discoveries in light; is the author of the celebrated "Principia" (Principles of Natural Philosophy), published in 1687

1000 centimeters per second has 1,000,000,000 units of momentum, etc. We shall always express momentum in C.G.S. units, that is, as a product of grams by centimeters per second.

129. Second law. Since a 2-gram mass is pulled toward the earth with twice as much force as is a 1-gram mass, and since both, when allowed to fall, acquire the same velocity in a second, it follows that in this case *the momentums produced in the two bodies by the two forces are exactly proportional to the forces themselves.* In all cases in which forces simply overcome inertias this rule is found to hold. Thus a 3000-pound pull on an automobile on a level road, where friction may be neglected, imparts in a second just twice as much velocity as does a 1500-pound pull. In view of this relation Newton's second law of motion was stated thus: *Rate of change of momentum is proportional to the force acting, and takes place in the direction in which the force acts.*

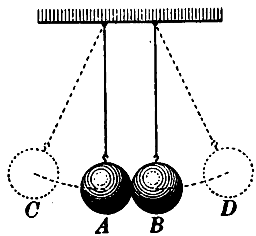


FIG. 96. Illustration of third law

130. The third law. When a man jumps from a boat to the shore, we all know that the boat experiences a backward thrust; when a bullet is shot from a gun, the gun recoils, or "kicks"; when a billiard ball strikes another, it loses speed, that is, is pushed back while the second ball is pushed forward. The following experiment will show how effects of this sort may be studied quantitatively.

Let a steel ball A (Fig. 96) be allowed to fall from a position C against another exactly similar ball B. In the impact A will lose practically all of its velocity, and B will move to a position D, which is at practically the same height as C. Hence the velocity acquired by B is almost exactly equal to that which A had before impact. These velocities would be exactly equal if the balls were perfectly elastic. It is found to be exactly true that the momentum acquired by B plus that retained by A is exactly equal to the momentum which A had before

the impact. The momentum acquired by B is therefore exactly equal to that lost by A. Since by the second law change in momentum is proportional to the force acting, this experiment shows that A pushed forward on B with exactly the same force with which B pushed back on A.

Now the essence of Newton's third law is the assertion that in the case of the man jumping from the boat the mass of the man times his velocity is equal to the mass of the boat times its velocity, and that in the case of the bullet and gun the mass of the bullet times its velocity is equal to the mass of the gun times its velocity. The truth of this assertion has been established by a great variety of experiments in addition to the one on impact given above.

Newton stated his third law thus: *To every action there is an equal and opposite reaction.*

Since force is measured by the rate at which momentum changes, this is only another way of saying that *whenever a body acquires momentum some other body acquires an equal and opposite momentum.*

It is not always easy to see at first that setting one body into motion involves imparting an equal and opposite momentum to another body. For example, when a gun is held against the earth and a bullet shot upward, we are conscious only of the motion of the bullet; the other body is in this case the earth, and its momentum is the same as that of the bullet. On account, however, of the greatness of the earth's mass its velocity is infinitesimal.

131. The dyne. Since the gram of force varies somewhat with locality, it has been found convenient for scientific purposes to take the second law as the basis for the definition of a new unit of force. It is called an absolute or C.G.S. unit, because it is based upon the fundamental units of length, mass, and time, and is therefore independent of gravity. It is named the *dyne*, and is defined as *the force which, acting for one second upon any mass, imparts to it one unit of momentum;* or *the force which, acting for one second upon a one-gram mass, produces a change in its velocity of one centimeter per second.*

132. A gram of force equivalent to 980 dynes. A gram of force was defined as the pull of the earth upon 1 gram of mass. Since this pull is capable of imparting to this mass in 1 second a velocity of 980 centimeters per second, that is, 980 units of momentum, and since a dyne is the force required to impart in 1 second 1 unit of momentum, it is clear that the gram of force is equivalent to 980 dynes of force. The dyne is therefore a very small unit, about equal to the force with which the earth attracts a cubic millimeter of water.

133. Algebraic statement of the second law. If a force F acts for t seconds on a mass of m grams, and in so doing increases its velocity v centimeters per second, then, since the total momentum imparted in a time t is mv, the momentum imparted per second is $\frac{mv}{t}$; and since force in dynes is equal to momentum imparted per second, we have

$$F = \frac{mv}{t}. \tag{8}$$

But since $\frac{v}{t}$ is the velocity gained per second, it is equal to the acceleration a. Equation (8) may therefore be written

$$F = ma. \tag{9}$$

This is merely stating in the form of an equation that force is measured by rate of change in momentum. Thus, if an engine, after pulling for 30 sec. on a train having a mass of 2,000,000 kg., has given it a velocity of 60 cm. per second, the force of the pull is $2{,}000{,}000{,}000 \times \frac{60}{30} = 4{,}000{,}000{,}000$ dynes. To reduce this force to grams we divide by 980, and to reduce it to kilos we divide further by 1000. Hence the pull exerted by the engine on the train $= \frac{4{,}000{,}000{,}000}{980{,}000} = 4081$ kg., or 4.081 metric tons.

QUESTIONS AND PROBLEMS

1. Balance a calling card on the finger and place a coin upon it. Snap out the card, leaving the coin balanced on the finger. What principle is illustrated?

2. Why does not the car C of Fig. 97 fall? What carries it from B to D?

3. Why does a flywheel cause machinery to run more steadily?

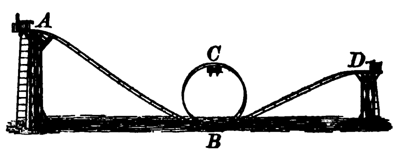


FIG. 97. A very ancient loop the loops

4. What principle is applied when one tightens the head of a hammer by pounding on the handle?

5. Is it any easier to walk toward the rear than toward the front of a rapidly moving train? Why?

6. Why does pounding a carpet free it from dust?

7. Suspend a weight by a string (Fig. 98). Attach a piece of the same string to the bottom of the weight. If the lower string is pulled with a sudden jerk, it breaks; but if the pull is steady, the upper string will break. Explain.

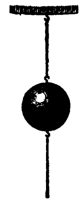


FIG. 98

8. If a weight is dropped from the roof to the floor of a moving car, will it strike the point on the floor which was directly beneath its starting point?

9. Why is a running track banked at the turns?

10. If the earth were to cease rotating, would bodies on the equator weigh more or less than now? Why?

11. How is the third law involved in rotary lawn sprinklers?

12. The modern way of drying clothes is to place them in a large cylinder with holes in the sides, and then to set it in rapid rotation. Explain.

13. If one ball is thrown horizontally from the top of a tower and another dropped at the same instant, which will strike the earth first? (Remember that the acceleration produced by a force is in the direction in which the force acts and proportional to it, whether the body is at rest or in motion. See second law.) If possible, try the experiment with an arrangement like that of Fig. 99.

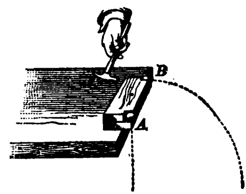


FIG. 99. Illustrating Newton's second law

14. If a rifle bullet is fired horizontally from a tower 19.6 m. high with a speed of 300 m., how far from the base of the tower would it strike the earth if there were no air resistance?

15. In a tug of war each team pulls with a force of 2000 lb. What is the strain on the rope?

16. If two men were together in the middle of a perfectly smooth (frictionless) pond of ice, how could they get off? Could one man get off if he were there alone?

17. If a 10-g. bullet is shot from a 5-kg. gun with a speed of 400 m. per second, what is the backward speed of the gun?

A laboratory exercise on the composition of forces should be performed during the study of this chapter. See, for example, Experiment 11 of the authors' manual.

CHAPTER VI

MOLECULAR FORCES*

MOLECULAR FORCES IN SOLIDS. ELASTICITY

134. Proof of the existence of molecular forces in solids. The fact that a gas will expand without limit as the volume of the containing vessel is increased, seems to show very conclusively that the molecules of gases do not exert any appreciable attractive forces upon one another. In fact, all of the experiments of Chapter IV showed that such substances certainly behave as they would if they consisted of *independent molecules* moving hither and thither with great velocities and influencing each other's motions only at the instants of collision. Between collisions the molecules doubtless move in straight lines. It must not, however, be thought that the distances moved by a single molecule between successive collisions are large. In ordinary air these distances probably do not average more than .0001 millimeter. Small, however, as this distance is, it is as much as one hundred times the radius of a molecule.

But that the molecules of *solids*, on the other hand, cling together with forces of great magnitude is proved by some of the simplest facts of nature; for solids not only do not expand indefinitely like gases, but it often requires enormous forces to pull their molecules apart. Thus a rod of cast steel 1 centimeter in diameter may be loaded with a weight of 7.8 tons before it will be pulled in two.

* This chapter should be preceded by a laboratory experiment in which Hooke's law is discovered by the pupil for certain kinds of deformation easily measured in the laboratory. See, for example, Experiment 13 of the authors' manual.

The following are the weights in kilograms necessary to break drawn wires of different materials, 1 square millimeter in cross section, — the so-called relative *tenacities* of the wires.

Lead, 2.6	Copper, 51	Iron, 77
Silver, 37	Platinum, 43	Steel, 91

135. Elasticity. We can obtain additional information about the molecular forces existing in different substances by studying what happens when the weights applied are not large enough to break the wires.

Thus let a long steel wire, for example No. 26 piano wire, be suspended from a hook in the ceiling, and let the lower end be wrapped tightly about one end of a meter stick, as in Fig. 100. Let a fulcrum c be placed in a notch in the stick at a distance of about 5 cm. from the point of attachment to the wire, and let the other end of the stick be provided with a knitting needle, one end of which is opposite the vertical mirror scale S. Let enough weights be applied to the pan P to place the wire under slight tension; then let the reading of the pointer p on the scale S be taken. Let 3 or 4 kilogram weights be added successively to the pan and the corresponding positions of the pointer read. Then let the readings be taken again as the weights are successively removed. In this last operation the pointer will probably be found to come back exactly to its first position.

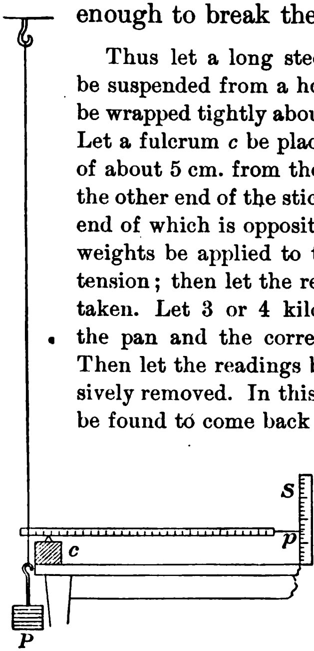


FIG. 100. Elasticity of a steel wire

This characteristic which the steel has shown in this experiment, of returning to its original length when the stretching weights are removed, is an illustration of a property possessed to a greater or less extent by all solid bodies. It is called *elasticity.*

136. Limits of perfect elasticity. If a sufficiently large weight is applied to the end of the wire of Fig. 100, it will be found that the pointer does not return exactly to its original position when the weight is removed. We say, therefore, that

steel is *perfectly elastic* only so long as the distorting forces are kept within certain limits, and that, as soon as these limits are overstepped, it no longer shows perfect elasticity. Different substances differ very greatly in the amount of distortion which they can sustain before they show this failure to return completely to the original shape.

137. Hooke's law. If we examine the stretches produced by the successive addition of kilogram weights in the experiment of § 135, Fig. 100, we shall find that these stretches are all equal, at least within the limits of observational error. Very carefully conducted experiments have shown that this law, namely, that the successive application of equal forces produces a succession of equal stretches, holds very exactly for all sorts of elastic displacements, so long, and only so long, as the limits of perfect elasticity are not overstepped. This law is known as *Hooke's law*, after the Englishman Robert Hooke (1635–1703). Another way of stating this law is the following: *Within the limits of perfect elasticity elastic deformations of any sort, be they twists or bends or stretches, are directly proportional to the forces producing them.*

138. Cohesion and adhesion. The preceding experiments have brought out the fact that in the solid condition, at least, molecules of the same kind exert attractive forces upon one another. That molecules of unlike substances also exert mutually attractive forces is equally true, as is proved by the fact that glue sticks to wood with tremendous tenacity, mortar to bricks, nickel plating to iron, etc.

The forces which bind *like* kinds of molecules together are commonly called *cohesive forces;* those which bind together molecules of *unlike* kind are called *adhesive* forces. Thus we say that mucilage sticks to wood because of *adhesion,* while wood itself holds together because of *cohesion.* Again, adhesion holds the chalk to the blackboard, while cohesion holds together the particles of the crayon.

139. Properties of solids depending on cohesion. Many of the physical properties in which solid substances differ from one another depend on differences in the cohesive forces existing between their molecules. Thus we are accustomed to classify solids with relation to their hardness, brittleness, ductility, malleability, tenacity, elasticity, etc. The last two of these terms have been sufficiently explained in the preceding paragraphs; but since confusion sometimes arises from failure to understand the first four, the tests for these properties are here given.

We test the relative *hardness* of two bodies by seeing which will *scratch* the other. Thus the diamond is the hardest of all substances, since it scratches all others and is scratched by none.

We test the relative *brittleness* of two substances by seeing which will *break* most easily under a blow from a hammer. Thus glass and ice are very brittle substances; lead and copper are not.

We test the relative *ductility* of two bodies by seeing which can be *drawn into the thinner wire.* Platinum is the most ductile of all substances. It has been drawn into wires but .00003 inch in diameter. Glass is also very ductile when sufficiently hot, as may be readily shown by heating it to softness in a Bunsen flame, when it may be drawn into threads which are so fine as to be almost invisible.

We test the relative *malleability* of two substances by seeing which can be *hammered into the thinner sheet.* Gold, the most malleable of all substances, has been hammered into sheets $\frac{1}{300000}$ inch in thickness.

QUESTIONS AND PROBLEMS

1. Why are springs made of steel rather than of copper?

2. If a given weight is required to break a given wire, how much force is required to break two such wires hanging side by side? How much to break one wire of twice the diameter?

3. What must be the cross section of a wire of copper if it is to have the same tensile strength (that is, break with the same weight) as a wire of iron 1 sq. mm. in cross section?

4. How many times greater must the diameter of one wire be than that of another of the same material if it is to have five times the tensile strength?

5. If the position of the pointer on a spring balance is marked when no load is on the spring, and again when the spring is stretched with a load of 10 g., and if the space between the two marks is then divided into ten equal parts, will each of these parts represent a gram? Why?

Molecular Forces in Liquids. Capillary Phenomena

140. Proof of the existence of molecular forces in liquids. The facility with which liquids change their shape might lead us to suspect that the molecules of such substances exert almost no forces upon one another, but a simple experiment will show that this is far from true.

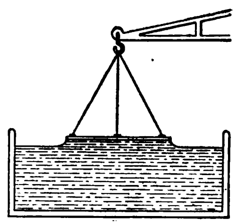


Fig. 101. Illustrating cohesion of water

By means of sealing wax and string let a glass plate be suspended horizontally from one arm of a balance, as in Fig. 101. After equilibrium is obtained let a surface of water be placed just beneath the plate and the beam pushed down until contact is made. It will be found necessary to add a considerable weight to the opposite pan in order to pull the plate away from the water. Since a layer of water will be found to cling to the glass, it is evident that the added force applied to the pan has been expended in pulling water molecules away from water molecules, not in pulling glass away from water. Similar experiments may be performed with all liquids. In the case of mercury the glass will not be found to be wet, showing that the cohesion of mercury is greater than the adhesion of glass and mercury.

141. Shape assumed by a free liquid. Since, then, every molecule of a liquid is pulling on every other molecule, any body of liquid which is free to take its natural shape, that is, which is acted on only by its own cohesive forces, must draw itself together until it has the smallest possible surface compatible with its volume; for, since every molecule in the surface is drawn toward the interior by the attraction of the molecules

within, it is clear that molecules must continually move toward the center of the mass until the whole has reached the most compact form possible. Now the geometrical figure which has the smallest area for a given volume is a sphere. We conclude, therefore, that if we could relieve a body of liquid from the action of gravity and other outside forces, it would at once take the form of a perfect sphere. This conclusion may be easily verified by the following experiment:

Let alcohol be added to water until a solution is obtained in which a drop of common lubricating oil will float at any depth. Then with a pipette insert a large globule of oil beneath the surface. The oil will be seen to float as a perfect sphere within the body of the liquid (Fig. 102). (Unless the drop is viewed from above, the vessel should have flat rather than cylindrical sides, otherwise the curved surface of the water will act like a lens and make the drop *appear* flattened.)

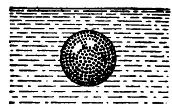


FIG. 102. Spherical globule of oil, freed from action of gravity

The reason that liquids are not more commonly observed to take the spherical form is that ordinarily the force of gravity is so large as to be more influential in determining their shape than are the cohesive forces. As verification of this statement we have only to observe that as a body of liquid becomes smaller and smaller, — that is, as the gravitational forces upon it become less and less, — it does indeed tend more and more to take the spherical form. Thus very small globules of mercury on a table will be found to be almost perfect spheres, and raindrops or minute *floating* particles of all liquids are quite accurately spherical.

142. Contractility of liquid films; surface tension. The tendency of liquids to assume the smallest possible surface furnishes a simple explanation of the contractility of liquid films.

Let a soap bubble 2 or 3 inches in diameter be blown on the bowl of a pipe and then allowed to stand. It will at once begin to shrink in size and in a few minutes will disappear within the bowl of the pipe.

The liquid of the bubble is simply obeying the tendency to reduce its surface to a minimum, a tendency which is due only to the mutual attractions which its molecules exert upon one another. A candle flame held opposite the opening in the stem of the pipe will be deflected by the current of air which the contracting bubble is forcing out through the stem.

Again, let a loop of fine thread be tied to the edge of a wire ring, as in Fig. 103. Let the ring be dipped into a soap solution so as to form a film across it, and then let a hot wire be thrust through the film inside the loop. The tendency of the film outside of the loop to contract will instantly snap out the thread into a perfect circle (Fig. 104). The reason that the thread takes the circular form is that since the film outside the

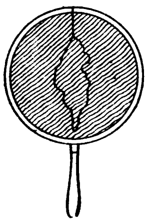


FIG. 103

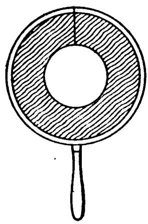


FIG. 104

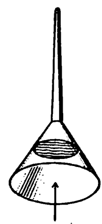


FIG. 105

Illustrating the contractility of soap films

loop is striving to assume the smallest possible surface, the area inside the loop must of course become as large as possible. The circle is the figure which has the largest possible area for a given perimeter.

Let a soap film be formed across the mouth of a clean 2-inch funnel, as in Fig. 105. The tendency of the film to contract will be sufficient to lift its weight against the force of gravity.

The tendency of a liquid to reduce its exposed surface to a minimum, that is, *the tendency of any liquid surface to act like a stretched elastic membrane is called surface tension.*

143. Ascension and depression of liquids in capillary tubes. It was shown in Chapter II that, in general, a liquid stands at the same level in any number of communicating vessels. The following experiments will show that this rule ceases to hold in the case of tubes of small diameter.

Let a series of capillary tubes of diameter varying from 2 mm. to .1 mm. be arranged as in Fig. 106.

When water is poured into the vessel it will be found to rise higher in the tubes than in the vessel, and it will be seen that the smaller the tube the greater the height to which it rises. If the water is replaced by mercury, however, the effects will be found to be just inverted. The mercury is depressed in all the tubes, the depression being greater in proportion as the tube is smaller [Fig. 107, (1)]. This depression is most easily observed with a U-tube like that shown in Fig. 107, (2).

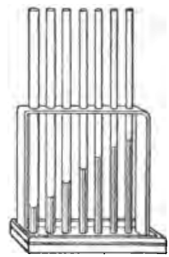


FIG. 106. Rise of liquids in capillary tubes

Experiments of this sort have established the following laws:

1. *Liquids rise in capillary tubes when they are capable of wetting them, but are depressed in tubes which they do not wet.*

2. *The elevation in the one case and the depression in the other are inversely proportional to the diameters of the tubes.*

It will be noticed, too, that when a liquid rises, its surface within the tube is concave upward, and when it is depressed its surface is convex upward.

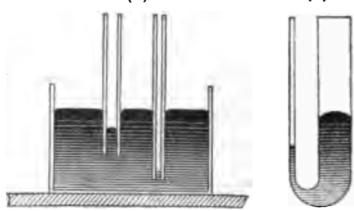


FIG. 107. Depression of mercury in capillary tubes

144. Cause of curvature of a liquid surface in a capillary tube. All of the effects presented in the last paragraph can be explained by a consideration of cohesive and adhesive forces. However, throughout the explanation we must keep in mind two familiar facts: first, that *the surface of a body of water at rest, for example a pond, is at right angles to the resultant force, that is, gravity, which acts upon it;* and second, that *the force of gravity acting on a minute amount of liquid is negligible in comparison with its own cohesive force* (see § 141).

Consider, then, a very small body of liquid close to the point *o* (Fig. 108), where water is in contact with the glass wall of the tube. Let the quantity of liquid considered be so minute that the force of gravity acting upon it may be disregarded. The force of *adhesion* of the wall will pull the

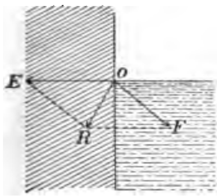


Fig. 108

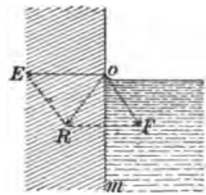


Fig. 109

Condition for elevation of a liquid near a wall

liquid particles at *o* in the direction *oE*. The force of *cohesion* of the liquid will pull these same particles in the direction *oF*. The resultant of these two pulls on the liquid at *o* will then be represented by *oR* (Fig. 108), in accordance with the parallelogram law of Chapter V. If, then, the adhesive force *oE* exceeds the cohesive force *oF*, the direction *oR* of the resultant force will lie to the left of the vertical *om* (Fig. 109), in which case, since the surface of a liquid always assumes a position at right angles to the resultant force, it must rise up against the wall as water does against glass (Fig. 109).

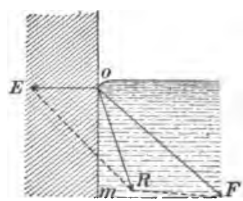


Fig. 110. Condition for the depression of a liquid near a wall

If the cohesive force *oF* (Fig. 110) is strong in comparison with the adhesive force *oE*, the resultant *oR* will fall to the right of the vertical, in which case the liquid must be depressed about *o*.

Whether, then, a liquid will rise against a solid wall or be depressed by it will depend only on the relative strengths of the adhesion of the wall for the liquid and the cohesion of the

liquid for itself. Since mercury does not wet glass, we know that cohesion is here relatively strong, and we should expect, therefore, that the mercury would be depressed, as indeed we find it to be. The fact that water will wet glass indicates that in this case adhesion is relatively strong, and hence we should expect water to rise against the walls of the containing vessel, as in fact it does.

It is clear that a liquid which is depressed near the edge of a vertical solid wall must assume within a tube a surface which is *convex upward*, while a liquid which rises against a wall must within such a tube be *concave upward.*

145. Explanation of ascension and depression in capillary tubes. As soon as the curvatures just mentioned are produced, the concave surface *aob* (Fig. 111) tends, by virtue of surface

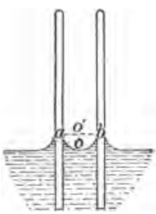


FIG. 111

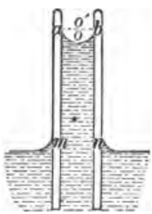


FIG. 112

A concave meniscus causes a rise in capillary tube

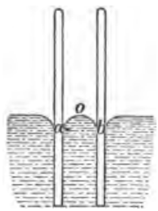


FIG. 113

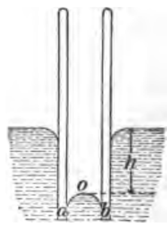


FIG. 114

A convex meniscus causes a fall

tension, to straighten out into the flat surface *ao'b*. But it no sooner thus begins to straighten out than adhesion again elevates it at the edges. It will be seen, therefore, that the liquid must continue to rise in the tube until the weight of the volume of liquid lifted, namely *amnb* (Fig. 112), balances the tendency of the surface *aob* to flatten out. That the liquid will rise higher in a small tube than in a large one is to be expected, since the weight of the column of liquid to be supported in the small tube is less.

Similarly, the convex mercury surface *aob* (Fig. 113) falls until the upward pressure at *o*, due to the depth *h* of mercury (Fig. 114), balances the tendency of the surface *aob* to flatten out.

146. Capillary phenomena in everyday life. Capillary phenomena play a very important part in the processes of nature and of everyday life. Thus the rise of oil in wicks of lamps, the complete wetting of a towel when one end of it is allowed to stand in a basin of water, the rapid absorption of liquid by a lump of sugar when one corner of it only is immersed, the taking up of ink by blotting paper, are all illustrations of precisely the same phenomena which we observe in the capillary tubes of Fig. 106.

147. Floating of small objects on water. Let a needle be laid very carefully on the surface of a dish of water. In spite of the fact that it is nearly eight times as dense as water it will be found to float. If the needle has been previously magnetized, it may be made to move about in any direction over the surface in obedience to the pull of a magnet held, for example, underneath the table.

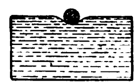


FIG. 115. Cross section of a floating needle

To discover the cause of this apparently impossible phenomenon, examine closely the surface of the water in the immediate neighborhood of the needle. It will be found to be depressed in the manner shown in Fig. 115. This furnishes at once the explanation. So long as the needle is so small that its own weight is no greater than the upward force exerted upon it by the tendency of the depressed (and therefore concave) liquid surface to straighten out into a flat surface, the needle could not sink in the liquid, no matter how great its density. If the water had wet the needle, that is, if it had risen about the needle instead of being depressed, the tendency of the liquid surface to flatten out would have pulled it down into the liquid instead of forcing it upward. Any body about

which a liquid is depressed will therefore float on the surface of the liquid if its mass is not too great. Even if the liquid tends to rise about a body when it is perfectly clean, an imperceptible film of oil upon the body will cause it to depress the liquid, and hence to float.

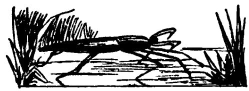


FIG. 116. Insect walking on surface of water

The above experiment explains the familiar phenomenon of insects walking and running on the surface of water (Fig. 116) in apparent contradiction to the law of Archimedes, in accordance with which they should sink until they displace their own weight of the liquid.

QUESTIONS AND PROBLEMS

1. Shot are made by pouring molten lead through a sieve on top of a tall tower and catching it in water at the bottom. Why are they spherical?

2. Would mercury ascend a lamp wick as oil and water do?

3. If water will rise 32 cm. in a tube .1 mm. in diameter, how high will it rise in a tube .01 mm. in diameter?

4. Candle grease may be removed from clothing by covering it with blotting paper and then passing a hot flatiron over the paper. Explain.

5. Why does a small stream of water break up into drops instead of falling as a continuous thread?

6. Why will a piece of sharp-cornered glass become rounded when heated to redness in a Bunsen flame?

7. The leads for pencils are made by subjecting powdered graphite to enormous pressures produced by hydraulic machines. Explain how the pressure changes the powder to a coherent mass.

8. Float two matches an inch apart. Touch the water between them with a hot wire. The matches will spring apart. What does this show about the effect of temperature on surface tension?

9. Repeat the experiment, touching the water with a wire moistened with alcohol. What do you infer as to the relative surface tensions of alcohol and water?

10. Rub a little soap on one end of half a toothpick and lay it upon the surface of a large vessel of clean still water. Explain the observed motion.

Absorption of Gases by Solids and Liquids

148. Absorption of gases by solids. Let a large test tube be filled with ammonia gas by heating aqua ammonia and causing the evolved gas to displace mercury in the tube, as in Fig. 117. Let a piece of charcoal an inch long and nearly as wide as the tube be heated to redness and then plunged beneath the mercury. When it is cool let it be slipped underneath the mouth of the test tube and allowed to rise into the gas. The mercury will be seen to rise in the tube, as in Fig. 118, thus showing that the gas is being absorbed by the charcoal. If the gas is unmixed with air, the mercury will rise to the very top of the tube, thus showing that all the ammonia has been absorbed by the charcoal.

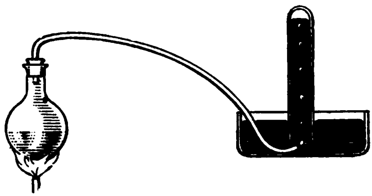


Fig. 117. Filling tube with ammonia

This property of absorbing gases is possessed to a notable degree by porous substances, such as meerschaum, gypsum, charcoal, etc., especially coconut charcoal. It is not improbable that all solids hold, closely adhering to their surfaces, thin layers of the gases with which they are in contact, and that the prominence of the phenomena of absorption in porous substances is due to the great extent of surface possessed by such substances.

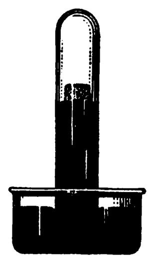


Fig. 118. Absorption of ammonia gas by charcoal

That the same substance exerts widely different attractions upon the molecules of different gases is shown by the fact that charcoal will absorb 90 times its own volume of ammonia gas, 35 times its volume of carbon dioxide, and but 1.7 times its volume of hydrogen. The usefulness of charcoal as a deodorizer is due to its enormous ability to absorb certain kinds of gases.

149. Absorption of gases in liquids. Let a beaker containing cold water be slowly heated. Small bubbles of air will be seen to collect

†

in great numbers upon the walls and to rise through the liquid to the surface. That they are indeed bubbles of air and not of steam is proved first by the fact that they appear when the temperature is far below boiling, and second by the fact that they do not condense as they rise into the higher and cooler layers of the water.

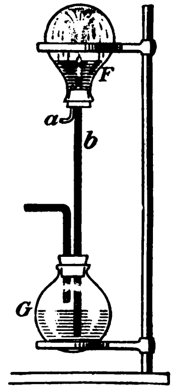


FIG. 119. Absorption of ammonia by water

The experiment shows two things: first, that water ordinarily contains considerable quantities of air dissolved in it; and second, that the amount of air which water can hold decreases as the temperature rises. The first point is also proved by the existence of fish life; for fishes obtain the oxygen which they need to support life, not immediately from the water, but from the air which is dissolved in it.

The amount of gas which will be absorbed by water varies greatly with the nature of the gas. At 0° C. and a pressure of 76 centimeters 1 cubic centimeter of water will absorb 1050 cubic centimeters of ammonia, 1.8 cubic centimeters of carbon dioxide, and but .04 cubic centimeter of oxygen. Ammonia itself is a gas under ordinary conditions. The commercial aqua ammonia is simply ammonia gas dissolved in water.

The following experiment illustrates the absorption of ammonia by water:

Let the flask *F* (Fig. 119) and tube *b* be filled with ammonia by passing a current of the gas in at *a* and out through *b*. Then let *a* be corked up and *b* thrust into *G*, a flask nearly filled with water which has been colored slightly red by the addition of litmus and a drop or two of acid. As the ammonia is absorbed the water will slowly rise in *b*, and as soon as it reaches *F* it will rush up very rapidly until the upper flask is nearly full. At the same time the color will change from red to blue because of the action of the ammonia upon the litmus.

Experiment shows that *in every case of absorption of a gas by a liquid or a solid, the quantity of gas absorbed decreases with*

an increase in temperature — a result which was to have been expected from the kinetic theory, since increasing the molecular velocity must of course increase the difficulty which the adhesive forces have in retaining the gaseous molecules.

150. Effect of pressure upon absorption. Soda water is ordinary water which has been made to absorb large quantities of carbon dioxide gas. This impregnation is accomplished by bringing the water into contact with the gas under high pressure. As soon as the pressure is relieved the gas passes rapidly out of solution. This is the cause of the characteristic effervescence of soda water. These facts show clearly that the amount of carbon dioxide which can be absorbed by water is greater for high pressures than for low. As a matter of fact, careful experiments have shown that the amount of any gas absorbed is directly proportional to the pressure, so that if carbon dioxide under a pressure of 10 atmospheres is brought into contact with water, ten times as much of the gas is absorbed as if it had been under a pressure of 1 atmosphere.

QUESTIONS AND PROBLEMS

1. Capillary action is much more effective in bringing moisture to the surface in tightly packed soil than in loose soil where the spaces between the earth particles are much greater. Why, then, is it advantageous to crops to keep the surface loose (dry farming)?

2. Why do fishes in an aquarium die if the water is not frequently renewed?

3. Explain the apparent generation of ammonia gas when aqua ammonia is heated.

4. Why in the experiment illustrated in Fig. 119 was the flow so much more rapid after the water began to run over into F?

5. How can you tell whether bubbles which rise from the bottom of a vessel which is being heated are bubbles of air or bubbles of steam?

CHAPTER VII

THERMOMETRY; EXPANSION COEFFICIENTS *

THERMOMETRY

151. Meaning of temperature. When a body feels hot to the touch we are accustomed to say that it has a *high temperature*, and when it feels cold that it has a *low temperature*. Thus the word "temperature" is used to denote the condition of hotness or coldness of the body whose state is being described.

152. Measurement of temperature. So far as we know, up to the time of Galileo no one had ever used any special instrument for the measurement of temperature. People knew how hot or how cold it was from their feelings only. But under some conditions this temperature sense is a very unreliable guide. For example, if the hand has been in hot water, tepid water will feel cold; while if it has been in cold water, the same tepid water will feel warm; a room may feel hot to one who has been running, while it will feel cool to one who has been sitting still.

Difficulties of this sort have led to the introduction in modern times of mechanical devices, called *thermometers*, for measuring temperature. These instruments depend for their operation upon the fact that practically all bodies expand as they grow hot.

153. Galileo's thermometer. It was in 1592 that Galileo, at the University of Padua in Italy, constructed the first thermometer. He was familiar with the facts of expansion

* It is recommended that this chapter be preceded by laboratory measurements on the expansions of a gas and a solid. See, for example, Experiments 14 and 15 of the authors' manual.

of solids, liquids, and gases; and since gases expand more than solids or liquids, he chose a gas as his expanding substance. His device was that shown in Fig. 120.

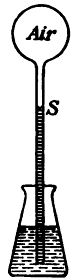


Fig. 120. Galileo's thermometer

The relative hotness of two bodies was compared by observing which one of the two, when placed in contact with the air bulb, caused the liquid to descend farther in the stem *S*. As a matter of fact, barometric as well as temperature changes cause changes in the height of the liquid in the stem of such an instrument, but Galileo does not seem to have been aware of this fact.

It was not until about 1700 that mercury thermometers were invented. On account of their extreme convenience these have now replaced all others for practical purposes.

154. The construction of a centigrade mercury thermometer. The meaning of a degree of temperature change is best understood from a description of the method of making and graduating a mercury thermometer.

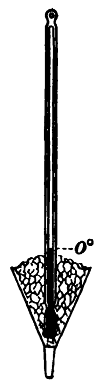


Fig. 121. Method of finding the 0° point of a thermometer

A bulb is blown at one end of a piece of thick-walled glass tubing of small, uniform bore. Bulb and tube are then filled with mercury, at a temperature slightly above the highest temperature for which the thermometer is to be used, and the tube is sealed off in a hot flame. As the mercury cools, it contracts and falls away from the top of the tube, leaving a vacuum above it.

The bulb is next surrounded with melting snow or ice, as in Fig. 121, and the point at which the mercury stands in the tube is marked 0°. Then the bulb and tube are placed in the steam

rising from boiling water, as in Fig. 122, and the new position of the mercury is marked 100°. The space between these two marks on the stem is then divided into 100 equal parts, and divisions of the same length are extended above the 100° mark and below the 0° mark.

One degree of change in temperature, measured on such a thermometer, means, then, such a temperature change as will cause the mercury in the stem to move over one of these divisions; that is, it is such a temperature change as will cause mercury contained in a glass bulb to expand $\frac{1}{100}$ of the amount which it expands in passing from the temperature of melting ice to that of boiling water. A thermometer in which the scale is divided in this way is called a centigrade thermometer.

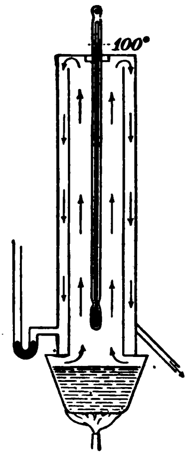


FIG. 122. Method of finding the 100° point of a thermometer.

Thermometers graduated on the centigrade scale are used almost exclusively in scientific work, and also for ordinary purposes in most countries which have adopted the metric system. This scale was first devised in 1742 by Celsius, of Upsala, Sweden. For this reason it is sometimes called the Celsius instead of the centigrade scale.

155. Fahrenheit thermometers. The common household thermometer in England and the United States differs from the centigrade only in the manner of its graduation. In its construction the temperature of melting ice is marked 32° instead of 0°, and that of boiling water 212° instead of 100°. The intervening stem is then divided into 180 parts. The zero of this scale is the temperature obtained by mixing equal weights of sal ammoniac (ammonium chloride) and snow. In 1714, when Fahrenheit, of Danzig, Germany, devised this scale, he chose this zero because he thought it represented the lowest possible temperature, that is, the entire absence of heat.

156. Comparison of centigrade and Fahrenheit thermometers. From the methods of graduation of the Fahrenheit and centigrade thermometers it will be seen that 100° on the centigrade scale denotes the same difference of temperature as 180° on the Fahrenheit scale (Fig. 123). Hence one Fahrenheit degree is equal to five ninths of a centigrade degree, and one centigrade degree is equal to nine fifths of a Fahrenheit degree. Hence *to reduce from the Fahrenheit to the centigrade scale, first find how many Fahrenheit degrees the given temperature is above or below the freezing temperature, and then multiply by five ninths.*

To reduce from centigrade to Fahrenheit, first multiply by nine fifths in order to find how many Fahrenheit degrees the given temperature is above or below the freezing temperature. Knowing how far it is from the freezing point, it will be very easy to find how far it is from 0° *F., which is* 32° *below the freezing point.*

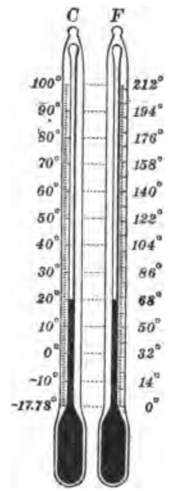


FIG. 123. The centigrade and Fahrenheit scales

157. The range of the mercury thermometer. Since mercury freezes at – 39° C., temperatures lower than this are very often measured by means of *alcohol* thermometers, for the freezing point of alcohol is – 130° C. Similarly, since the boiling point of mercury is 360° C., mercury thermometers cannot be used for measuring very high temperatures. For both very high and very low temperatures, in fact for all temperatures, a *gas* thermometer is the standard instrument.

158. The standard hydrogen thermometer. The modern gas thermometer (Fig. 124) is, however, widely different from that devised by Galileo (Fig. 120). It is not usually the increase in the volume of a gas kept under constant pressure which is taken as the measure of temperature change, but

rather the increase in pressure which the molecules of a confined gas exert against the walls of a vessel whose volume is kept constant. The essential features of the method of calibration and use of the standard hydrogen thermometer at the International Bureau of Weights and Measures at Paris are as follows:

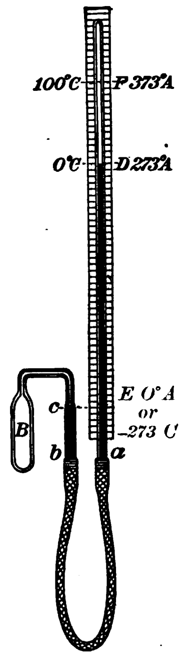


FIG. 124. The standard gas thermometer

The bulb *B* (Fig. 124) is first filled with hydrogen and the space above the mercury in the tube *a* made as nearly a perfect vacuum as possible. *B* is then surrounded with melting ice (as in Fig. 121) and the tube *a* raised or lowered until the mercury in the arm *b* stands exactly opposite the fixed mark *c* on the tube. Now, since the space above *D* is a vacuum, the pressure exerted by the hydrogen in *B* against the mercury surface at *c* just supports the mercury column *ED*. The point *D* is marked on a strip of metal behind the tube *a*. The bulb *B* is then placed in a steam bath like that shown in Fig. 122. The increased pressure of the gas in *B* at once begins to force the mercury down at *c* and up at *D*. But by raising the arm *a* the mercury in *b* is forced back again to *c*, the increased pressure of the gas on the surface of the mercury at *c* being balanced by the increased height of the mercury column supported, which is now *EF* instead of *ED*. When the gas in *B* is thoroughly heated to the temperature of the steam, the arm *a* is very carefully adjusted so that the mercury in *b* stands very exactly at *c*, its original level. A second mark is then placed on the metal strip exactly opposite the new level of the mercury, that is, at *F*. *D* is then marked 0° C., and *F* is marked 100° C. The vertical distance between these marks is divided into 100 exactly equal parts. Divisions of exactly the same length are carried above the 100° mark and below the 0° mark. One degree of change in temperature is then defined as any change in temperature which will cause the pressure of the gas in *B* to change by the amount represented by the distance between any two of these divisions. This distance is found to be $\frac{1}{273}$ of the height *ED*.

In other words, *one degree of change in temperature is such a temperature change as will cause the pressure exerted by a confined gas to change by* $\frac{1}{273}$ *of its value at the temperature of melting ice* (0° C.).

159. Absolute temperature. Since, then, cooling the hydrogen through 1° C., as defined above, reduces the pressure $\frac{1}{273}$ of its value at 0° C., it is clear that cooling it 273° below 0° C. would reduce its pressure to nothing. But from the standpoint of the kinetic theory this would be the temperature at which all motions of the hydrogen molecules would cease. This temperature is called the *absolute zero* and the temperature measured from this zero is called *absolute temperature.* Thus, if A is used to denote the absolute scale, we have 0° C. = 273° A., 100° C. = 373° A., 15° C. = 288° A., etc. It is customary to indicate temperatures on the centigrade scale by t, on the absolute scale by T. We have then

$$T = t + 273. \qquad (1)$$

160. Comparison of gas and mercury thermometers. Since an international committee has chosen the hydrogen thermometer described in § 158 as the standard of temperature measurement, it is important to know whether mercury thermometers, graduated in the manner described in § 154, agree with gas thermometers at temperatures other than 0° and 100°, where, of course, they must agree, since these temperatures are in each case the starting points of the graduation. A careful comparison has shown that although they do not agree exactly, yet fortunately the disagreements at ordinary temperatures are small, not amounting to more than .2° anywhere between 0° and 100°. At 300° C., however, the difference amounts to about 4°. (Mercury thermometers are actually used for measuring temperatures above the boiling point of mercury, 360° C. They are then filled with nitrogen, the pressure of which prevents boiling.)

Hence for all ordinary purposes mercury thermometers are sufficiently accurate, and no special standardization of them is necessary. But in all scientific work, if mercury thermometers are used at all, they must first be compared with a gas thermometer and a table of corrections obtained. The errors of an alcohol thermometer are considerably larger than those of a mercury thermometer.

161. Low temperatures. The absolute zero of temperature can, of course, never be attained, but in recent years rapid strides have been made toward it. Forty years ago the lowest temperature which had ever been measured was −110° C., the temperature attained by Faraday in 1845 by causing a mixture of ether and solid carbon dioxide to evaporate in a vacuum. But in 1880 air was first liquefied, and found, by means of a gas thermometer, to have a temperature of −180° C. When liquid air evaporates into a space which is kept exhausted by means of an air pump, its temperature falls to about −220° C. Recently hydrogen has been liquefied and found to have a temperature at atmospheric pressure of −243° C. All of these temperatures have been measured by means of hydrogen thermometers. By allowing liquid hydrogen to evaporate into a space kept exhausted by an air pump, Dewar in 1900 attained a temperature of −260°. In 1911 Kamerlingh Onnes liquefied helium and attained a temperature of −271.3° C., only 1.7° above absolute zero (see § 92).

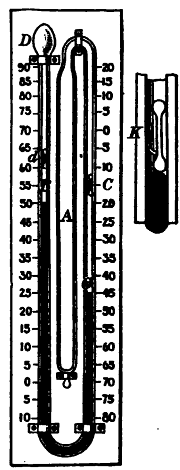
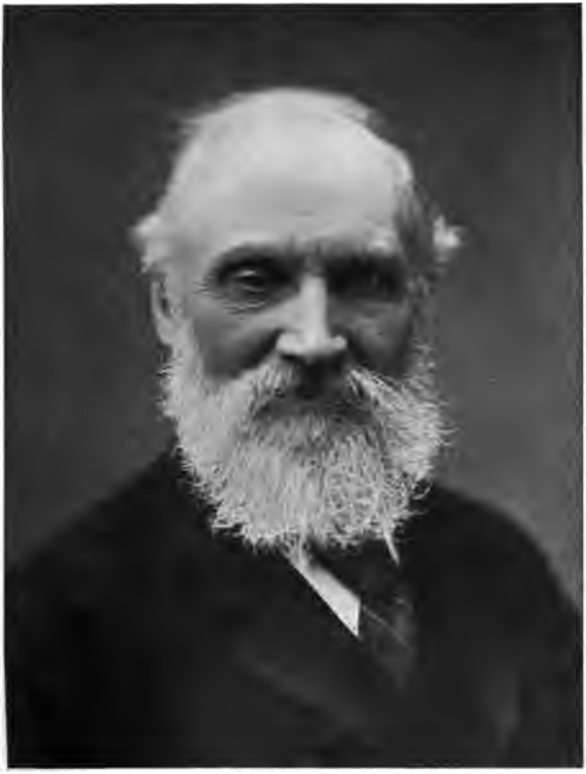


FIG. 125. The maximum and minimum thermometer

162. Maximum and minimum thermometers. In all weather bureaus the lowest temperature reached during the night, and the highest temperature reached during the day, are automatically recorded by a special device called a maximum and minimum thermometer. The construction of one form of this instrument is shown in Fig. 125. The bulb *A* and the stem down to the point *G* are filled with alcohol, from *G* to *B* the stem is filled with mercury, while the liquid above *B* is again alcohol. The bulb *D* contains only alcohol and its vapor. The two indices *d* and *C* move with slight friction in the stem. As the temperature falls, the alcohol in *A* contracts and the mercury pushes up the index on the right and leaves it opposite the mark corresponding to the lowest temperature reached. As the temperature



Sir William Thomson, Lord Kelvin (1824–1907)

One of the best known and most prolific of nineteenth-century physicists; born in Belfast, Ireland; professor of physics in Glasgow University, Scotland, for more than fifty years; especially renowned for his investigations in heat and electricity; originator of the absolute thermodynamic scale of temperature; formulator of the second law of thermodynamics; inventor of the electrometer, the mirror galvanometer, and many other important electrical devices

rises, the alcohol in A expands and the mercury pushes up the index on the left and leaves it opposite the mark corresponding to the highest temperature reached. In order to obtain the right amount of friction, a small steel spring is attached to the indices, as in K. After each observation the observer pulls the index back to contact with the mercury by means of a small magnet.

QUESTIONS AND PROBLEMS

1. Normal room temperature is 68° F. What is it centigrade?
2. The normal temperature of the human body is 98.4° F. What is it centigrade?
3. What temperature centigrade corresponds to 0° F.?
4. Mercury freezes at − 40° F. What is this centigrade?
5. The temperature of liquid air is −180° C. What is it Fahrenheit?
6. The lowest temperature attainable by evaporating liquid helium is − 271.4° C. What is it Fahrenheit?
7. What is the absolute zero of temperature on the Fahrenheit scale?
8. Why is a fever thermometer made with a very long cylindrical bulb, instead of a spherical one?
9. When the bulb of a thermometer is placed in hot water, it at first falls a trifle and then rises. Why?
10. How does the distance between the 0° mark and the 100° mark vary with the size of the bore, the size of the bulb remaining the same?
11. What is meant by the absolute zero of temperature?
12. Why is the temperature of liquid air lowered if it is placed under the receiver of an air pump and the air exhausted?
13. Two thermometers have bulbs of equal size. The bore of one has a diameter twice that of the other. What are the relative lengths of the stems between 0° and 100°?

EXPANSION COEFFICIENTS

163. The laws of Charles and Gay-Lussac. When, as in the experiment described in § 158, we keep the volume of a gas constant and observe the rate at which the pressure increases with rise in temperature, we obtain *the pressure coefficient of expansion, which is defined as the ratio between the increase in pressure per degree and the value of the pressure at* 0° *C.* This was first done for different gases by a Frenchman, Charles,

in 1787, who found that *the pressure coefficients of expansion of all gases are the same.* This is known as *the law of Charles.*

When we arrange the experiment so that the gas can expand as the temperature rises, the pressure remaining constant, we obtain *the volume coefficient of expansion, which is defined as the ratio between the increase in volume per degree and the total volume of the gas at 0° C.* This was first done for different gases in 1802 by another Frenchman, Gay-Lussac, who found that *all gases have the same volume coefficient of expansion,* this coefficient being the same as the pressure coefficient, namely 1/273. This is known as *the law of Gay-Lussac.*

From the definition of absolute temperature and Charles's law we learn that for all gases at constant volume, *pressure is proportional to absolute temperature;* that is,

$$\frac{P_1}{P_2}=\frac{T_1}{T_2}. \tag{2}$$

Also from Gay-Lussac's law we learn that for all gases at constant pressure, *volume is proportional to absolute temperature;* that is,

$$\frac{V_1}{V_2}=\frac{T_1}{T_2}. \tag{3}$$

If pressure, temperature, and volume all vary,* we have

$$\frac{P_1V_1}{P_2V_2}=\frac{T_1}{T_2}. \tag{4}$$

Any one of these six quantities may be found if the other five are known.

If the volume remains constant, that is, if $V_1 = V_2$, equation (4) reduces to (2); that is, to Charles's law. If the pressure

* If this is not clear to the student, let him recall that if the speeds of two runners are the same, then their distances are proportional to their times, that is, $D_1/D_2 = t_1/t_2$; but if their times are the same and the speeds different, $D_1/D_2 = s_1/s_2$. If now one runs both twice as fast and twice as long, he evidently goes 4 times as far, that is, if time and speed both vary, $D_1/D_2 = t_1s_1/t_2s_2$.

remains constant, $P_1 = P_2$ and equation (4) reduces to (3); that is, to Gay-Lussac's law. If the temperature does not change, $T_1 = T_2$ and equation (4) reduces to $P_1 V_1 = P_2 V_2$; that is, to Boyle's law. If the relation of densities instead of volumes are sought, it is only necessary to replace $\frac{V_1}{V_2}$ in (3) and (4) by $\frac{D_2}{D_1}$.

QUESTIONS AND PROBLEMS

1. What fractional part of the air in a room passes out when the air in it is heated from $-15°$ C. to $20°$ C.? ($-15°$ C. = $258°$ A.; $20°$ C. = $293°$ A.)

2. Why is it unsafe to let a pneumatic inkstand like that of Fig. 32, p. 32, remain in the sun? What changes will occur in the volume of the confined gas if it is heated from 15° C. to 40° C.?

3. To what temperature must a cubic foot of gas initially at 0° C. be raised in order to double its volume, the pressure remaining constant?

4. If the air within a bicycle tire is under a pressure of 2 atmospheres, that is, 152 cm. of mercury, when the temperature is 10° C., what pressure will exist within the tube when the temperature changes to 35° C.?

5. If the pressure to which 15 cc. of air is subjected changes from 76 cm. to 40 cm., the temperature remaining constant, what does its volume become? (See Boyle's law, p. 35.) If, then, the temperature of the same gas changes from 15° C. to 100° C., the pressure remaining constant, what will be the final volume?

6. If the volume of a gas at 20° C. and 76 cm. pressure is 500 cc., what is its volume at 50° C. and 70 cm. pressure?

Expansion of Liquids and Solids

164. The expansion of liquids. The expansion of liquids differs from that of gases in that

1. The coefficients of expansion of liquids are all considerably smaller than those of gases.

2. Different liquids expand at wholly different rates; for example, the coefficient of alcohol between 0° and 10° C. is .0011; of ether it is .0015; of petroleum, .0009.

3. The same liquid often has different coefficients at different temperatures; that is, the expansion is irregular. Thus,

if the coefficient of alcohol is obtained between 0° and 60° C., instead of between 0° and 10° C., it is .0013 instead of .0011.

The coefficient of mercury, however, is very nearly constant through a wide range of temperature, which indeed might have been inferred from the fact that mercury thermometers agree so well with gas thermometers.

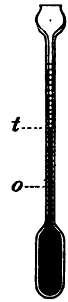


FIG. 126. Bulb for investigating expansions of liquids

165. Method of measuring the expansion coefficients of liquids. One of the most convenient and common methods of measuring the coefficients of liquids is to place them in bulbs of known volume, provided with capillary necks of known diameter, like that shown in Fig. 126, and then to watch the rise of the liquid in the neck for a given rise in temperature. A certain allowance must be made for the expansion of the bulb, but this can readily be done if the coefficient of expansion of the substance of which the bulb is made is known.

166. Maximum density of water. When water is treated in the way described in the preceding paragraph, it reaches its lowest position in the stem at 4° C. As the temperature falls from that point down to 0° C., water exhibits the peculiar property of *expanding* with a *decrease* in temperature.

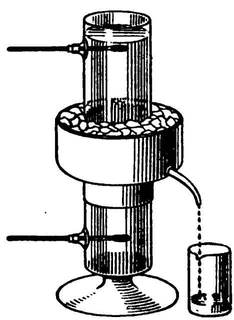


FIG. 127. Maximum density of water

This may be shown experimentally by surrounding for an hour or more a vessel of water with a freezing mixture of ice and salt as in Fig. 127. The lower thermometer will first fall to 4° C. and remain there, after which the upper thermometer will fall rapidly, showing that water colder than 4° C. is now rising instead of falling.

We learn, therefore, that *water has its maximum density at a temperature of* 4° *C.*

167. The cooling of a lake in winter. The preceding paragraph makes it easy to understand the cooling of any large body of water with the approach of winter. The surface layers are first cooled and contract. Hence, being then heavier than the lower layers, they sink and are replaced by the warmer water from beneath. This process of cooling at the surface, and sinking, goes on until the whole body of water has reached a temperature of 4° C. After this condition has been reached, further cooling of the surface layers makes them *lighter* than the water beneath, and they now remain on top until they freeze. Thus, before any ice whatever can form on the surface of a lake, the whole mass of water to the very bottom must be cooled to 4° C. This is why it requires a much longer and more severe period of cold to freeze deep bodies of water than shallow ones. Further, since the circulation described above ceases at 4° C., practically all of the unfrozen water will be at 4° C. even in the coldest weather. Only the water which is in the immediate neighborhood of the ice will be lower than 4° C. This fact is of vital importance in the preservation of aquatic life.

168. Linear coefficients of expansion of solids. It is often more convenient to measure the increase in *length* of one edge of an expanding solid than to measure its increase in *volume. The ratio between the increase in length per degree rise in temperature and the total length is called the linear coefficient of expansion of the solid.* Thus, if l_1 represent the length of a bar at $t_1°$, and l_2 its length at $t_2°$, the equation which defines the linear coefficient k is

$$k = \frac{l_2 - l_1}{l_1 (t_2 - t_1)}. \tag{5}$$

Fig. 128 illustrates the method now in use at the International Bureau of Weights and Measures for obtaining these coefficients. The two microscopes which are mounted in fixed

positions upon heavy piers are focused upon scratches near the ends of the bar whose coefficient is to be obtained. The temperature of the water is then changed from, say, 0° C. to 10° C., and the amount of elongation of the bar is determined from the observed amounts of motion of its ends as seen through the microscopes.

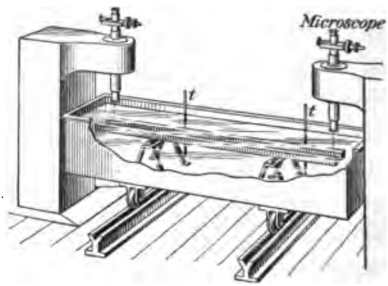


FIG. 128. Apparatus for determination of linear coefficients of expansion

The linear coefficients of a few common substances are given in the following table:

Aluminium	.000023	Glass	.000009	Silver	.000019
Brass	.000018	Iron	.000012	Steel	.000011
Copper	.000017	Lead	.000029	Tin	.000022
Gold	.000014	Platinum	.000009	Zinc	.000029

APPLICATIONS OF EXPANSION

169. Compensated pendulum. Since a long pendulum vibrates more slowly than a short one, the expansion of the rod which carries the pendulum bob causes an ordinary clock to run too slowly in summer, and its contraction causes it to run too fast in winter. For this reason very accurate clocks are provided with *compensated* pendulums, which are so constructed that the distance of the bob beneath the point of support is independent of the temperature. This is accomplished by suspending the bob by means of two sets of rods of different material, in such a way that the expansion of one set raises the bob, while the expansion of the other set lowers it. Such a pendulum is shown in Fig. 129.

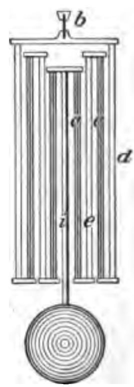


FIG. 129. The compensated pendulum

The expansion of the iron rods *b*, *d*, *e*, and *i* tends to lower the bob, while that of the copper rods *c* tends to raise it. In order to produce complete compensation it is only necessary to make the total lengths of iron and copper rods inversely proportional to the coefficients of expansion of iron and copper.

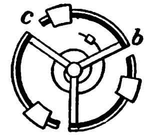


FIG. 130. The compensated balance wheel

170. Compensated balance wheel. In the balance wheel of an accurate watch (Fig. 130) another application of the unequal expansion of metals is made. Increase in temperature both increases the radius of the wheel and weakens the elasticity of the spring which controls it. Both of these effects tend to make the watch lose time. This tendency may be counteracted by bringing the mass of the rotating parts in toward the center of the wheel. This is accomplished by making the arcs *bc* of metals of different expansion coefficients, the inner metal, shown in black in the figure, having the smaller coefficient. The weighted ends of the arcs are then sufficiently pulled in by a rise in temperature to counteract the retarding effects.

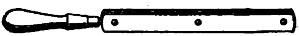


FIG. 131


FIG. 132

Unequal expansion of metals

The principle is precisely the same as that which finds simple illustration in the compound bar shown in Fig. 131. This bar consists of two strips, one of brass and one of iron, riveted together. When the bar is placed edgewise in a Bunsen flame, so that both metals are heated equally, it will be found to bend in such a way that the more expansible metal, namely the brass, is on the outside of the curve, as shown in Fig. 132. When it is cooled with snow or ice it bends in the opposite direction.

FIG. 133. The thermostat

The common thermostat (Fig. 133) is precisely such a bar which is arranged so as to open the drafts

†

through closing an electrical circuit at *a* when it is too cold, and to close the drafts through making contact at *b* when it is too warm.

171. The dial thermometer. The dial thermometer is a compound metallic ribbon wound in helical form. One end *a* of the helix [Fig. 134, (2)] is fixed, while the other end is attached to a lever arm *b*, the motion of which rotates the pointer *d* over the dial [Fig. 134, (1)], which is graduated by comparison with a mercury thermometer. The more expansible metal is on the outside. Hence rise in temperature causes the helix to wind up closer, the index moving to the right.

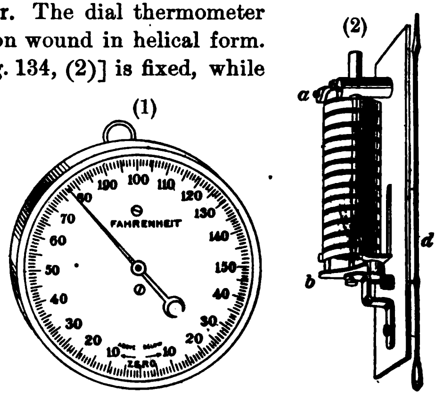


FIG. 134. The dial thermometer

QUESTIONS AND PROBLEMS

1. What is the temperature of the water at the bottom of a lake in very cold weather? Test the temperature of your tap water in winter. Explain.

2. Why is a thick tumbler more likely to break when hot water is poured into it than a thin one?

3. Why may a glass stopper sometimes be loosened by pouring hot water on the neck of a bottle?

4. If an iron steam pipe is 60 ft. long at 0° C., what is its length when steam passes through it at 100° C.?

5. Pendulums are often compensated by using cylinders of mercury as in Fig. 135. Explain.

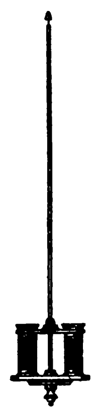


FIG. 135

6. The steel cable from which Brooklyn Bridge hangs is more than a mile long. By how many feet does a mile of its length vary between a winter day when the temperature is – 20° C., and a summer day when it is 30° C.?

7. The changes in temperature to which long lines of steam pipes are subjected make it necessary to introduce "expansion joints." These joints consist of brass collars fitted tightly by means of packing over the separated ends of two adjacent lengths of pipe. If the pipe is of iron, and such a joint is inserted every 200 ft., and if the range of temperature which must be allowed for is from – 30° C. to 125° C., what is the minimum play which must be allowed for at each expansion joint?

CHAPTER VIII

WORK AND MECHANICAL ENERGY*

Definition and Measurement of Work

172. Definition of work. Whenever a force *moves* a body on which it acts, it is said to do work upon that body; and the amount of the work accomplished is measured by the product of the force acting and the distance through which it moves the body. Thus, if 1 gram of mass is lifted 1 centimeter in a vertical direction, 1 gram of force has acted, and the distance through which it has moved the body is 1 centimeter. We say, therefore, that the lifting force has accomplished 1 *gram centimeter* of work. If the gram of force had lifted the body upon which it acted through 2 centimeters, the work done would have been 2 gram centimeters. If a force of 3 grams had acted and the body had been lifted through 3 centimeters, the work done would have been 9 gram centimeters, etc. Or, in general, if W represent the work accomplished, F the value of the acting force, and s the distance through which its point of application moves, then the definition of work is given by the equation

$$W = F \times s. \tag{1}$$

In the scientific sense no work is ever done unless the force succeeds in *producing motion* in the body on which it acts. A pillar supporting a building does no work; a man tugging at a stone, but failing to move it, does no work. In the popular

*It is recommended that this chapter be preceded by an experiment in which the student discovers for himself the law of the lever, that is, the principle of moments (see, for example, Experiment 16, authors' manual), and that it be accompanied by a study of the principle of work as exemplified in at least one of the other simple machines (see, for example, Experiment 17, authors' manual).

sense we sometimes say that we are doing work when we are simply holding a weight, or doing anything else which results in fatigue; but in physics the word "work" is used to describe not the effort put forth but the *effect accomplished*, as represented in equation (1).

173. Units of work. There are two common units of work in the metric system, the *gram centimeter* and the *kilogram meter.* As the names imply, the gram centimeter is the work done by a force of 1 gram when it moves the point on which it acts 1 centimeter. The kilogram meter is the work done by a kilogram of force when it moves the point on which it acts 1 meter. The *gram meter* is also sometimes used.

Corresponding to the English unit of force, the pound, is the unit of work, the *foot pound.* It is the work done by a "pound of force" when it moves the point on which it acts 1 foot. Thus it takes a foot pound of work to lift a pound of mass 1 foot high.

In the absolute system of units the dyne is the unit of force and the dyne centimeter, or *erg*, is the corresponding unit of work. The erg is the amount of work done by a force of 1 dyne when it moves the point on which it acts 1 centimeter. To raise 1 liter of water from the floor to a table 1 meter high would require $1000 \times 980 \times 100 = 98{,}000{,}000$ ergs of work. It will be seen, therefore, that the erg is an exceedingly small unit. For this reason it is customary to employ a unit which is equal to 10,000,000 ergs. It is called a *joule*, in honor of the great English physicist James Prescott Joule (1818–1889). The work done in lifting a liter of water 1 meter is therefore 9.8 joules.

QUESTIONS AND PROBLEMS

1. Analyze several types of manual labor and see if the above definition ($W = Fs$) holds for each. Is not $F \times s$ the thing *paid for* in every case?

2. How many foot pounds of work does a 150-lb. man do in climbing to the top of Mt. Washington, which is 6300 ft. high?

3. A horse pulls a metric ton of coal to the top of a hill 30 m. high. Express the work accomplished, first in kg. m., then in joules.

4. If the 20,000 inhabitants of a city use an average of 20 l. of water per day per capita, how many kilogram meters of work must the engines do per day, if the water has to be raised to a height of 75 m.?

Work expended upon and accomplished by Systems of Pulleys

174. The single fixed pulley. Let the force of the earth's attraction upon a mass F' be overcome by pulling upon a spring balance S, in the manner shown in Fig. 136, until F' moves slowly upward. If F' is 100 grams, the spring balance will also be found to register a force of 100 grams.

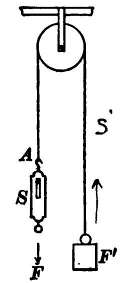


Fig. 136. The single fixed pulley

Experiment therefore shows that in the use of the single fixed pulley the acting force F which is producing the motion is equal to the resisting force F' which is opposing the motion.

Again, since the length of the string is always constant, the distance s through which the point A, at which F is applied, must move, is always equal to the distance s' through which the weight F' is lifted. Hence, if we consider the work put into the system at A, namely $F \times s$, and the work accomplished by the system at F', namely $F' \times s'$, we find obviously, since $F' = F$ and $s = s'$, that

$$Fs = F's'; \qquad (2)$$

that is, in the case of the single fixed pulley, *the work done by the acting force F (the effort) is equal to the work done against the resisting force F' (the resistance)*; or the work *put into* the machine at A is equal to the work *accomplished by* the machine at F'.

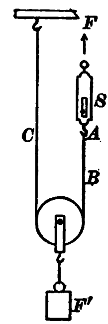


Fig. 137. The single movable pulley

175. The single movable pulley. Let now the force of the earth's attraction upon the mass F' be overcome by a single movable pulley, as shown in Fig. 137. Since the weight of F' (F' representing in this case the weight of both the pulley and the suspended mass) is now supported half by the strand C and half by the strand B, the force F which must act at A to hold the weight in place, or to move it slowly upward if there is no friction, should be only one half of F'. A reading of the balance will show that this is indeed the case.

Experiment thus shows that *in the case of the single movable pulley the effort F is just one half as great as the resistance F'.*

But when again we consider the *work* which the force F must do to lift the weight F' a distance s', we see that A must move upward 2 inches in order to raise F' 1 inch. For when F' moves up 1 inch both of the strands B and C must be shortened 1 inch. As before, therefore, since $F' = 2F$, and $s' = \frac{1}{2}s$,

$$F \times s = F' \times s';$$

that is, in the case of the single movable pulley, as in the case of the fixed pulley, *the work put into the machine by the effort F is equal to the work accomplished by the machine against the resistance F'.*

176. Combinations of pulleys. Let a weight F' be lifted by means of such a system of pulleys as is shown in Fig. 138, either (1) or (2). Here, since F' is supported by 6 strands of the cord, it is clear that the force which must be applied at A in order to hold F' in place, or to make it move slowly upward if there is no friction, should be but $\frac{1}{6}$ of F'.

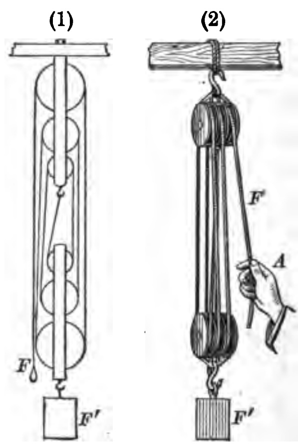


FIG. 138. Combinations of pulleys

The experiment will show this to be the case if the effects of friction, which are often very considerable, are eliminated by taking the mean of the forces which must be applied at F to cause it to move first slowly upward and then slowly downward. The law of any combination of movable pulleys may then be stated thus: *If n represent the number of strands between which the weight is divided,*

$$F = F'/n. \tag{3}$$

But when again we consider the work which the force F must do in order to lift the weight F' through a distance s', we see that, in order that the weight F' may be moved up through 1 inch, each of the strands must be shortened 1 inch, and hence the point A must move through n inches; that is, $s' = s/n$. Hence, ignoring friction, in this case also we have

$$F \times s = F' \times s';$$

that is, although the effort F is only $\frac{1}{n}$ of the resistance F', *the work put into the machine by the effort F is equal to the work accomplished by the machine against the resistance F'.*

177. Mechanical advantage. The above experiments show that it is sometimes possible, by applying a small force F, to overcome a much larger resisting force F'. *The ratio of the resistance F' to the effort F is called the mechanical advantage of the machine.* Thus the mechanical advantage of the single fixed pulley is 1, that of the single movable pulley is 2, that of the systems of pulleys shown in Fig. 138 is 6, etc.

If the acting force is applied at F' instead of at F, the mechanical advantage of the systems of pulleys of Fig. 138 is $\frac{1}{6}$; for it requires an application of 6 pounds at F' to lift 1 pound at F. But it will be observed that the resisting force at F now moves six times as fast and six times as far as the acting force at F'. We can thus either sacrifice speed to gain force, or sacrifice force to gain speed; but in every case, whatever we gain in the one we lose in the other. Thus in the hydraulic elevator shown in Fig. 13, p. 18, the cage moves only as fast as the piston; but in that shown in Fig. 14 it moves four times as fast. Hence the force applied to the piston in the latter case must be four times as great as in the former if the same load is to be lifted. This means that the diameter of the latter cylinder must be twice as great.

QUESTIONS AND PROBLEMS

1. Since the mechanical advantage of a single fixed pulley is only 1, why is it ever used?

2. If the hydraulic elevator of Fig. 14, p. 18, is to carry a total load of 20,000 lb., what force must be applied to the piston? If the water pressure is 70 lb. per square inch, what must be the area of the cross section of the piston?

3. Draw a diagram of a set of pulleys by which a force of 50 lb. can support a load of 200 lb.

4. Draw a diagram of a set of pulleys by which a force of 50 lb. can support 250 lb. What would be the mechanical advantage of this arrangement?

Work and the Lever

178. The law of the lever. The lever is a rigid rod free to turn about some point P called the *fulcrum* (Fig. 139).

Let a meter stick be first balanced as in the figure, and then let a mass, of, say, 300 g., be hung by a thread from a point 15 cm. from the fulcrum. Then let a point be found on the other side of the fulcrum at which a weight of 100 g. will just support the 300 g. This point will be found to be 45 cm. from the fulcrum. It will be seen at once that the product of 300×15 is equal to the product of 100×45.

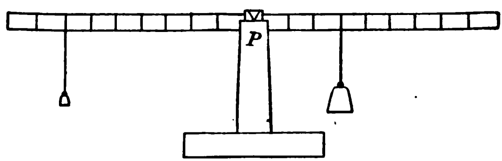


Fig. 139. The simple lever

Next let the point be found at which 150 g. just balance the 300 g. This will be found to be 30 cm. from the fulcrum. Again, the products 300×15 and 150×30 are equal.

No matter where the weights are placed, or what weights are used on either side of the fulcrum, the product of the effort F by its distance l from the fulcrum (Fig. 140) will be found to be equal to the product of the resistance F' by its distance l' from the fulcrum. Now the distances l and l' are called the *lever arms* of the forces F and F', and the product

of a force by its lever arm is called the *moment* of that force. The above experiments on the lever may then be generalized in the following law: *The moment of the effort is equal to the moment of the resistance.* Algebraically stated, it is

$$Fl = F'l'. \qquad (4)$$

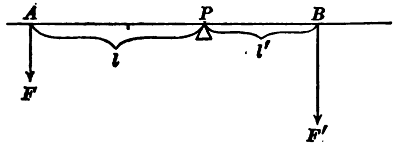


FIG. 140. Illustrating the law of moments, namely $Fl = F'l'$

It will be seen that the *mechanical advantage* of the lever, namely F'/F, is equal to l/l'; that is, *to the lever arm of the effort divided by the lever arm of the resistance.*

179. General laws of the lever. If parallel forces are applied at several points on a lever, as in Figs. 141 and 142, it will be found, in the particular cases illustrated, that for equilibrium

$$200 \times 30 = 100 \times 20 + 100 \times 40$$

and $$300 \times 20 + 50 \times 40 = 100 \times 15 + 200 \times 32.5;$$

that is, *the sum of all the moments which are tending to turn the beam in one direction is equal to the sum of all the moments tending to turn it in the opposite direction.*

If, further, we support the levers of Figs. 141 and 142 by spring balances attached at P, we shall find,

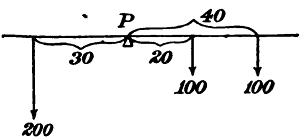


FIG. 141

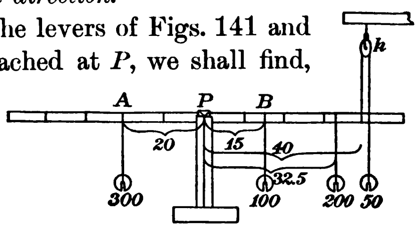


FIG. 142

Condition of equilibrium of a bar acted upon by several forces

after allowing for the weight of the meter stick, that the two forces indicated by the balances are respectively $200 + 100 + 100 = 400$ and $300 + 100 + 200 - 50 = 550$; that is, *the sum of all the forces acting in one direction on the lever is equal to the sum of all the forces acting in the opposite direction.*

These two laws may be combined as follows: If we think of the force exerted by the spring balance as the equilibrant of all the other forces acting on the lever, then we find that *the resultant of any number of parallel forces is their algebraic sum, and its point of application is the point about which the algebraic sum of the moments is zero.*

180. The couple. There is one case, however, in which parallel forces can have no single force as their resultant, namely, the case represented in Fig. 143. Such *a pair of equal and opposite forces acting at different points on a lever is called a couple* and can be neutralized only by another couple tending to produce rotation in the opposite direction. The moment of such a couple is evidently $F_1 \times oa + F_2 \times ob = F_1 \times ab$; that is, it is one of the forces times the total distance between them.

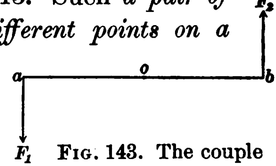


FIG. 143. The couple

181. Work expended upon and accomplished by the lever. We have just seen that when the lever is in equilibrium — that is, when it is *at rest* or is *moving uniformly* — the relation between the effort F and the resistance F' is expressed in the equation of moments, namely $Fl = F'l'$. Let us now suppose, precisely as in the case of the pulleys, that the force F raises the weight F' through a small distance s'. To accomplish this, the point A to which F is attached must move through a distance s (Fig. 144). From the similarity of the triangles APn and BPm it will be seen that l/l' is equal to s/s'. Hence equation (4), which represents the law of the lever, and which may be written $F/F' = l'/l$, may also be written in the form

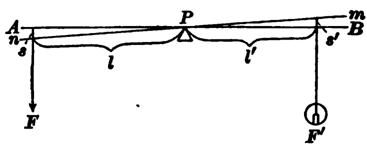


FIG. 144. Showing that the equation of moments, $Fl = F'l'$, is equivalent to $Fs = F's'$

$$F/F' = s'/s, \text{ or } Fs = F's'.$$

Now Fs represents the work done by the effort F, and $F's'$ the work done against the resistance F'. Hence the law of moments, which has just been found by experiment to be the law of the lever, is equivalent to the statement that *whenever work is accomplished by the use of the lever, the work expended upon the lever by the effort F is equal to the work accomplished by the lever against the resistance F'.*

182. The three classes of levers. It is customary to divide levers into three classes, as follows:

1. In levers of the first class the fulcrum P is between the acting force F and the resisting force F' (Fig. 145). The

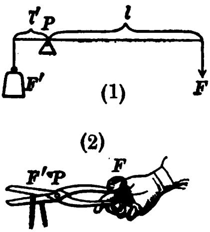


FIG. 145. Levers of first class

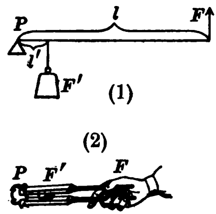


FIG. 146. Levers of second class

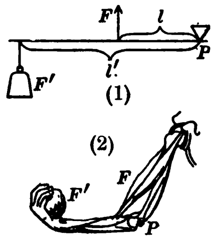


FIG. 147. Levers of third class

mechanical advantage of levers of this class is greater or less than unity according as the lever arm l of the effort is greater or less than the lever arm l' of the resistance.

2. In levers of the second class the resistance F' is between the effort F and the fulcrum P (Fig. 146). Here the lever arm of the effort, that is, the distance from F to P, is necessarily greater than the lever arm of the resistance, that is, the distance from F' to P. Hence the mechanical advantage is always greater than 1.

3. In levers of the third class the acting force is between the resisting force and the fulcrum (Fig. 147). The mechanical advantage is then obviously less than 1, that is, in this type of lever force is always sacrificed for the sake of gaining speed.

QUESTIONS AND PROBLEMS

1. Two boys are carrying a bag of walnuts at the middle of a long stick. Will it make any difference whether they walk close to the bag or farther away, so long as each is at the same distance?

2. When a load is carried on a stick over the shoulder, why does the pressure on the shoulder become greater as the load is moved farther out on the stick?

3. Explain the use of the rider in weighing (see Fig. 23).

4. In which of the three classes of levers does the wheelbarrow belong? grocer's scales? pliers? sugar tongs? a claw hammer? a pump handle?

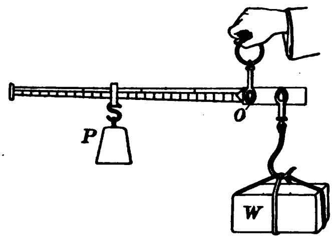


FIG. 148. Steelyards

5. Explain the principle of weighing by the steelyards (Fig. 148). What must be the weight of the bob P if, at a distance of 40 cm. from the fulcrum O, it balances a weight of 10 kg. placed at a distance of 2 cm. from O?

6. If you knew your own weight, how could you determine the weight of a companion if you had only a teeter board and a foot rule?

7. How would you arrange a crowbar to use it as a lever of the first class in overturning a heavy object? as a lever of the second class?

8. If 3 horses are to pull equally on a load, how should the whiffletree be designed?

9. Two boys carry a load of 60 lb. on a pole between them. If the load is 4 ft. from one boy and 6 ft. from the other, how many pounds does each boy carry? (Consider the force exerted by one of the boys as the effort, the load as the resistance, and the second boy as the fulcrum.)

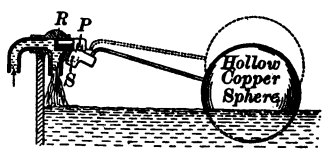


FIG. 149. The automatic float valve

10. Why is it that a couple cannot be balanced by a single force?

11. Why do tinners' shears have long handles and short blades and tailors' shears just the opposite?

12. If the ball of the float valve (Fig. 149) has a diameter of 10 cm., and if the distance from the center of the ball to the pivot S is 20 times the distance from S to the pin P, with what force is the valve R held shut when the ball is half immersed?

13. In the Yale lock (Fig. 150) the cylinder G rotates inside the fixed cylinder F and works the bolt through the arm H. The *right* key raises the pins a', b', c', d', e', until their tops are just even with the top of G. What mechanical principles do you find involved in this device?

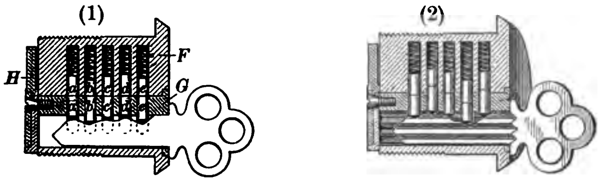


FIG. 150. Yale lock

(1) The right key ; (2) the wrong key

THE PRINCIPLE OF WORK

183. Statement of the principle of work. The study of pulleys led us to the conclusion that in all cases where such machines are used, the work done by the effort is equal to the work done against the resistance, provided always that friction may be neglected, and that the motions are uniform so that none of the force exerted is used in overcoming inertia. The study of levers led to precisely the same result. In Chapter II the study of the hydraulic press showed that the same law applied in this case also, for it was shown that the force on the small piston times the distance through which it moved was equal to the force on the large piston times the distance through which it moved. Similar experiments upon all sorts of machines have shown that in all cases where friction may be neglected the following is an absolutely general law: *In all mechanical devices of whatever sort the work expended upon the machine is equal to the work accomplished by it.*

This important generalization, called "the principle of work," was first stated by Newton in 1687. It has proved to be one of the most fruitful principles ever put forward in the history of physics. By its application it is easy to deduce the relation between the force applied and the force overcome in

any sort of machine, provided only that friction is negligible, and that the motions take place slowly. It is only necessary to produce, or imagine, a displacement at one end of the machine, and then to measure or calculate the corresponding displacement at the other end. The ratio of the second displacement to the first is the ratio of the force acting to the force overcome.

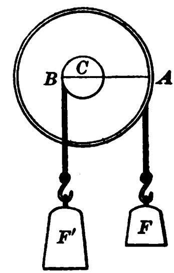


Fig. 151. The wheel and axle

184. The wheel and axle. Let us apply the work principle to discover the law of the wheel and axle (Fig. 151). When the large wheel has made one revolution, the point A moves down a distance equal to the circumference of this wheel. During this time the weight F' is lifted a distance equal to the circumference of the axle. Hence the equation $Fs = F's'$ becomes $F \times 2\pi R = F' \times 2\pi r$, where R and r are the radii of the wheel and axle respectively. This equation may be written in the form

$$F'/F = R/r; \tag{5}$$

that is, *the weight lifted on the axle is as many times the force applied to the wheel as the radius of the wheel is times the radius of the axle.* Otherwise stated, *the mechanical advantage of the wheel and axle is equal to the radius of the wheel divided by the radius of the axle.*


Fig. 152. The capstan

The *capstan* (Fig. 152) is a special case of the wheel and axle, the length of the lever arm taking the place of the radius of the wheel, and the radius of the barrel corresponding to the radius of the axle.

185. The work principle applied to the inclined plane. The work done against gravity in lifting a weight F' (Fig. 153) om the bottom to the top of a plane is evidently equal

to F' times the height h of the plane. But the work done by the acting force F, while the carriage of weight F' is being pulled from the bottom to the top of the plane, is equal to F times the length l of the plane. Hence the principle of work gives

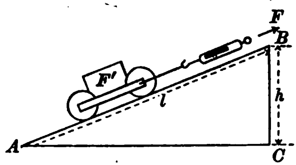


FIG. 153. The inclined plane

$$Fl = F'h, \text{ or } F'/F = l/h; \quad (6)$$

that is, *the mechanical advantage of the inclined plane, or the ratio of the weight lifted to the force acting parallel to the plane, is the ratio of the length of the plane to the height of the plane.* This is precisely the conclusion at which we arrived in another way in Chapter V, p. 80.

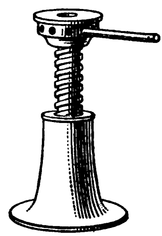


FIG. 154. The jackscrew

186. The screw. The screw (Fig. 154) is a combination of the inclined plane and the lever. Its law is easily obtained from the principle of work. When the force which acts on the end of the lever has moved this point through one complete revolution, the weight F', which rests on top of the screw, has evidently been lifted through a vertical distance equal to the distance between two adjoining threads. This distance d is called *the pitch* of the screw. Hence, if we represent by l the length of the lever, the work principle gives

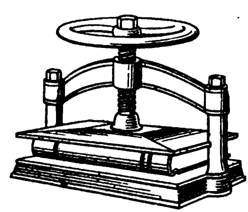


FIG. 155. The letter press

$$F \times 2\,\pi l = F'd; \quad (7)$$

that is, *the mechanical advantage of the screw, or ratio of the weight lifted to the force applied, is equal to the ratio of the circumference of the circle moved over by the end of the lever, to the distance between the threads of the screw.* In actual practice the friction in such

an arrangement is always very great, so that the mechanical advantage is considerably less than its full theoretical value. The common jackscrew just described (and used chiefly for raising buildings), the letter press (Fig. 155), and the vise (Fig. 156) are all familiar forms of the screw.

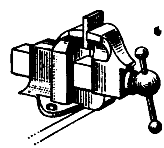


FIG. 156. The vise

187. A train of gear wheels. A form of machine capable of very high mechanical advantage is the train of gear wheels shown in Fig. 157. Let the student show from the principle of work, namely $Fs = F's'$, that the mechanical advantage, that is, $\frac{F'}{F}$, of such a device is

$$\frac{\text{circum. of } a \times \frac{\text{no. cogs in } d}{\text{no. cogs in } c} \times \frac{\text{no. cogs in } f}{\text{no. cogs in } b}}{\text{circum. of } e}. \qquad (8)$$

188. The worm wheel. Another device of high mechanical advantage is the worm wheel (Fig. 158). Show that if l is the length of the crank arm C, n the number of teeth in the cogwheel W, and r the radius of the axle, the mechanical advantage is given by

$$\frac{2\pi ln}{2\pi r} = n\frac{l}{r}. \qquad (9)$$

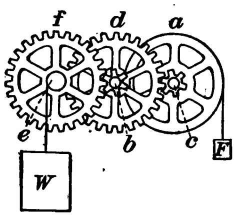


FIG. 157. Train of gear wheels

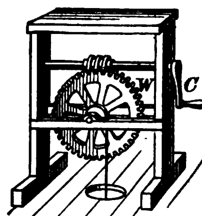


FIG. 158. The worm gear

This device is used most frequently when the primary object is to decrease speed rather than to multiply force. It will be seen that the crank handle must make n turns while the cogwheel is making one.

189. The differential pulley. In the differential pulley (Fig. 159) an endless chain passes first over the fixed pulley A, then down and around the movable pulley C, then up again over the fixed pulley B, which is rigidly attached to A, but differs slightly from it in diameter. On the circumference of all the pulleys are projections which fit between the links, and thus keep the chains from slipping. When the chain is pulled down at F, as in Fig. 159 (2), until the upper rigid system of

pulleys has made one complete revolution, the chain between the upper and lower pulleys has been shortened by the difference between the circumferences of the pulleys A and B, for the chain has been pulled up a distance equal to the circumference of the larger pulley and let down a distance equal to the circumference of the smaller pulley. Hence the load F' has been lifted by half the difference between the circumferences of A and B. The mechanical advantage is therefore equal to the circumference of A divided by one half the difference between the circumferences of A and B.

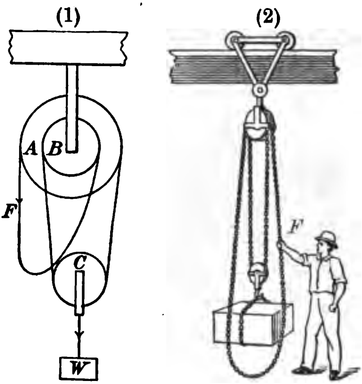


FIG. 159. The differential pulley

QUESTIONS AND PROBLEMS

1. A 300-lb. barrel was rolled up a plank 12 ft. long into a doorway 3 ft. high. What force was applied parallel to the plank?

2. A 1500-lb. safe must be raised 5 ft. The force which can be applied is 250 lb. What is the shortest inclined plane which can be used for the purpose?

3. In the differential wheel and axle (Fig. 160) the rope is wound in opposite directions on two axles of different diameter. For a complete revolution of the axle the weight is lifted by a distance equal to one half the difference between the circumferences of the two axles. If the crank has a radius of 2 ft., the larger axle a diameter of 6 in., and the smaller one a diameter of 4 in., find the mechanical advantage of the arrangement.

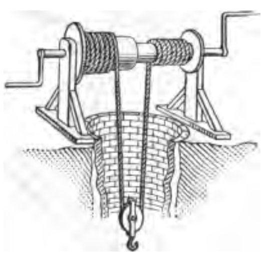


FIG. 160. Differential windlass

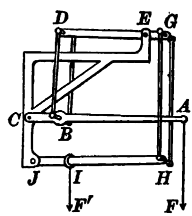


FIG. 161. The compound lever

4. If, in the compound lever of Fig. 161, $AC = 6$ ft., $BC = 1$ ft., $DE = 4$ ft., $EG = 8$ in., $HJ = 5$ ft., and $IJ = 2$ ft., what force applied at F will support a weight of 2000 lb. at F'?

†

5. The hay scales shown in Fig. 162 consist of a compound lever with fulcrums at F, F', F'', F'''. If Fo and $F'o'$ are lengths of 6 in., FE and $F'E$ 5 ft., $F''n$ 1 ft., $F''m$ 6 ft., rF''' 2 in., and $F'''S$ 20 in., how many pounds at W will be required to balance a weight of a ton on the platform?

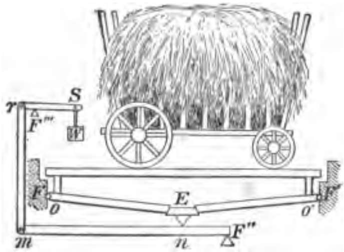


Fig. 162. Hay scales

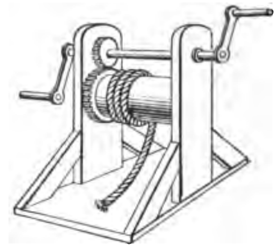


Fig. 163. Windlass with gears

6. If the capstan of a ship is 12 in. in diameter and the levers are 6 ft. long, what force must be exerted by each of 4 men in order to raise an anchor weighing 2000 lb.?

7. In the windlass of Fig. 163 the crank handle has a length of 2 ft., and the barrel a diameter of 8 in. There are 20 cogs in the small cogwheel and 60 in the large one. What is the mechanical advantage of the arrangement?

8. A force of 80 kg. on a wheel whose diameter is 3 m. balances a weight of 150 kg. on the axle. Find the diameter of the axle.

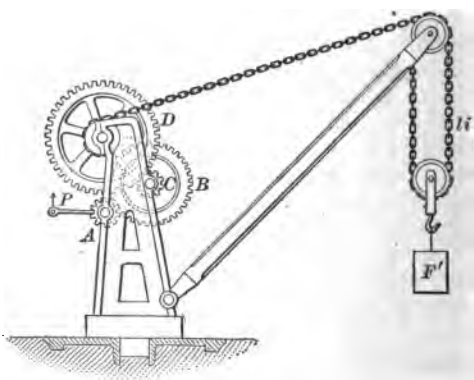


Fig. 164. The crane

9. Eight jackscrews each of which has a pitch of $\frac{1}{4}$ in. and a lever arm of 18 in. are being worked simultaneously to raise a building weighing 100,000 lb. What force would have to be exerted at the end of each lever if there were no friction? What if 75% were wasted in friction?

10. If a worm wheel (Fig. 158) has 30 teeth, and the crank is 25 cm. long, while the radius of the axle is 3 cm., what is the mechanical advantage of the arrangement?

11. If in the crane of Fig. 164 the crank arm has a length of 1/2 m., and the gear wheels A, B, C, and D have respectively 12, 48, 12, and 60 cogs, while the axle over which the chain runs has a radius of 10 cm., what is the mechanical advantage of the crane?

12. With the aid of Fig. 165 describe the process of winding and setting a watch. The rocker R is pivoted at S. C carries the mainspring and E the hands. $S.P.$ is a light spring which normally keeps the wheel A in mesh with C. Pressing down on P, however, releases A from C and engages B with D. What mechanical principles do you find involved? What happens when M is turned backward?

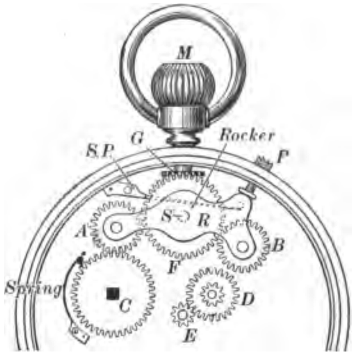


Fig. 165. Winding and setting mechanism of a stem-winding watch

Power and Energy

190. Definition of power. When a given load has been raised a given distance a given amount of work has been done, whether the time consumed in doing it is small or great. Time is therefore not a factor which enters into the determination of work; but it is often as important to know the *rate* at which work is done as to know the *amount* of work accomplished. *The rate of doing work is called power, or activity.* Thus, if P represent power, W the work done, and t the time required to do it,

$$P = \frac{W}{t}. \tag{10}$$

191. Horse power. James Watt (1736–1819), the inventor of the steam engine, considered that an average horse could do 33,000 foot pounds of work per minute, or 550 foot pounds per second. The metric equivalent is 76.05 kilogram meters per second. This number is probably considerably too high, but it has been taken ever since, in English-speaking countries,

as the unit of power, and named the *horse power* (H.P.). The power of steam engines has usually been rated in horse power. The horse power of an ordinary railroad locomotive is from 500 to 1000. Stationary engines and steamboat engines of the largest size often run from 5000 to 20,000 H.P. The power of an average horse is about 3/4 H.P., and that of an ordinary man about 1/7 H.P.

192. The kilowatt. In the metric system the erg has been taken as the absolute unit of work. The corresponding unit of power is an erg per second. This is, however, so small that it is customary to take as the practical unit 10,000,000 ergs per second; that is, one joule per second (see § 173, p. 132). This unit is called the *watt*, in honor of James Watt. The power of dynamos and electric motors is almost always expressed in kilowatts, a kilowatt representing 1000 watts, and in modern practice even steam engines are being increasingly rated in kilowatts rather than in horse power. A horse power is equivalent to 746 watts; it may therefore in general be considered to be 3/4 of a kilowatt.

193. Definition of energy. The *energy* of a body is defined as its *capacity for doing work.* In general, inanimate bodies possess energy only because of work which has been done upon them at some previous time. Thus, suppose a kilogram weight is lifted from the first position in Fig. 166 through a height of 1 m., and placed upon the hook *H* at the end of a cord which passes over a frictionless pulley *p* and is attached at the other end to a second kilogram weight *B*. The operation of lifting *A* from position *1* to position *2* has required an expenditure upon it of 1 kg. m. (100,000 g. cm., or 98,000,000 ergs) of work. But in position *2*, *A* is itself possessed of a certain capacity for doing work which it did

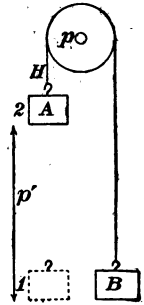


FIG. 166. Illustration of potential energy

James Prescott Joule
(1818–1889)

English physicist, born at Manchester; most prominent figure in the establishment of the doctrine of the conservation of energy; studied chemistry as a boy under John Dalton, and became so interested that his father, a prosperous Manchester brewer, fitted out a laboratory for him at home; conducted most of his researches either in a basement of his own house or in a yard adjoining his brewery; discovered the law of heating a conductor by an electric current; carried out, in connection with Lord Kelvin, epoch-making researches upon the thermal properties of gases; did important work in magnetism; first proved experimentally the identity of various forms of energy

James Watt (1736–1819)

The Scotch instrument maker at the University of Glasgow, who may properly be considered the inventor of the steam engine; for, although a crude and inefficient type of steam engine was known before his time, he left it in essentially its present form. The modern industrial era may be said to begin with Watt

not have before. For if it is now started downward by the application of the slightest conceivable force, it will, of its own accord, return to position *1*, and will in so doing raise the kilogram weight *B* through a height of 1 m. In other words, it will do upon *B* exactly the same amount of work which was originally done upon it.

194. Potential and kinetic energy. A body may have a capacity for doing work not only because it has been given an elevated position, but also because it has in some way acquired velocity; for example, a heavy flywheel will keep machinery running for some time after the power has been shut off; a bullet shot upward will lift itself a great distance against gravity because of the velocity which has been imparted to it. Similarly, any body which is in motion is able to rise against gravity, or to set other bodies in motion by colliding with them, or to overcome resistances of any conceivable sort. Hence, in order to distinguish between the energy which a body may have because of an *advantageous position*, and the energy which it may have because it is in *motion*, the two terms "potential" and "kinetic" energy are used. Potential energy includes the energy of lifted weights, of coiled or stretched springs, of bent bows, etc.; in a word, *potential energy is energy of position, while kinetic energy is energy of motion.*

195. Transformations of potential and kinetic energy. The swinging of a pendulum and the oscillation of a weight attached to a spring illustrate well the way in which energy which has once been put into a body may be transformed back and forth between the potential and kinetic varieties. When the pendulum bob is at rest at the bottom of its arc it possesses no energy of either type, since, on the one hand, it is as low as it can be, and, on the other, it has no velocity. When we pull it up the arc to the position *A* (Fig. 167), we do an amount of work upon it which is equal in gram centimeters to its weight in grams times the distance *AD* in centimeters;

that is, we store up in it this amount of potential energy. As now the bob falls to C this potential energy is completely transformed into kinetic. That this kinetic energy at C is exactly equal to the potential energy at A is proved by the fact that if friction is completely eliminated, the bob rises to a point B such that BE is equal to AD. We see, therefore, that at the ends of its swing the energy of the pendulum is all potential, while in the middle of the swing its energy is all kinetic. In intermediate positions the energy is part potential and part kinetic, but the sum of the two is equal to the original potential energy.

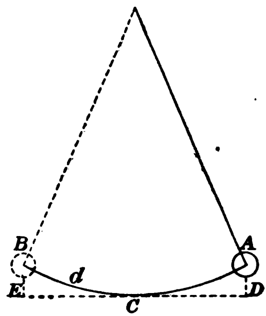


FIG. 167. Transformation of potential and kinetic energy

196. General statement of the law of frictionless machines. In our development of the law of machines, which led us to the conclusion that the work of the acting force is always equal to the work of the resisting force, we were careful to make two important assumptions: first, that friction was negligible; and second, that the motions were all either uniform or so slow that no appreciable velocities were imparted. In other words, we assumed that the work of the acting force was expended simply in lifting weights or compressing springs; that is, in storing up potential energy. If now we drop the second assumption, a very simple experiment will show that our conclusion must be somewhat modified. Suppose, for instance, that instead of lifting a 500-g. weight slowly by means of a balance, we jerk it up suddenly. We shall now find that the initial pull indicated by the balance, instead of being 500 g., will be considerably more — perhaps as much as several thousand grams if the pull is sufficiently sudden. This is obviously because the acting force is now overcoming

not merely the 500 g. which represents the resistance of gravity, but also the inertia of the body, since velocity is being imparted to it. Now work done in imparting velocity to a body, that is, in overcoming its inertia, always appears as *kinetic* energy, while work done in overcoming gravity appears as the *potential* energy of a lifted weight. Hence, whether the motions produced by machines are slow or fast, if friction is negligible, the law for all devices for transforming work may be stated thus: *The work of the acting force is equal to the sum of the potential and kinetic energies stored up in the mass acted upon.* In machines which work against gravity the body usually starts from rest and is left at rest, so that the kinetic energy resulting from the whole operation is zero. Hence in such cases the work done is the weight lifted times the height through which it is lifted, whether the motion is slow or fast. The kinetic energy imparted to the body in starting is all given up by it in stopping.

197. The measure of potential and kinetic energy. The measure of the potential energy of any lifted body, such as a lifted pile driver, is equal to the work which has been spent in lifting the body. Thus if h is the height in centimeters and M the weight in grams, then the potential energy P.E. of the lifted mass is

$$\text{P.E.} = Mh \text{ gram centimeters.} \tag{11}$$

Since the force of the earth's attraction for M grams is Mg dynes, if we wish to express the potential energy in ergs instead of in gram centimeters, we have

$$\text{P.E.} = Mgh \text{ ergs.} \tag{12}$$

Since this energy is all transformed into kinetic energy when the mass falls the distance h, the product Mgh also represents the number of ergs of kinetic energy which the moving weight has when it strikes the pile.

If we wish to express this kinetic energy in terms of the velocity with which the weight strikes the pile, instead of the height from which it has fallen, we have only to substitute for h its value in terms of g and

the velocity acquired [see equation (3), p. 93], namely $h = v^2/2\,g$. This gives the kinetic energy K.E. in the form

$$\text{K.E.} = \tfrac{1}{2}\,Mv^2 \text{ ergs.} \tag{13}$$

Since it makes no difference how a body has acquired its velocity, this represents the general formula for the kinetic energy *in ergs* of any moving body, in terms of its mass and its velocity.

Thus the kinetic energy of a 100-g. bullet moving with a velocity of 10,000 cm. per second is

$$\text{K.E.} = \tfrac{1}{2} \times 100 \times (10{,}000)^2 = 5{,}000{,}000{,}000 \text{ ergs.}$$

Since 1 g. cm. is equivalent to 980 ergs, the energy of this bullet is $\frac{5{,}000{,}000{,}000}{980} = 5{,}102{,}000$ g. cm., or 51.02 kg. m.

We know, therefore, that the powder pushing on the bullet as it moved through the rifle barrel did 51.02 kg. m. of work upon the bullet in giving it the velocity of 100 m. per second.

In general terms, if M is in grams and v in centimeters per second, $\text{K.E.} = \dfrac{Mv^2}{2 \times 980}$ g. cm.; if M is in pounds and v in feet per second, $\text{K.E.} = \dfrac{Mv^2}{2 \times 32.16}$ ft. lb.

QUESTIONS AND PROBLEMS

1. A 150-lb. man runs up a flight of stairs 60 ft. high in 10 sec. What is his horse power while doing it? How do you account for the result?

2. If a rifle bullet can just pass through a plank, how many planks will it pass through if its speed is doubled?

3. What must be the horse power of an engine which is to pump 10,000 l. of water per second from a mine 150 m. deep?

4. What must be the power in kilowatts of the engines supplying the city water in Problem 4, p. 132? Express the power also in horse power. (Assume a 24-hour day.)

5. A water motor discharges 100 l. of water per minute when fed from a reservoir in which the water surface stands 50 m. above the level of the motor. If all of the potential energy of the water were transformed into work in the motor, what would be the horse power of the motor? (The potential energy of the water is the amount of work which would be required to carry it back to the top of the reservoir.)

CHAPTER IX

WORK AND HEAT ENERGY

FRICTION

198. Friction always results in wasted work. All of the experiments mentioned in the last chapter were so arranged that *friction* could be neglected or eliminated. So long as this condition was fulfilled it was found that the result of universal experience could be stated thus: *The work done by the acting force is equal to the sum of the kinetic and potential energies stored up.*

But wherever friction is present this law is found to be inexact, for the work of the acting force is then always somewhat greater than the sum of the kinetic and potential energies stored up. If, for example, a block is pulled over the horizontal surface of a table, at the end of the motion no velocity has been imparted to the block, and hence no kinetic energy has been stored up. Further, the block has not been lifted nor put into a condition of elastic strain, and hence no potential energy has been communicated to it. We cannot in any way obtain from the block more work after the motion than we could have obtained before it was moved. It is clear, therefore, that all of the work which was done in moving the block against the friction of the table was *wasted work.* Experience shows that, in general, where work is done against friction it can never be regained. Before considering what becomes of this wasted work we shall consider some of the factors on which friction depends, and some of the laws which are found by experiment to hold in cases in which friction occurs.

199. Coefficient of friction. It is found that if F represents the force parallel to a plane which is necessary to maintain uniform motion in a body which is pressed against the plane with a force F', then the ratio $\frac{F}{F'}$ depends only on the nature of the surfaces in contact, and not at all on the area or on the velocity of the motion. The ratio $\frac{F}{F'}$ is called the *coefficient of friction* for the given materials. Thus, if F is 300 g. and F' 800 g., the coefficient of friction is $\frac{300}{800} = .375$. The coefficient of iron on iron is about .2, of oak on oak about .4.

200. Rolling friction. The chief cause of sliding friction is the interlocking of minute projections (shown greatly magnified at *a, b, c,* and *d* in Fig. 168). When a round solid *rolls* over a smooth surface the frictional resistance is generally much less than when it slides; for example, the coefficient of friction of cast-iron wheels rolling on iron rails may be as low as .002; that is, $\frac{1}{100}$ of the sliding friction of iron on iron. This means that a pull of 1 pound will keep a 500-pound car in motion. Sliding friction is not, however, entirely dispensed with in ordinary

FIG. 168. Illustrating friction of rubbing surfaces

(1) (2)

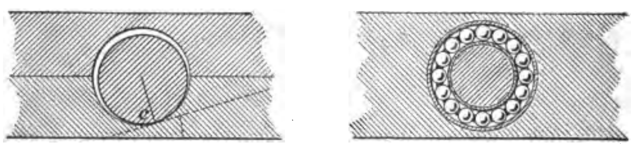


FIG. 169. Friction in bearings

(1) Common bearing; (2) ball bearing

wheels, for although the rim of the wheel rolls on the track, the axle slides continuously at some point *c* [Fig. 169, (1)] upon the surface of the journal.

The great advantage of the ball bearing [Fig. 169, (2)] is that the sliding friction in the hub is almost completely replaced by rolling friction. The manner in which ball bearings are used in a bicycle

pedal is illustrated in Fig. 170. The "free wheel" ratchet is shown in Fig. 171. The "pawls" a, b, enable the pedals and chain wheel W to stop while the rear axle continues to revolve.

201. Fluid friction. When a solid moves through a fluid, as when a bullet moves through the air or a ship through the water, the resistance encountered is not at all independent of velocity, as in the case of solid friction, but increases

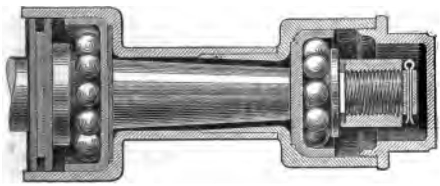


FIG. 170. The bicycle pedal

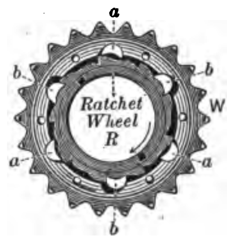


FIG. 171. Free wheel ratchet

for slow speeds nearly as the square of the velocity, and for high speeds at a rate considerably greater. This explains why it is so expensive to run a fast train; for the resistance of the air, which is a small part of the total resistance so long as the train is moving slowly, becomes the predominant factor at high speeds. The resistance offered to steamboats running at high speeds is usually considered to increase as the cube of the velocity. Thus the *Cedric*, of the White Star Line, having a speed of 17 knots, has a horse power of 14,000, and a total weight when loaded of about 38,000 tons, while the *Kaiser Wilhelm II*, of the North German Lloyd Line, having a speed of 24 knots, has engines of 40,000 horse power, although the total weight when loaded is only 26,000 tons.

QUESTIONS AND PROBLEMS

1. In what respects is friction an advantage, and in what a disadvantage, in everyday life? Could we get along without it?

2. Why is a stream swifter at the center than at the banks?

3. A smooth block is $10 \times 8 \times 3$ in. Compare the distances which it will slide when given a certain initial velocity on smooth ice, if resting first on a 10×8 face; second, on a 10×3 face; and third, on an 8×3 face.

4. Why is sand often placed on a track in starting a heavy train?

5. What is the coefficient of friction of brass on brass if a force of 25 lb. is required to maintain uniform motion in a brass block weighing 200 lb., when it slides horizontally on a brass bed?

6. Why does a team have to keep pulling after a load is started?

7. The coefficient of friction between a block and a table is .3. What force will be required to keep a 500-g. block in uniform motion?

8. In what way is friction an advantage in lifting buildings with a jackscrew? In what way is it a disadvantage?

EFFICIENCY

202. Definition of efficiency. Since it is only in an ideal machine that there is no friction, in all actual machines the work done by the acting force always exceeds, by the amount of the work done against friction, the amount of potential and kinetic energy stored up. We have seen that the former is wasted work in the sense that it can never be regained. Since the energy stored up represents work which can be regained, it is termed *useful work.* In most machines an effort is made to have the useful work as large a fraction of the total work expended as possible. *The ratio of the useful work to the total work done by the acting force is called the* EFFICIENCY *of the machine.* Thus

$$\text{Efficiency} = \frac{\text{Useful work accomplished}}{\text{Total work expended}}. \tag{1}$$

Thus, if in the system of pulleys shown in Fig. 138 it is necessary to add a weight of 50 g. at F in order to pull up slowly an added weight of 240 g. at W, the work done by the 50 g. while F is moving over 1 cm. will be 50×1 g. cm. The useful work accomplished in the same time is $240 \times \frac{1}{6}$ g. cm. Hence the efficiency is equal to $\frac{240 \times \frac{1}{6}}{50 \times 1} = \frac{4}{5} = 80\%$.

203. Efficiencies of some simple machines. In simple levers the friction is generally so small as to be negligible; hence the efficiency of such machines is approximately 100%. When inclined planes are used as machines the friction is also small, so that the efficiency generally lies between 90% and 100%. The efficiency of the commercial block and tackle (Fig. 138), with several movable pulleys, is usually considerably less, varying between 40% and 60%. In the jackscrew there is

necessarily a very large amount of friction, so that although the mechanical advantage is enormous, the efficiency is often as low as 25%. The differential pulley of Fig. 159 has also a very high mechanical advantage with a very small efficiency. Gear wheels such as those shown in Fig. 157, or chain gears such as those used in bicycles, are machines of comparatively high efficiency, often utilizing between 90% and 100% of the energy expended upon them.

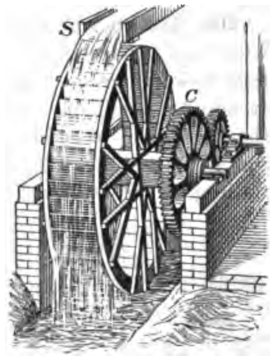


FIG. 172. Overshot water wheel

204. Efficiency of overshot water wheels. The overshot water wheel (Fig. 172) utilizes chiefly the potential energy of the water at S; for the wheel is turned by the weight of the water in the buckets. The work expended on the wheel per second, in foot pounds or gram centimeters, is the product of the weight of the water which passes over it per second by the distance through which it falls. The efficiency is the work which the wheel can accomplish in a second, divided by this quantity. Such wheels are very common in mountainous regions, where it is easy to obtain considerable fall, but where the streams carry a small *volume* of water. The efficiency is high, being often between 80% and 90%. The loss is due not only to the friction in the bearings and gears (see C), but also to the fact that some of the water is spilled from the buckets, or passes over without entering them at all. This may still be regarded as a frictional loss, since the energy disappears in internal friction when the water strikes the ground.

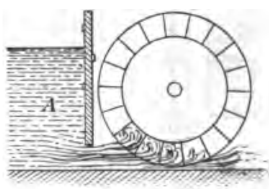


FIG. 173. The undershot wheel

205. Efficiency of undershot water wheels. The old-style undershot wheel (Fig. 173), so common in flat countries where there is little fall but abundance of water, utilizes only the kinetic energy of the water running through the race from A. It seldom transforms into useful work more than 25% or 30% of the potential energy of the water above the dam. There are, however, certain modern forms of

undershot wheel which are extremely efficient. For example, the *Pelton wheel* (Fig. 174), developed since 1880, and now very commonly used for small-power purposes in cities supplied with waterworks, sometimes has an efficiency as high as 83%. The water is delivered from a nozzle *O* against cup-shaped buckets arranged as in the figure.

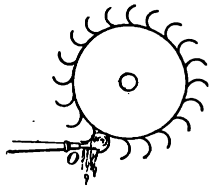


FIG. 174. The Pelton water motor

206. Efficiency of water turbines. The turbine wheel was invented in France in 1833, and is now used more than any other form of water wheel. It stands completely under water in a case at the bottom of a *turbine pit*, rotating in a horizontal plane. Fig. 175 shows the method of installing a turbine at Niagara. *C* is the outer case into which the water enters from the penstock *p*. Fig. 176, (1), shows the outer case with contained turbine; Fig. 176, (2), is the inner case in which are the fixed guides *G*, which direct the water at the most advantageous angle against the blades

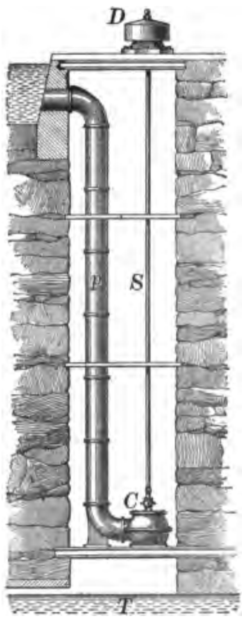


FIG. 175. A turbine installed

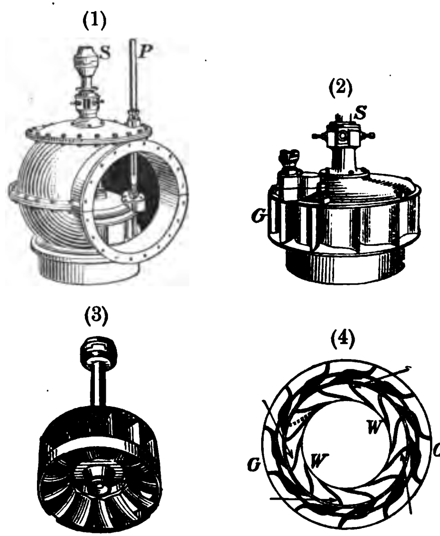


FIG. 176. The turbine wheel

(1) Outer case; (2) inner case; (3) rotating part; (4) section

of the wheel inside; Fig. 176, (3), is the wheel itself; and Fig. 176, (4), is a section of wheel and inner case, showing how the water enters through the guides and impinges upon the blades *W*. The spent water simply falls down from the blades into the tailrace *T* (Fig. 175). The amount of water which passes through the turbine can be controlled by means of the rod *P* [Fig. 176, (1)], which can be turned so as to increase or decrease the size of the openings between the guides *G* [Fig. 176, (2)]. The energy expended upon the turbine per second is the product of the mass of water which passes through it by the height of the turbine pit. Efficiencies as high as 90% have been attained with such wheels. One of the most powerful turbines in existence is at Shawenegan Falls, Quebec, Canada. The pit is 135 feet deep, the wheel 10 feet in diameter, and the horse power developed 10,500.

QUESTIONS AND PROBLEMS

1. If it is necessary to pull on a block and tackle with a force of 100 lb. in order to lift a weight of 300 lb., and if the force must move 6 ft. to raise the weight 1 ft., what is the efficiency of the system?

2. If the efficiency had been 65%, what force would have been necessary in the preceding problem?

3. The largest overshot water wheel in existence is at Laxey, on the Isle of Man. It has a horse power of 150, a diameter of 72.5 ft., and an efficiency of 85%. How many cubic feet of water pass over it per second?

4. The Niagara turbine pits are 136 ft. deep and their average horse power is 5000. Their efficiency is 85%. How much water does each turbine discharge per minute?

5. There is a Pelton wheel at the Sutro tunnel in Nevada which is driven by water supplied from a reservoir 2100 ft. above the level of the motor. The diameter of the nozzle is about $\frac{1}{2}$ in., and that of the wheel but 3 ft., yet 100 H.P. is developed. If the efficiency is 80%, how many cubic feet of water are discharged per second?

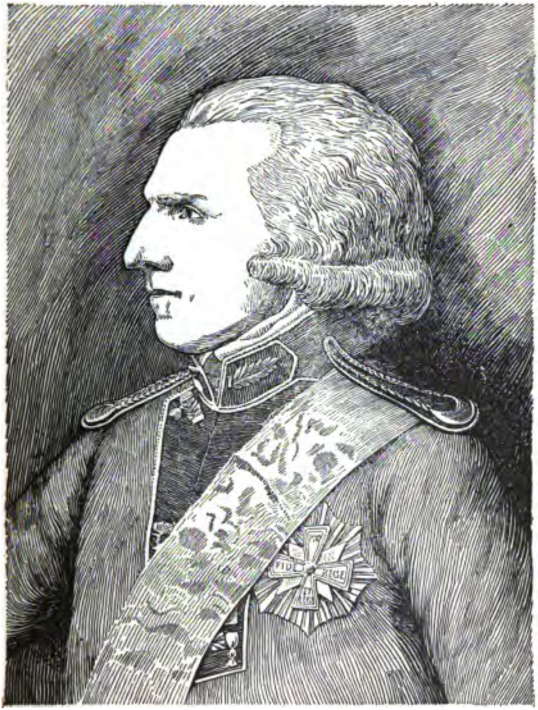
Mechanical Equivalent of Heat *

207. What becomes of wasted work? In all of the devices for transforming work which we have considered we have found that on account of frictional resistances a certain per cent of the work expended upon the machine is wasted. The

* This subject should be preceded by a laboratory experiment upon the "law of mixtures," and either preceded or accompanied by experiments upon specific heat and mechanical equivalent. See authors' manual, Experiments 18, 19, and 20.

question which at once suggests itself is, "What becomes of this wasted work?" The following familiar facts suggest an answer. When two sticks are vigorously rubbed together they become hot; augers and drills often become too hot to hold; matches are ignited by friction; if a strip of lead be struck a few sharp blows with a hammer, it is appreciably warmed. Now since we learned in Chapter IV that, according to modern notions, increasing the temperature of a body means simply increasing the average velocity of its molecules, and therefore their average kinetic energy, the above facts point strongly to the conclusion that in each case the mechanical energy expended has been simply transformed into the energy of molecular motion. This view was first brought into prominence in 1798 by Benjamin Thompson, Count Rumford, an American by birth, who was led to it by observing that in the boring of cannon heat was continuously developed. It was first carefully tested by the English physicist, James Prescott Joule (1818–1889), in a series of epoch-making experiments extending from 1842 to 1870. In order to understand these experiments we must first learn how heat quantities are measured.

208. Unit of heat — the calorie. A unit of heat is defined as *the amount of heat which is required to raise the temperature of 1 gram of water through 1° centigrade.* This unit is called *the calorie.* Thus, for example, when a hundred grams of water has its temperature raised four degrees, we say that four hundred calories of heat have entered the water. Similarly, when a hundred grams of water has its temperature lowered ten degrees, we say that a thousand calories have passed out of the water. If, then, we wish to measure, for instance, the amount of heat developed in a lead bullet when it strikes against a target, we have only to let the spent bullet fall into a known weight of water and to measure the number of degrees through which the temperature of the water rises. The product of the number of grams of water by its rise in



Count Rumford (Benjamin Thompson, 1753–1814)

"An eminent scientist, enlightened philanthropist, and sagacious public administrator," was born at Woburn, Massachusetts; was a Tory during the Revolution; removed to England, and afterward to Munich, where he lived for eleven years as minister of war, minister of police, and grand chamberlain to the elector. He reorganized the Bavarian army, suppressed beggary, provided employment for the poor, and established industrial schools. He was one of the earliest and most influential advocates of the view that heat is a mode of molecular motion. He invented the Rumford photometer (see § 445, p. 367)

temperature is then, by definition, the number of calories of heat which have passed into the water.

It will be noticed that in the above definition we make no assumption whatever as to what *heat* is. Previous to the nineteenth century physicists generally held it to be an invisible, weightless fluid, the passage of which into or out of a body caused it to grow hot or cold. This view accounts well enough for the heating which a body experiences when it is held in contact with a flame or other hot body, but it has difficulty in explaining the heating produced by rubbing or pounding. Rumford's view accounts easily for this, as we have seen, while it accounts no less easily for the heating of cold bodies by contact with hot ones; for we have only to think of the hotter and therefore more energetic molecules of the hot body as communicating their energy to the molecules of the colder body in much the same way in which a rapidly moving billiard ball transfers part of its kinetic energy to a more slowly moving ball against which it strikes.

209. Joule's experiment on the heat developed by friction. Joule argued that if the heat produced by friction, etc. is indeed merely mechanical energy which has been transferred to the molecules of the heated body, then the same number of calories must always be produced by the disappearance of a given amount of mechanical energy. And this must be true, no matter whether the work is expended in overcoming the friction of wood on wood, of iron on iron, in percussion, in compression, or in any other conceivable way. To see whether or not this were so, he caused mechanical energy to disappear in as many ways as possible and measured in every case the amount of heat developed.

In his first experiment he caused paddle wheels to rotate in a vessel of water by means of falling weights W (Fig. 177). The amount of work done by gravity upon the weights in causing them to descend through any distance d was equal to their weight W times this distance.

If the weights descended slowly and uniformly, this work was all expended in overcoming the resistance of the water to the motion of the paddle wheels through it; that is, it was wasted in eddy currents in the water. Joule measured the rise in the temperature of the water and found that the mean of his three best trials gave 427 gram meters as the amount of work required to develop enough heat to raise a gram of water one degree. He then repeated the experiment, substituting mercury for water, and obtained 425 gram meters as the work necessary to produce a calorie of heat. The difference between these numbers is less than was to have been expected from the unavoidable errors in the observations. He then devised an arrangement in which the heat was developed by the friction of iron on iron, and again obtained 425.

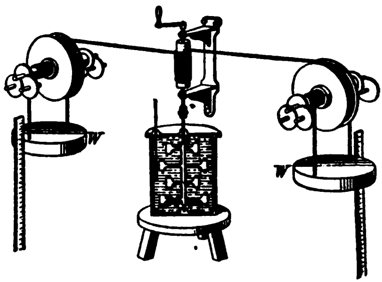


FIG. 177. Joule's first experiment on the mechanical equivalent of heat

210. Heat produced by collision. A Frenchman by the name of Hirn was the first to make a careful determination of the relation between the heat developed by *collision* and the kinetic energy which disappears. He allowed a steel cylinder to fall through a known height and crush a lead ball by its impact upon it. The amount of heat developed in the lead was measured by observing the rise in temperature of a small amount of water into which the lead was quickly plunged. As the mean of a large number of trials he also found that 425 gram meters of energy disappeared for each calorie of heat which appeared.

211. Heat produced by the compression of a gas. Another way in which Joule measured the relation between heat and work was by compressing a gas and comparing the amount of work done in the compression with the amount of heat developed.

Every bicyclist is aware of the fact that when he inflates his tires the pump grows hot. This is due partly to the friction of

the piston against the walls, but chiefly to the fact that the downward motion of the piston is transferred to the molecules which come in contact with it, so that the velocity of these molecules is increased. The principle is precisely the same as that involved in the velocity communicated to a ball by a bat. If the bat is held rigidly fixed and a ball thrown against it, the ball rebounds with a certain velocity; but if the bat is moving rapidly forward to meet the ball, the latter rebounds with a much greater velocity. So the molecules which in their natural motions collide with an advancing piston, rebound with greater velocity than they would if they had impinged upon a fixed wall. This increase in the molecular velocity of a gas on compression is so great that when a mass of gas at 0° centigrade is compressed to one half its volume the temperature rises to 87° centigrade.

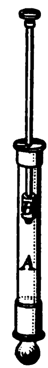


FIG. 178. The fire syringe

The effect may be strikingly illustrated by the fire syringe (Fig. 178). Let a few drops of carbon bisulphide be placed on a small bit of cotton, dropped to the bottom of the tube A, and then removed; then let the piston B be inserted and very suddenly depressed. Sufficient heat will be developed to ignite the vapor and a flash will result. (If the flash does not result from the first stroke, withdraw the piston completely, then reinsert, and compress again.)

To measure the heat of compression Joule surrounded a small compression pump with water, took 300 strokes on the pump, and measured the rise in temperature of the water. As the result of these measurements he obtained 444 gram meters as the "mechanical equivalent" of the calorie. The experiment, however, could not be performed with great exactness.

212. Cooling by expansion. Joule also obtained the relation between heat and work from experiments on the cooling produced by expansion. This process is exactly the converse

of heating by compression. If a compressed gas is allowed to expand and force out a piston, or merely to expand against atmospheric pressure, it is always found to be cooled by the process. This is because the kinetic energy of the molecules is transferred to the piston, so that the former rebound from the latter with less velocity than they had before the blow. The refrigerators used on shipboard are good illustrations of this principle. Air is compressed by an engine to perhaps one fourth its natural volume. The heat generated by the compression is then removed by causing the air to circulate about pipes kept cool by the flow of cold water through them. This compressed air is then allowed to expand into the refrigerating chamber, the temperature of which is thus lowered many degrees.

Joule's determination of the mechanical equivalent of heat from the amount of work done by an expanding gas and the amount of heat lost in expansion gave 437 gram meters. This experiment also was one for which no great amount of exactness could be claimed.

213. Significance of Joule's experiments. Joule made three other determinations of the relation between heat and work by methods involving electrical measurements. He published as the mean of all his determinations 426.4 gram meters as the mechanical equivalent of the calorie. But the value of his experiments does not lie primarily in the accuracy of the final results, but rather in the proof which they for the first time furnished that *whenever a given amount of work is wasted, no matter in what particular way this waste may occur, there is always an appearance of the same definite invariable amount of heat.*

The most accurate determination of the mechanical equivalent of heat was made by Rowland (1848–1901) in 1880 at Johns Hopkins University. He obtained 427 gram meters (4.19×10^7 ergs). We shall generally take it as 42,000,000 ergs.

214. The conservation of energy. We are now in a position to state the law of all machines in its most general form; that is, in such a way as to include even the cases where friction is present. It is: *The work done by the acting force is equal to the sum of the kinetic and potential energies stored up plus the mechanical equivalent of the heat developed.*

In other words, *whenever energy is expended on a machine or device of any kind, an exactly equal amount of energy always appears either as useful work or as heat.* The useful work may be represented in the potential energy of a lifted mass, as when water is pumped up to a reservoir; or in the kinetic energy of a moving mass, as when a stone is thrown from a sling; or in the potential energies of molecules whose positions with reference to one another have been changed, as when a spring has been bent; or in the molecular potential energy of chemically separated atoms, as when an electric current separates a compound substance. The *wasted* work always appears in the form of increased molecular motion; that is, in the form of heat. This important generalization has received the name of the *Principle of the Conservation of Energy.* It may be stated thus: *Energy may be transformed, but it can never be created or destroyed.*

215. Perpetual motion. In all ages there have been men who have spent their lives in trying to invent a machine out of which work could be continually obtained, without the expenditure of an equivalent amount of work upon it. Such devices are called perpetual-motion machines. Even to this day the United States patent office annually receives scores of applications for patents on such devices. The possibility of the existence of such a device is absolutely denied by the statement of the principle of the conservation of energy. For only in case there is no heat developed, that is, in case there are no frictional losses, can the work taken out be equal to the work put in, and in no case can it be greater. Since, in

fact, there are always some frictional losses, the principle of the conservation of energy asserts that it is impossible to make a machine which will keep itself running forever, even though it does no useful work; for no matter how much kinetic or potential energy is imparted to the machine to begin with, there must always be a continual drain upon this energy to overcome frictional resistances; so that, as soon as the wasted work has become equal to the initial energy, the machine must stop.

The first man to make a formal and complete statement of the principle of the conservation of energy was the German physician Robert Mayer, whose work was published in 1842. Twenty years later, partly through the theoretical writings of Helmholtz and Clausius in Germany, and of Kelvin and Rankine in England, but more especially through the experimental work of Joule, the principle had gained universal recognition and had taken the place which it now holds as the corner stone of all physical science.

216. Transformations of energy in a power plant. The transformations of energy which take place in any power plant, such as that at Niagara, are as follows: The energy first exists as the potential energy of the water at the top of the falls. This is transformed in the turbine pits into the kinetic energy of the rotating wheels. These turbines drive dynamos in which there is a transformation into the energy of electric currents. These currents travel on wires as far as Syracuse, 150 miles away, where they run street cars and other forms of motors. The principle of conservation of energy asserts that the work which gravity did upon the water in causing it to descend from the top to the bottom of the turbine pits is exactly equal to the work done by all the motors, plus the heat developed in all the wires and bearings, and in the eddy currents in the water.

Let us next consider where the water at the top of the falls obtained its potential energy. Water is being continually evaporated at the surface of the ocean by the sun's heat. This heat imparts sufficient kinetic energy to the molecules to enable them to break away from the attractions of their fellows and to rise above the surface in the form of vapor. The lifted vapor is carried by winds over the continents and precipitated in the form of rain or snow. Thus the potential energy of the water

above the falls at Niagara is simply transformed heat energy of the sun. If, in this way, we analyze any available source of energy at man's disposal, we find in practically every case that it is directly traceable to the sun's heat as its source. Thus the energy contained in coal is simply the energy of separation of the oxygen and carbon which were separated in the processes of growth. This separation was effected by the sun's rays.

We can form some conception of the enormous amount of energy which the sun radiates in the form of heat by reflecting that of this heat the earth receives not more than $\frac{1}{2,000,000,000}$ part. Of the amount received by the earth not more than $\frac{1}{1000}$ part is stored up in animal and vegetable life and lifted water. This is practically all of the energy which is available on the earth for man's use.

QUESTIONS AND PROBLEMS

1. Analyze the transformations of energy which occur when a bullet is fired vertically upward.
2. Show that the energy of a water fall is merely transformed solar energy.
3. How many calories of heat are generated by the impact of a 200-kilo bowlder when it falls vertically through 100 m.?
4. Thousands of meteorites are falling into the sun with enormous velocities every minute. From a consideration of the preceding example account for a portion, at least, of the sun's heat.
5. The Niagara Falls are 160 ft. high. How much warmer is the water at the bottom of the falls than at the top?
6. Why does air escaping from a bicycle tire feel cold?

Specific Heat

217. Definition of specific heat. When we experiment upon different substances we find that it requires wholly different amounts of heat energy to produce in one gram of mass one degree of change in temperature.

Let 100 g. of lead shot be placed in one test tube, 100 g. of bits of iron wire in another, and 100 g. of aluminium wire in a third. Let them all be placed in a pail of boiling water for ten or fifteen minutes, care being taken not to allow any of the water to enter any of the tubes. Let three small vessels be provided, each of which contains 100 g. of

water at the temperature of the room. Let the heated shot be poured into the first beaker, and after thorough stirring let the rise in the temperature of the water be noted. Let the same be done with the other metals. The aluminium will be found to raise the temperature about twice as much as the iron, and the iron about three times as much as the lead. Hence, since the three metals have cooled through approximately the same number of degrees, we must conclude that about six times as much heat has passed out of the aluminium as out of the lead; that is, each gram of aluminium in cooling 1° C. gives out about six times as many calories as a gram of lead.

The number of calories taken up by 1 gram of a substance when its temperature rises through 1° C., or given up when it falls through 1° C., is called the specific heat of that substance.

It will be seen from this definition, and the definition of the calorie, that the specific heat of water is 1.

218. Determination of specific heat by the method of mixtures. The preceding experiments illustrate a method for measuring accurately the specific heats of different substances. For, in accordance with the principle of the conservation of energy, when hot and cold bodies are mixed, as in these experiments, so that heat energy passes from one to the other, *the gain in the heat energy of one must be just equal to the loss in the heat energy of the other.*

This method is by far the most common one for determining the specific heats of substances. It is known as the *method of mixtures.*

Suppose, to take an actual case, that the initial temperature of the shot used in § 217 was 95° C., and that of the water 19.7°, and that, after mixing, the temperature of the water and shot was 22°. Then, since 100 g. of water has had its temperature raised through $22° - 19.7° = 2.3°$, we know that 230 calories of heat have entered the water. Since the temperature of the shot fell through $95° - 22° = 73°$, the number of calories given up by the 100 g. of shot in falling 1° was $\frac{230}{73} = 3.15$. Hence the specific heat of lead, that is, *the number of calories of heat given up by 1 gram of lead when its temperature falls 1° C.* is $\frac{3.15}{100} = .0315$.

Or again, we may work out the problem algebraically as follows: Let x equal the specific heat of lead. Then the number of calories which come out of the shot is *its mass times its specific heat times its change in temperature;* that is, $100 \times x \times (95-22)$, and, similarly, the number which enter the water is the same, namely $100 \times 1 \times (22-19.7)$. Hence we have

$$100\,(95-22)\,x = 100\,(22-19.7) \quad \text{or} \quad x = .0315.$$

By experiments of this sort the specific heats of some of the common substances have been found to be as follows:

TABLE OF SPECIFIC HEATS

Aluminium	.218	Iron	.113
Brass	.094	Lead	.0315
Copper	.095	Mercury	.0333
Glass	.2	Platinum	.032
Gold	.0316	Silver	.0568
Ice	.504	Zinc	.0935

QUESTIONS AND PROBLEMS

1. Why is a liter of hot water a better foot warmer than an equal volume of any substance in the preceding table?

2. Which would be heated more, a lead or a steel bullet, if they were fired against a target with equal speeds?

3. If 100 g. of mercury at 95° C. are mixed with 100 g. of water at 15° C., and if the resulting temperature is 17.6° C., what is the specific heat of mercury?

4. A 10-g. bullet of lead is shot from a gun with a velocity of 400 m. per second. Through how many degrees centigrade is its temperature raised when it strikes a target? (Assume that all of the heat stays in the bullet.)

5. From what height must a block of lead fall in order to have its temperature raised through 1° C.?

6. If 200 g. of water at 80° C. are mixed with 100 g. of water at 10° C., what will be the temperature of the mixture? (Let x equal the final temperature; then $100\,(x-10)$ calories are gained by the cold water, while $200\,(80-x)$ calories are lost by the hot water.)

7. What temperature will result if 300 g. of copper at 100° C. are placed in 200 g. of water at 15° C.?

8. The specific heat of water is much greater than that of any other liquid or of any solid. Explain how this accounts for the fact

that an island in mid-ocean undergoes less extremes of temperature than an inland region.

9. How many grams of ice-cold water must be poured into a tumbler weighing 300 g., to cool it from 60° C. to 20° C., the specific heat of glass being .2?

10. If you put a 20 g. silver spoon at 20° C. into a 150-cc. cup of tea at 70° C., how much do you cool the tea?

Change of State — Fusion*

219. Heat of fusion. If on a cold day in winter a quantity of snow is brought in from out of doors, where the temperature is below 0° C., and placed over a source of heat, a thermometer plunged into the snow will be found to rise slowly until the temperature reaches 0° C., when it will become stationary and remain so during all the time that the snow is melting, provided only that the contents of the vessel are continuously and vigorously stirred. As soon as the snow is all melted the temperature will begin to rise again.

Since the temperature of ice at 0° C. is the same as the temperature of water at 0° C., it is evident from this experiment that when ice is being changed to water, the entrance of heat energy into it does not produce any change in the average kinetic energy of its molecules. This energy must therefore all be expended in pulling apart the molecules of the crystals of which the ice is composed and thus reducing it to a form in which the molecules are held together less intimately; that is, to the liquid form. In other words, the energy which existed in the flame as the kinetic energy of molecular motion has been transformed, upon passage into the melting solid, into the potential energy of molecules which have been pulled apart against the force of their mutual attraction. *The number of calories of heat energy required to melt one gram of any substance without producing any change in its temperature is called the heat of fusion of that substance.*

*This subject should be preceded by a laboratory exercise on the curve of cooling through the point of fusion, and followed by a determination of the heat of fusion of ice. See, for example, Experiments 21 and 22 of the authors' manual.

220. Numerical value of heat of fusion of ice. Since it is found to require about 80 times as long for a given flame to melt a quantity of snow as to raise the melted snow through 1° C., we conclude that it requires about 80 calories of heat to melt 1 g. of snow or ice. This constant is, however, much more accurately determined by the method of mixtures. Thus, suppose that a piece of ice weighing, for example, 131 g. is dropped into 500 g. of water at 40° C., and suppose that after the ice is all melted the temperature of the mixture is found to be 15° C. The number of calories which have come out of the water is $500 \times (40 - 15) = 12{,}500$. But $131 \times 15 = 1965$ calories of this heat must have been used in raising the ice from 0° C. to 15° C. after it, by melting, became water at 0°. The remainder of the heat, namely $12{,}500 - 1965 = 10{,}535$, must have been used in melting the 131 g. of ice. Hence the number of calories required to melt 1 g. of ice is $\frac{10535}{131} = 80.4$.

To state the problem algebraically, let $x =$ the heat of fusion of ice. Then we have

$$131\,x + 1965 = 12{,}500;\ \text{that is, } x = 80.4.$$

According to the most careful determinations the heat of fusion of ice is 80.0 *calories.*

221. Heat given out when water freezes. Let snow and salt be added to a beaker of water until the temperature of the liquid mixture is as low as −10° C. or −12° C. Then let a test tube containing a thermometer and a quantity of pure water be thrust into the cold solution. If the thermometer is kept very quiet, the temperature of the water in the test tube will fall four or five, or even ten, degrees below 0° C. without producing solidification. But as soon as the thermometer is stirred, or a small crystal of ice is dropped into the neck of the tube, the ice crystals will form with great suddenness, and at the same time the thermometer will rise to 0° C., where it will remain until all the water is frozen.

The experiment shows in a very striking way that the process of freezing is a heat-evolving process. This was to have been expected from the principle of the conservation of energy;

for *since it takes* 80 *calories of heat energy to turn a gram of ice at* 0° *C. into water at* 0° *C., this amount of energy must reappear when the water turns back to ice.*

222. Utilization of heat evolved in freezing. The heat given off by the freezing of water is often turned to practical account; for example, tubs of water are sometimes placed in vegetable cellars to prevent the vegetables from freezing. The effectiveness of this procedure is due to the fact that the temperature at which the vegetables freeze is slightly lower than 0° C. As the temperature of the cellar falls the water therefore begins to freeze first, and in so doing evolves enough heat to prevent the temperature of the room from falling as far below 0° C. as it otherwise would.

It is partly because of the heat evolved by the freezing of large bodies of water that the temperature never falls so low in the vicinity of large lakes as it does in inland localities.

223. Latent heat. Before the time of Joule, when heat was supposed to be a weightless fluid, the heat which disappears in a substance when it melts and reappears again when it solidifies was called *latent* or *hidden* heat. Thus water was said to have a latent heat of 80 calories. This expression is still in common use, although, with the change which has taken place in our views of the nature of heat, its appropriateness is entirely gone. For the heat energy which is required to change a substance from a solid to a liquid does not exist within the liquid as concealed or hidden heat energy, but has, instead, *ceased to exist as heat energy at all*, having been transformed into the potential energy of partially separated molecules; that is, *latent heat represents the work which has been done in effecting the change of state.*

224. Melting points of crystalline substances. If a piece of ice is placed in a vessel of boiling water for an instant and then removed and wiped, it will not be found to be in the slightest degree warmer than a piece of ice which has not been

exposed to the heat of the warm water. The melting point of ice is therefore a perfectly fixed, definite temperature, above which the ice can never be raised so long as it remains ice, no matter how fast heat is applied to it. All crystalline substances are found to behave exactly like ice in this respect, each substance of this class having its characteristic melting point. The following table gives the melting points of some of the commoner crystalline substances:

Mercury .	− 39.5° C.	Sulphur . .	114° C.	Silver . .	954° C.
Ice . . .	0 "	Tin	233 "	Copper . . .	1100 "
Benzine .	7 "	Lead . . .	330 "	Cast iron .	1200 "
Acetic acid	17 "	Zinc . . .	433 "	Platinum .	1775 "
Paraffin . .	54 "	Aluminium .	650 "	Iridium . .	1950 "

We may summarize the experiments upon melting points of crystalline substances in the two following laws:

1. *The temperatures of solidification and of fusion are the same.*

2. *The temperature of the melting or solidifying substance remains constant from the moment at which melting or solidification begins until the process is completed.*

225. Fusion of noncrystalline or amorphous substances. Let the end of a glass rod be held in a Bunsen flame. Instead of changing suddenly from the solid to the liquid state, it will gradually grow softer and softer until, if the flame is sufficiently hot, a drop of molten glass will finally fall from the end of the rod.

If the temperature of the rod had been measured during this process, it would have been found to be continually rising. This behavior, so completely unlike that of crystalline substances, is characteristic of tar, wax, resin, glue, gutta-percha, alcohol, carbon, and in general of all amorphous substances. Such substances cannot be said to have any definite melting points at all, for they pass through all stages of viscosity both in melting and in solidifying. It is in virtue of this property that glass and other similar substances can be heated to softness and then molded or rolled into any desired shapes.

226. Change of volume on solidifying. One has only to reflect that ice floats, or that bottles or crocks of water burst when they freeze, in order to know that water expands upon solidifying. In fact, 1 cubic foot of water becomes 1.09 cubic feet of ice, thus expanding more than one twelfth of its initial volume when it freezes. This may seem strange in view of the fact that the molecules are certainly more closely knit together in the solid than in the liquid state; but the strangeness disappears when we reflect that in freezing the molecules of water group themselves into crystals, and that this operation presumably leaves comparatively large free spaces between different crystals, so that, although groups of individual molecules are more closely joined than before, the total volume occupied by the whole assemblage of molecules is greater.

But the great majority of crystalline substances are unlike water in this respect, for, with the exception of antimony and bismuth, they all contract upon solidifying and expand on liquefying. It is only from substances which expand, or which, like cast iron change in volume very little on solidifying, that sharp castings can be made. For it is clear that contracting substances cannot retain the shape of the mold. It is for this reason that gold and silver coins must be stamped rather than cast. Any metal from which type is to be cast must be one which expands upon solidifying, for it need scarcely be said that perfectly sharp outlines are indispensable to good type. Ordinary type metal is an alloy of lead, antimony, and copper which fulfills these requirements.

227. Effect of the expansion which water undergoes on freezing. If water were not unlike most substances in that it expands on freezing, many, if not all, of the forms of life which now exist on the earth would be impossible. For in winter the ice would sink in ponds and lakes as fast as it froze, and soon our rivers, lakes, and perhaps our oceans also would become solid ice.

The force exerted by the expansion of freezing water is very great. Steel bombs have been burst by filling them with water and exposing them on cold winter nights. One of the chief agents in the disintegration of rocks is the freezing and consequent expansion of water which has percolated into them.

228. Pressure lowers the melting point of substances which expand on solidifying. Since the outside pressure acting on the surface of a body tends to prevent its expansion, we should expect that any increase in the outside pressure would tend to prevent the solidification of substances which expand upon freezing. It ought therefore to require a lower temperature to freeze ice under a pressure of two atmospheres than under a pressure of one. Careful experiments have verified this conclusion, and have shown that the melting point of ice is lowered .0075° C. for an increase of one atmosphere in the outside pressure. Although this lowering is so small a quantity, its existence may be shown as follows:

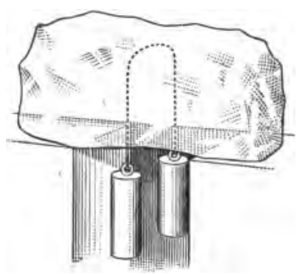


FIG. 179. Regelation

Let two pieces of ice be pressed firmly together beneath the surface of a vessel full of warm water. When taken out they will be found to be frozen together, in spite of the fact that they have been immersed in a medium much warmer than the freezing point of water. The explanation is as follows:

At the points of contact the pressure reduces the freezing point of the ice below 0° C., and hence it melts and gives rise to a thin film of water the temperature of which is slightly below 0° C. When this pressure is released the film of water at once freezes, for its temperature is below the freezing point corresponding to ordinary atmospheric pressure. The same phenomenon may be even more strikingly illustrated by the following experiment:

Let two weights of from 5 to 10 kg. be hung by a wire over a block of ice as in Fig. 179. In half an hour or less the wire will be found to have cut completely through the block, leaving the ice, however, as

†

solid as at first. The explanation is as follows: Just below the wire the ice melts because of the pressure; as the wire sinks through the layer of water thus formed, the pressure on the water is relieved and it immediately freezes again above the wire.

This process of melting under pressure and freezing again as soon as the pressure is relieved is known as *regelation*.

229. Pressure raises the freezing point of substances which contract on solidifying. Substances like paraffin, zinc, and lead which contract on solidifying have their melting points raised by an increase in pressure, for in this case the outside pressure tends to assist the molecular forces which are pulling the body out of the larger liquid form into the smaller solid form; hence this result can be accomplished at a higher temperature with pressure than without it.

We may therefore summarize the effects of pressure on the melting points of crystalline substances in the following law:

Substances which expand on solidifying have their melting points lowered by pressure, and those which contract on solidifying have their melting points raised by pressure.

QUESTIONS AND PROBLEMS

1. What is the temperature of a mixture of ice and water? What determines whether it is freezing or melting?

2. Why does ice cream seem so much colder to the teeth than ice water?

3. What is the meaning of the statement that the heat of fusion of mercury is 2.8?

4. Why will a snowball "pack" if the snow is melting, but not if it is much below 0° C.?

5. If water were like gold in contracting on solidification, what would happen to lakes and rivers during a cold winter?

6. Equal weights of hot water and ice are mixed and the result is water at 0° C. What was the temperature of the hot water?

7. Which is the more effective as a cooling agent, 100 lb. of ice at 0° C. or 100 lb. of water at the same temperature? Why?

8. How many grams of ice must be put into 500 g. of water at 50° C. to lower the temperature to 10° C.?

CHANGE OF STATE — VAPORIZATION *

230. Heat of vaporization defined. The experiments performed in Chapter IV, on Molecular Motions, led us to the conclusion that, at the free surface of any liquid, molecules frequently acquire velocities sufficiently high to enable them to lift themselves beyond the range of attraction of the molecules of the liquid and to pass off as free gaseous molecules into the space above. They taught us further that since it is only such molecules as have unusually high velocities which are able thus to escape, the *average kinetic energy* of the molecules left behind is continually diminished by this loss from the liquid of the most rapidly moving molecules, and consequently the temperature of an evaporating liquid constantly falls until the rate at which it is losing heat is equal to the rate at which it receives heat from outside sources. Evaporation, therefore, always takes place at the expense of the heat energy of the liquid. *The number of calories of heat which disappear in the formation of one gram of vapor is called the heat of vaporization of the liquid.*

231. Heat due to condensation. When molecules pass off from the surface of a liquid they rise against the downward forces exerted upon them by the liquid, and in so doing exchange a part of their kinetic energy for the potential energy of separated molecules in precisely the same way in which a ball thrown upward from the earth exchanges its kinetic energy in rising for the potential energy which is represented by the separation of the ball from the earth. Similarly, just as when the ball falls back, it regains in the descent all of the kinetic energy lost in the ascent, so when the molecules of the vapor

* It is recommended that this subject be accompanied by a laboratory determination of the boiling point of alcohol by the direct method, and by the vapor-pressure method, and that it be followed by an experiment upon the fixed points of a thermometer and the change of boiling point with pressure. See, for example, Experiments 23 and 24 of the authors' manual.

reënter the liquid, they must regain all of the kinetic energy which they lost when they passed out of the liquid. We may expect, therefore, that *every gram of steam which condenses will generate in this process the same number of calories which was required to vaporize it.*

232. Measurement of heat of vaporization. To find accurately the number of calories expended in the vaporization, or released in the condensation, of a gram of water at 100° C., we pass steam rapidly for two or three minutes from an arrangement like that shown in Fig. 180 into a vessel containing, say, 500 g. of water. We observe the initial and final temperatures and the initial and final weights of the water. If, for example, the gain in weight of the water is 16.5 g., we know that 16.5 g. of steam have been condensed. If the rise in temperature of the water is from 10° C. to 30° C., we know that $500 \times (30 - 10) = 10{,}000$ calories of heat have entered the water. If x represents the number of calories given up by 1 g. of steam in condensing, then the total heat imparted to the water by the condensation of the steam is $16.5\,x$ calories. This condensed steam is at first water at 100° C., which is then cooled to 30° C. In this cooling process it gives up $16.5 \times (100 - 30) = 1155$ calories. Therefore, equating the heat gained by the water to the heat lost by the steam, we have

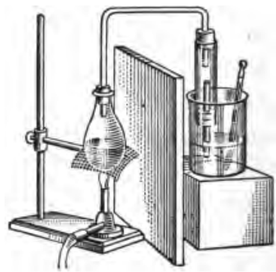


FIG. 180. Heat of vaporization of water

$$10{,}000 = 16.5\,x + 1155, \text{ or } x = 536.1.$$

This is the method usually employed for finding the heat of vaporization. The now accepted value of this constant is 536.

233. Boiling temperature defined. If a liquid is heated by means of a flame, it will be found that there is a certain temperature above which it cannot be raised, no matter how rapidly the heat is applied. This is the temperature which exists when bubbles of vapor form at the bottom of the vessel and rise to the surface, growing in size as they rise. This temperature is commonly called the *boiling temperature.*

But a second and more exact definition of the boiling point may be given. It is clear that a bubble of vapor can exist within the liquid only when the pressure exerted by the vapor within the bubble is at least equal to the atmospheric pressure pushing down on the surface of the liquid. For if the pressure within the bubble were less than the outside pressure, the bubble would immediately collapse. Therefore, *the boiling point is the temperature at which the pressure of the saturated vapor first becomes equal to the pressure existing outside.*

234. Variation of the boiling point with pressure. Since the pressure of a saturated vapor varies rapidly with the temperature, and since the boiling point has been defined as the temperature at which the pressure of the saturated vapor is equal to the outside pressure, it follows that *the boiling point must vary as the outside pressure varies.*

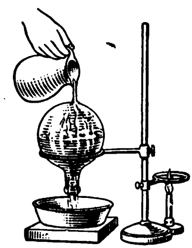


FIG. 181. Lowering the boiling point by diminishing the pressure

Thus let a round-bottomed flask be half filled with water and boiled. After the boiling has continued for a few minutes, so that the steam has driven out most of the air from the flask, let a rubber stopper be inserted, and the flask removed from the flame and inverted as shown in Fig. 181. The temperature will fall rapidly below the boiling point. But if cold water is poured over the flask, the water will again begin to boil vigorously, for the cold water, by condensing the steam, lowers the pressure within the flask, and thus enables the water to boil at a temperature lower than 100° C. The boiling will cease, however, as soon as enough vapor is formed to restore the pressure. The operation may be repeated many times without reheating.

At the city of Quito, Ecuador, water boils at 90° C., and on the top of Mt. Blanc it boils at 84° C. On the other hand, in the boiler of a steam engine in which the pressure is 100 lb. to the square inch, the boiling point of the water is 155° C.

235. Evaporation and boiling. The only essential difference between evaporation and boiling is that the former consists in the passage of molecules into the vaporous condition *from the free surface only*, while the latter consists in the passage of the molecules into the vaporous condition both at the free surface and at the surface of bubbles which exist within the body of the liquid. The only reason that vaporization takes place so much more rapidly at the boiling temperature than just below it is that the evaporating *surface* is enormously increased as soon as the bubbles form. The reason the temperature cannot be raised above the boiling point is that the surface always increases, on account of the bubbles, to just such an extent that the loss of heat because of evaporation is exactly equal to the heat received from the fire.

236. Distillation. Let water holding in solution some aniline dye be boiled in B (Fig. 182). The vapor of the liquid will pass into the tube T, where it will be condensed by the cold water which is kept in continual circulation through the jacket J. The condensed water collected in P will be seen to be free from all traces of the color of the dissolved aniline.

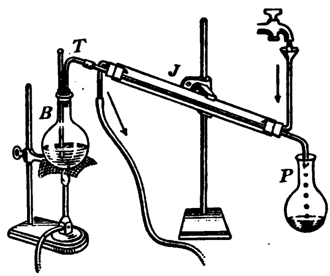


FIG. 182. Distillation

We learn then that *when solids are dissolved in liquids the vapor which rises from the solution contains none of the dissolved substance.* Sometimes it is the pure liquid in P which is desired, as in the manufacture of alcohol, and sometimes the solid which remains in B, as in the manufacture of sugar. In the white-sugar industry it is necessary that the evaporation take place at a low temperature, so that the sugar may not be scorched. Hence the boiler is kept partially exhausted by means of an air pump, thus enabling the solution to boil at considerably reduced temperatures.

237. Fractional distillation. When both of the constituents of a solution are volatile, as in the case of a mixture of alcohol and water, the vapor of both will rise from the liquid. But the one which has the lower boiling point, that is, the higher vapor pressure, will predominate. Hence, if we have in B (Fig. 182) a solution consisting of 50% alcohol and 50% water, it is clear that we can obtain in P, by evaporating and condensing, a solution containing a much larger percentage of alcohol. By repeating this operation a number of times we can increase the purity of the alcohol. This process is called *fractional distillation.* The boiling point of the mixture lies between the boiling points of alcohol and water, being higher the greater the percentage of water in the solution.

QUESTIONS AND PROBLEMS

1. After water has been "brought to a boil," will eggs become hard any quicker when the flame is high than when it is low?

2. The hot water which leaves a steam radiator may be as hot as the steam which entered it. How then has the room been warmed?

3. How much heat is given up by 30 g. of steam at 100° C. in condensing to water at the same temperature?

4. A vessel contains 300 g. of water at 0° C. and 130 g. of ice. If 25 g. of steam are condensed in it, what will be the resulting temperature?

5. How many calories are given up by 30 g. of steam at 100° C. in condensing and then cooling to 20° C.? How much water will this steam raise from 10° C. to 20° C.?

6. Why do fine bubbles rise in a vessel of water which is being heated long before the boiling point is reached? How can you distinguish between this phenomenon and boiling?

7. When water is boiled in a deep vessel it will be noticed that the bubbles rapidly increase in size as they approach the surface. Give two reasons for this.

8. Why does steam produce so much more severe burns than hot water of the same temperature?

9. Why in winter does not all the snow melt at once as soon as the temperature of the air rises above 0° C.?

10. Explain how freezing and thawing disintegrate rocks.

11. In the fall we expect frost on clear nights when the dew point is low, but not on cloudy nights when the dew point is high. Can you see any reason why a large deposit of dew will prevent the temperature of the air from falling very low?

12. Why does the distillation of a mixture of alcohol and water always result to some extent in a mixture of alcohol and water?

13. A fall of 1° C. in the boiling point is caused by rising 960 ft. How hot is boiling water at Denver, 5000 ft. above sea level?

14. How may we obtain pure drinking water from sea water?

ARTIFICIAL COOLING

238. Cooling by solution. Let a handful of common salt be placed in a small beaker of water at the temperature of the room and stirred with a thermometer. The temperature will fall several degrees. If equal weights of ammonium nitrate and water at 15° C. are mixed, the temperature will fall as low as −10° C. If the water is nearly at 0° C. when the ammonium nitrate is added, and if the stirring is done with a test tube partly filled with ice-cold water, the water in the tube will be frozen.

These experiments show that the breaking up of the crystals of a solid requires an expenditure of heat energy, as well when this operation is effected by solution as by fusion. The reason for this will appear at once if we consider the analogy between solution and evaporation. For just as the molecules of a liquid tend to escape from its surface into the air, so the molecules at the surface of the salt are tending, because of their velocities, to pass off, and are only held back by the attractions of the other molecules in the crystal to which they belong. If, however, the salt is placed in water, the attraction of the water molecules for the salt molecules aids the natural velocities of the latter to carry them beyond the attraction of their fellows. As the molecules escape from the salt crystals two forces are acting on them, the attraction of the water molecules tending to increase their velocities, and the attraction of the remaining salt molecules tending to diminish these velocities. If the latter force has a greater resultant effect than the former, the mean velocity of the molecules after they have escaped will

be diminished and the solution will be cooled. But if the attraction of the water molecules amounts to more than the backward pull of the dissolving molecules, as when caustic potash or sulphuric acid is dissolved, the mean molecular velocity is increased and the solution is heated.

239. Freezing points of solutions. If a solution of one part of common salt to ten of water is placed in a test tube and immersed in a "freezing mixture" of water, ice, and salt, the temperature indicated by a thermometer in the tube will not be zero when ice begins to form, but several degrees below zero. *The ice which does form, however, will be found, like the vapor which rises above a salt solution, to be free from salt,* and it is this fact which furnishes a key to the explanation of why the freezing point of the salt solution is lower than that of pure water. For cooling a substance to its freezing point simply means reducing its temperature, and therefore the mean velocity of its molecules, sufficiently to enable the cohesive forces of the liquid to pull the molecules together into the crystalline form. Since in the freezing of a salt solution the cohesive forces of the water are obliged to overcome the attractions of the salt molecules as well as the molecular motions, the motions must be rendered less, that is, the temperature must be made lower, than in the case of pure water in order that crystallization may occur. We should expect from this reasoning that the larger the amount of salt in solution the lower would be the freezing point. This is indeed the case. The lowest freezing point obtainable with common salt in water is $-22°$ C. This is the freezing point of a saturated solution.

240. Freezing mixtures. If snow or ice is placed in a vessel of water, the water melts it, and in so doing its temperature is reduced to the freezing point of pure water. Similarly, if ice is placed in salt water, it melts and reduces the temperature of the salt water to the freezing point of the solution. This may be one, or two, or twenty-two degrees below zero, according

to the concentration of the solution. Whether then we put the ice in pure water or in salt water, enough of it always melts to reduce the whole mass to the freezing point of the liquid, and each gram of ice which melts uses up 80 calories of heat. *The efficiency of a mixture of salt and ice in producing cold is therefore due simply to the fact that the freezing point of a salt solution is lower than that of pure water.*

The best proportions are three parts of snow or finely shaved ice to one part of common salt. If three parts of calcium chloride are mixed with two parts of snow, a temperature of $-55°$ C. may be produced. This is low enough to freeze mercury.

241. Intense cold by evaporation. If, instead of utilizing as above the heats of fusion, we utilize the larger heats of vaporization, still lower temperatures may be produced (see §§ 92 and 161).

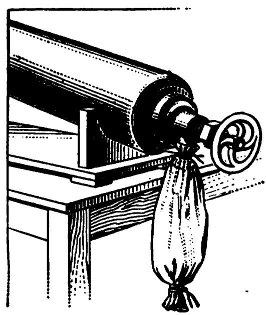


FIG. 183. Cold by rapid evaporation of carbon dioxide

Thus, if a cylinder of liquid carbon dioxide is placed as in Fig. 183 and the stopcock opened, such intense cold is produced by the rapid evaporation of the liquid which rushes out into the bag that the liquid freezes to a snowy solid. The solid itself evaporates so rapidly that it maintains, as long as it lasts, a temperature of $-80°$ C. If a little of this solid is placed in a beaker containing ether, and the mixture is stirred with a test tube filled with mercury, the mercury will be frozen solid. The chief function of the ether is to insure intimate contact between the cold solid and the test tube.

QUESTIONS AND PROBLEMS

1. Explain why salt is sometimes thrown on icy sidewalks on cold winter days.

2. When salt water freezes the ice formed is practically free from salt. What effect, then, does freezing have on the concentration of a salt solution?

3. A partially concentrated salt solution which has a freezing point of $-5°$ C. is placed in a room which is kept at $-10°$ C. Will it all freeze?

4. Give two reasons why the ocean freezes less easily than the lakes.

5. Why does pouring H_2SO_4 into water produce heat, while pouring the same substance upon ice produces cold?

6. It sometimes happens that a liquid which is unable to dissolve a solid at a low temperature will do so at a higher temperature. Why? (See § 238.)

7. When the salt in an ice-cream freezer unites with the ice to form brine, about how many calories of heat are used for each gram of ice melted? Where does it come from? If the freezing point of the salt solution were the same as that of the cream, would the latter freeze?

Industrial Applications

242. The modern steam engine. Thus far in our study of the transformations of energy we have considered only cases in which mechanical energy was transformed into heat energy. In all heat engines we have examples of exactly the reverse operation, namely, the transformation of heat energy back into mechanical energy. How this is done may best be understood from a study of various modern forms of heat engines. The invention of the form of the steam engine which is now in use is due to James Watt, who, at the time of the invention (1768), was an instrument maker in the University of Glasgow.

The operation of such a machine can best be understood from the ideal diagram shown in Fig. 184. Steam generated in the boiler *B* by the fire *F* passes through the pipe *S* into the steam chest *V*, and thence through the passage *N* into the cylinder *C*, where its pressure forces the piston *P* to the left. It will be seen from the figure that, as the driving rod *R* moves toward the left, the so-called eccentric rod *R′*, which controls the valve *V*, moves toward the right. Hence, when the piston has reached the left end of its stroke the passage *N* will have been closed, while the passage *M* will have been opened, thus throwing the pressure from the right to the left side of the piston, and at the same time putting the right end of the cylinder, which is full of spent steam, into connection with the exhaust pipe *E*. This

operation goes on continually, the rod R' opening and closing the passages M and N at just the proper moments to keep the piston moving back and forth throughout the length of the cylinder. The shaft carries a heavy flywheel W, the great

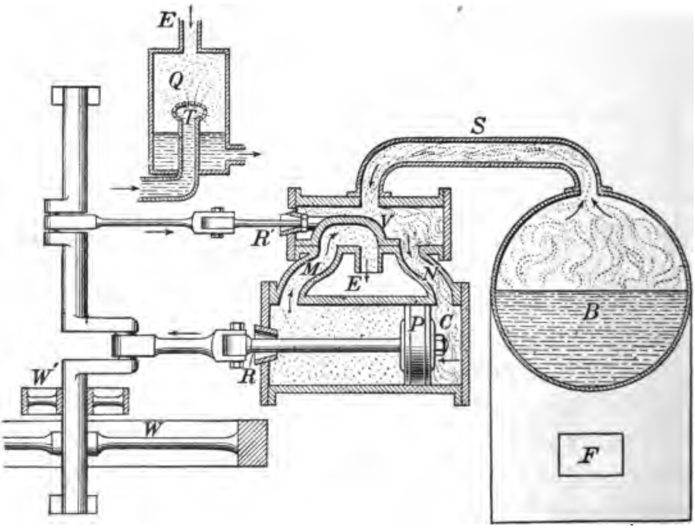


FIG. 184. Ideal diagram of a steam engine

inertia of which insures constancy in speed. The motion of the shaft is communicated to any desired machinery by means of a belt which passes over the pulley W'.

243. Condensing and noncondensing engines. In most stationary engines the exhaust E leads to a condenser which consists of a chamber Q, into which plays a jet of cold water T, and in which a partial vacuum is maintained by means of an air pump. In the best engines the pressure within Q is not more than from 3 to 5 centimeters of mercury; that is, not more than a pound to the square inch. Hence the condenser reduces the back pressure against that end of the piston which is open to the atmosphere from 15 pounds down to 1 pound, and thus

increases the effective pressure which the steam on the other side of the piston can exert. Since, however, the addition of the condenser makes the engine more expensive, more heavy, and more complicated, it is generally omitted on locomotives, and on other engines in which simplicity, compactness, and stability are of more importance than economy of fuel. It is obvious that if a noncondensing engine is to have the same effective pressure on the piston head as a condensing engine, the pressure maintained within the boiler must be about 15 pounds higher. For this reason noncondensing engines are often called high-pressure engines. Such engines can easily be recognized by the puffs of exhaust steam which they send out into the atmosphere at each stroke of the piston.

244. The eccentric. In practice the valve rod R' is not attached as in the ideal engine indicated in Fig. 184, but motion is communicated to it by a so-called *eccentric.* This consists of a circular disk K (Fig. 185) rigidly attached to the axle, but so set that its center does not coincide with the center of the axle A. The disk K rotates inside the collar C and thus communicates to the eccentric rod R' a back-and-forth motion which operates the valve V in such a way as to admit steam through M and N at the proper time.

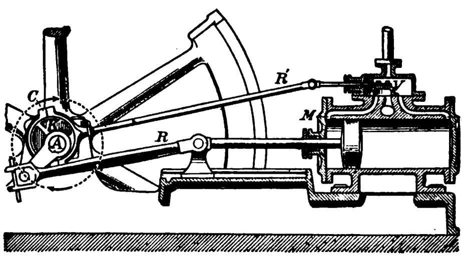


FIG. 185. The eccentric

245. The boiler. When an engine is at work steam is being removed very rapidly from the boiler; for example, a railway locomotive consumes from 3 to 6 tons of water per hour. It is therefore necessary to have the fire in contact with as large a surface as possible. In the tubular boiler this end is accomplished by causing the flames to pass through a large number of metal tubes immersed in water. The arrangement of the furnace and the boiler may be seen from the diagram of a locomotive shown in Fig. 186.

246. The draft. In order to suck the flames through the tubes B of the boiler a powerful draft is required. In locomotives this is obtained by running the exhaust steam from the cylinder C (Fig. 186) into the smokestack E through the blower F. The strong current through F

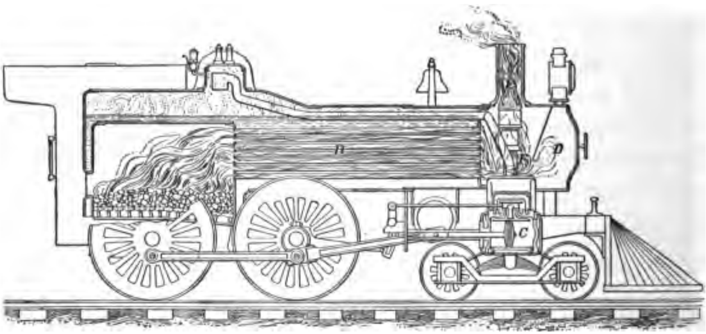


FIG. 186. Diagram of locomotive

draws with it a portion of the air from the smoke box D, thus producing within D a partial vacuum into which a powerful draft rushes from the furnace through the tubes B. The coal consumption of an ordinary locomotive is from one-fourth ton to one ton per hour.

In stationary engines a draft is obtained by making the smokestack very high. Since in this case the pressure which is forcing the air through the furnace is equal to the difference in the weights of columns of air of unit cross section inside and outside the chimney, it is evident that this pressure will be greater the greater the height of the smokestack. This is the reason for the immense heights given to chimneys in large power plants.

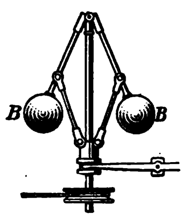


FIG. 187. The governor

247. The governor. Fig. 187 shows an ingenious device of Watt's, called a *governor*, for regulating automatically the speed with which a stationary engine runs. If it runs too fast, the heavy rotating balls B move apart and upward, and in so doing operate a valve which partially shuts off the supply of steam from the cylinder.

248. Compound engines. In an engine which has but a single cylinder the full force of the steam has not been spent when the cylinder is opened to the exhaust. The waste of energy which this entails is

obviated in the compound engine by allowing the partially spent steam to pass into a second cylinder of larger area than the first. The most efficient of modern engines have three and sometimes four cylinders of this sort, and the engines are accordingly called *triple* or *quadruple expansion engines*. Fig. 188 shows the relation between any two successive cylinders of a compound engine. By automatic devices not differing in principle from the eccentric, valves C^1, D^2, and E^2 open simultaneously and thus permit steam from the boiler to enter the small cylinder A, while the partially spent steam in the other end of the same cylinder passes through D^2 into B, and the more fully exhausted steam in the upper end of B passes out through E^2. At the upper end of the stroke of the pistons P and P', C^1, D^2, and E^2 automatically close, while C^2, D^1, and E^1 simultaneously open and thus reverse the direction of motion of both pistons. These pistons are attached to the same shaft.

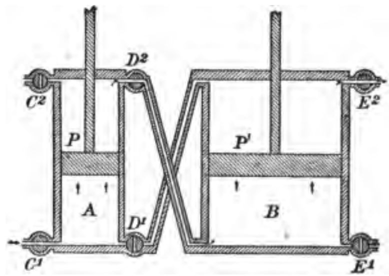


FIG. 188. Compound engine cylinders

249. Efficiency of a steam engine. We have seen that it is possible to transform completely a given amount of mechanical energy into heat energy. This is done whenever a moving body is brought to rest by means of a frictional resistance. But the inverse operation, namely, that of transforming heat energy into mechanical energy, differs in this respect, that it is only a comparatively small fraction of the heat developed by combustion which can be transformed into work. For it is not difficult to see that in every steam engine at least a part of the heat must of necessity pass over with the exhaust steam into the condenser or out into the atmosphere. This loss is so great that even in an ideal engine not more than about 23% of the heat of combustion could be transformed into work. In practice the very best condensing engines of the quadruple-expansion type transform into mechanical work not more than 17% of the heat of combustion. Ordinary locomotives utilize

at most not more than 8%. *The efficiency of a heat engine is defined as the ratio between the heat utilized, or transformed into work, and the total heat expended.* The efficiency of the best steam engines is therefore about $\frac{17}{23}$, or 75%, of that of an ideal heat engine, while that of the ordinary locomotive is only about $\frac{6}{23}$, or 26%, of the ideal limit.

250. The principle of the gas engine. Within the last decade gas engines have begun to replace steam engines to a very great extent, especially for small-power purposes. These engines are driven by properly timed explosions of a mixture of gas and air occurring within the cylinder.

Fig. 189 is a diagram illustrating the four stages into which it is convenient to divide the complete cycle of operations which goes on within such an engine. Suppose that the heavy flywheels *W* have already been set in motion. As the piston *p* moves to the right in the first stroke (see *1*) the valve *E* opens and an explosive mixture of gas and air is drawn into the cylinder through *E*.

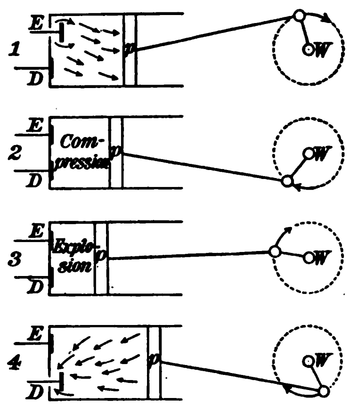


FIG. 189. Principle of the gas engine

As the piston returns to the left (see *2*) valve *E* closes, and the mixture of gas and air is compressed into a small space in the left end of the cylinder. An electric spark ignites the explosive mixture, and the force of the explosion drives the piston violently to the right (see *3*). At the beginning of the return stroke (see *4*) the exhaust valve *D* opens, and as the piston moves to the left, the spent gaseous products of the explosion are forced out of the cylinder. The initial condition is thus restored and the cycle begins over again.

Since it is only during the third stroke that the engine is receiving energy from the exploding gas, the flywheel is always made very heavy so that the energy stored up in it in the third stroke may keep the machine running with little loss of speed during the other three parts of the cycle.

251. Mechanism of the gas engine. The mechanism by which the above operations are carried out in one type of modern gas engine may be seen from a study of Figs. 190 and 191. Fig. 191 is a section of the

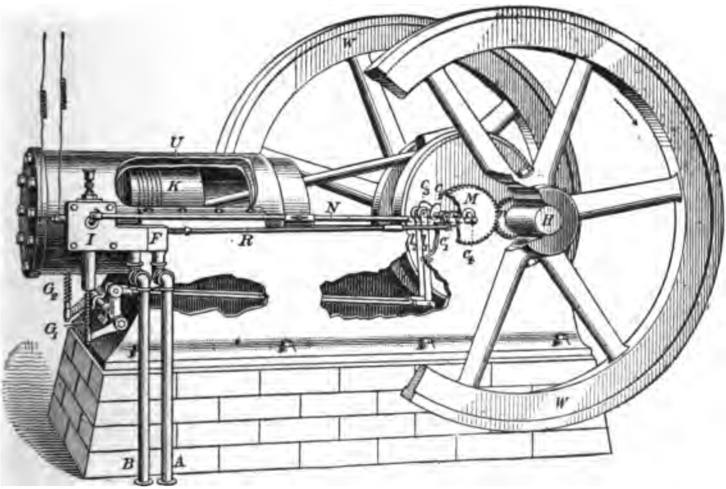


FIG. 190. The gas engine

left end of the engine shown in perspective in Fig. 190. Suppose that the flywheels W are first set in motion by hand. When the cam, or eccentric c_1 (Fig. 190), drives the rod R to the left, it opens a valve in F through which gas passes from the inlet pipe A into the mixing chamber I (Figs. 190 and 191). Here it mixes with air which entered through the pipe B. As soon as the cam c_2 has moved about to the position in which it throws the lever arm l_1 to the left, the rod G_1 is forced upward and the inlet valve E (Fig. 191) is therefore opened. This happens at the beginning of stage *1* (§ 250) when the piston K is beginning to move to the right. Hence the explosive mixture is at once drawn into C (Fig. 191). At the beginning of stage No. 3 a third eccentric rod N operated by an eccentric c_4 (Fig. 190) breaks

†

an electric contact at i (Fig. 191), and thus produces a spark which explodes the gas. At the beginning of stage *4* the cam c_3 drives the lever arm l_2 (Fig. 190) to the left, and thus with the aid of G_2 (Figs. 190 and 191) opens the exhaust valve D (Fig. 191) and thus permits the spent gases to escape. This completes the cycle.

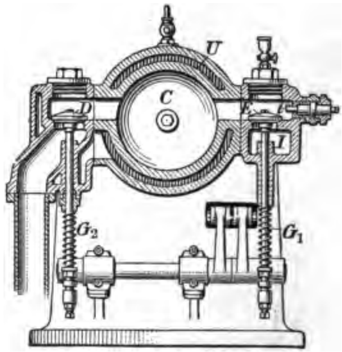


FIG. 191. Section through end of gas engine

Since each of the four cams, c_1, c_2, c_3, c_4, must open its valve once in two revolutions of the flywheel, all four of these cams are placed not on the main shaft H, but on the axle of the gear wheel M, which has twice as many teeth as has the gear wheel n on the main shaft. M therefore revolves but once while the main shaft is revolving twice. In order that the cylinder may be kept cool, it is surrounded by a jacket U through which water is kept continually circulating.

The efficiency of the gas engine is often as high as 25%, or nearly double that of the best steam engines. Furthermore, it is free from smoke, is very compact, and may be started at a moment's notice. On the other hand, the fuel, gas or gasoline, is comparatively expensive. Most automobiles are run by gasoline engines, chiefly because the lightness of the engine and of the fuel to be carried are here considerations of great importance.

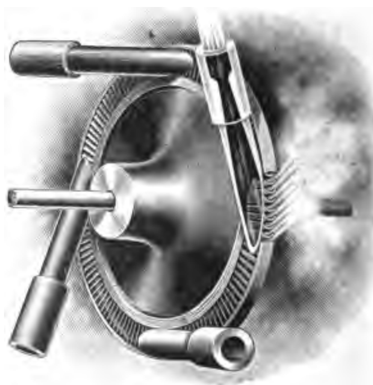


FIG. 192. The principle of the steam turbine

It has been the development of the light and efficient gas engine which has made possible man's recent conquest of the air through the use of the aëroplane and airship.

252. The steam turbine. The steam turbine represents the latest development of the heat engine. In principle it is very much like the common windmill, the chief difference being that it is steam instead of air which is driven at a high velocity

against a series of blades which are arranged radially about the circumference of the wheel which is to be set into rotation. The steam, however, unlike the wind, is always directed by nozzles at the angle of greatest efficiency against the blades (see Fig. 192). Furthermore, since the energy of the steam is not nearly spent after it has passed through one set of blades, such as that shown in Fig. 192, it is in practice always passed through a whole series of such sets (Fig. 193), every alternate

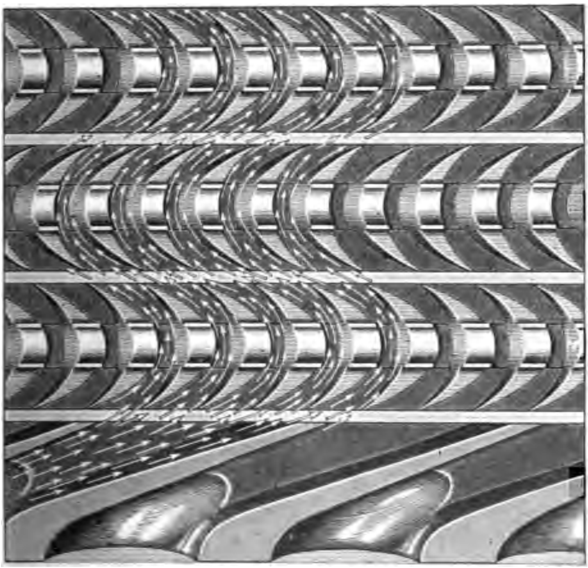


FIG. 193. Path of steam in Curtis's turbine

row of which is rigidly attached to the rotating shaft, while the intermediate rows are fastened to the immovable outer jacket of the engine, and only serve as guides to redirect the steam at the most favorable angle against the next row of movable blades. In this way the steam is kept alternately bounding from fixed to movable blades till its energy is expended. The number of rows of blades is often as high as sixteen.

Turbines are at present coming rapidly into use, chiefly for large-power purposes. Their advantages over the reciprocating steam engine lie first in the fact that they run with almost no jarring, and therefore

require much lighter and less expensive foundations; and second, in the fact that they occupy less than one tenth the floor space of ordinary engines of the same capacity. Their efficiency is fully as high as that of the best reciprocating engines. The highest speeds attained by vessels at sea, namely about 40 miles per hour, have been made with the aid of steam turbines. The largest vessel which has thus far ever been launched, the Hamburg-American steamer *Imperator*, 919 feet long, 98 feet wide, 100 feet high (from the keel to the top of her ninth deck), having a total "displacement" of 70,000 tons and a speed of $22\frac{1}{2}$ knots, is driven by four steam turbines having a total horse power of 72,000. One of the immense rotors contains 50,000 blades and develops 22,000 horse power.

253. The liquid-air machine. In the actual manufacture of liquid air a pressure pump *P* (Fig. 194) forces the air into a spiral coil *C* under a pressure of about 200 atmospheres. The heat produced by this compression is carried off by running water which circulates through the tank *R*. The cock *c* is then opened and the air expands from 200 atmospheres down to 1 atmosphere. In this expansion the temperature falls. This cooled air returns through a larger spiral *S* which incloses the high-pressure spiral *s*, and thus cools off the air which is coming down to the expansion valve through the inner spiral. In this process the temperature of the air issuing from the valve *c* continuously falls until it reaches the temperature of liquefaction. Liquid air can then be drawn off through the stopcock *R*. The air which escapes liquefaction returns to the compressor, where it is again forced into the inner spiral *s*, together with a certain amount of air which enters from the outside at *o*.

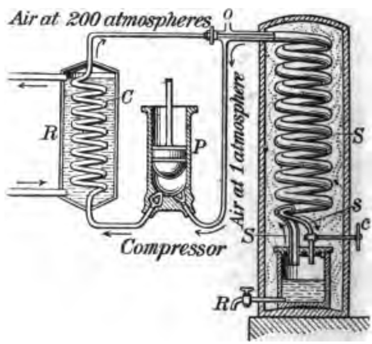


FIG. 194. The liquefaction of air

254. Manufactured ice. In the great majority of modern ice plants the low temperature required for the manufacture of the ice is produced by the rapid evaporation of liquid ammonia. At ordinary temperatures ammonia is a gas, but it may be liquefied by pressure alone. At 80° F. a pressure of 155 pounds per square inch, or about 10 atmospheres, is required to produce its liquefaction. Fig. 195 shows the essential parts

THE FISK STREET STATION OF THE COMMONWEALTH-EDISON COMPANY, CHICAGO, ILLINOIS

The largest steam turbo-generating station in the world; contains ten 12,000-kilowatt, 25-cycle, 9000-volt, alternating-current generators, driven at the rate of 750 revolutions per minute by vertical-shaft Curtis steam turbines (turbines below, generators above). The total capacity of the station is 160,000 horse power. This one station supplies nearly one half of the total electric energy used by the city of Chicago. Larger individual machines are found, however, at the Northwest Station, Chicago, where 20,000-kilowatt turbo-generators are installed. None larger than these have thus far been built. The floor space occupied by the Curtis steam turbines is about one tenth as much as would be required by reciprocating engines of the same horse power.

of an ice plant. The compressor, which is usually run by a steam engine, forces the gaseous ammonia under a pressure of 155 pounds into the condenser coils shown on the right, and there liquefies it. The heat of condensation of the ammonia is carried off by the running water which constantly circulates about the condenser coils. From the condenser the liquid ammonia is allowed to pass very slowly through the regulating valve *V* into the coils of the evaporator, from which the evaporated ammonia is pumped out so rapidly that the pressure within the coils does not rise above 34 pounds. It will be noted from the figure that the same pump which is there labeled the compressor exhausts the

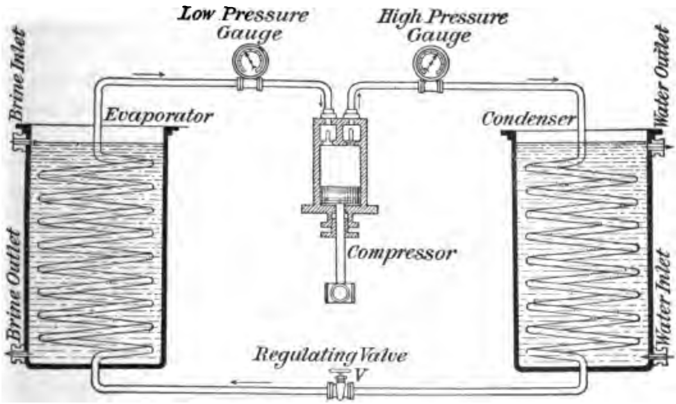


FIG. 195. Compression system of ice manufacture

ammonia from the evaporating coils and compresses it in the condensing coils; for, just as in Fig. 194, the valves are so arranged that the pump acts as an exhaust pump on one side and as a compression pump on the other. The rapid evaporation of the liquid ammonia under the reduced pressure existing within the evaporator cools these coils to a temperature of about 5° F. for every gram which evaporates. The brine with which these coils are surrounded has its temperature thus reduced to about 16° or 18° F. This brine is made to circulate about the cans containing the water to be frozen.

255. Cold storage. The artificial cooling of factories and cold-storage rooms is accomplished in a manner exactly similar to that employed in the manufacture of ice. The brine is cooled exactly as described above, and is then pumped through coils placed in the rooms to be

cooled. In some systems carbon dioxide is used in place of ammonia, but the principle is in no way altered. Sometimes, too, the brine is dispensed with, and the air of the rooms to be cooled is forced by means of fans directly over the cold coils containing the evaporating ammonia or carbon dioxide. It is in this way that theaters and hotels are cooled.

QUESTIONS AND PROBLEMS

1. Why is not the boiling point of water in the boiler of a steam engine 100° C.?

2. If the average pressure in the cylinder of a steam engine is 10 kilos to the square centimeter, and the area of the piston is 427 sq. cm., how much work is done by the piston in a stroke of length 50 cm.? How many calories did the steam lose in this operation?

3. The total efficiency of a certain 600 H.P. locomotive is 6%; 8000 calories of heat are produced by the burning of 1 g. of the best anthracite coal; how many kilos of such coal are consumed per hour by this engine? (Take 1 H.P. = 746 watts and 1 calorie per second 4.2 watts.)

4. It requires a force of 300 kilos to drive a given boat at a speed of 15 knots (25 km.). How much coal will be required to run this boat at this speed across a lake 300 km. wide, the efficiency of the engines being 7% and the coal being of a grade to furnish 6000 calories per gram?

5. What total pushing force do the propellers of the *Lusitania* exert when she is using her maximum horse power (70,000) and is running at 25 knots (46.4 km.) per hour?

6. The best triple-expansion marine engines consume 1.5 lb. of coal per hour per H.P., or .91 kg. per indicated kilowatt (see Ency. Brit., "Steam Engine"). If the coal furnishes 6000 calories per gram, what is the efficiency?

7. If liquid air is placed in an open vessel, its temperature will not rise above −182° C. Why not? Suggest a way in which its temperature could be made to rise above −182° C., and a way in which it could be made to fall below that temperature.

8. The average locomotive has an efficiency of about 6%. What horse power does it develop when it is consuming 1 ton of coal per hour? (See Problem 6, above.)

9. What pull does a 1000 H.P. locomotive exert when it is running at 25 miles per hour and exerting its full horse power?

CHAPTER X

THE TRANSFERENCE OF HEAT

CONDUCTION

256. Conduction in solids. If one end of a short metal bar be held in the fire, the other end soon becomes too hot to hold. But if the metal rod is replaced by one of wood or glass, the end away from the flame will not be appreciably heated.

This experiment and others like it show that nonmetallic substances possess a much smaller ability to conduct heat than do metallic substances. But although all metals are good conductors as compared with nonmetals, they differ widely among themselves in their conducting powers.

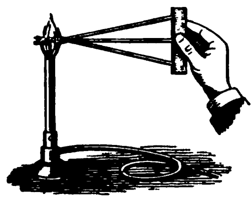


FIG. 196. Differences in heat conductivities of metals

Let copper, iron, and German silver wires 50 cm. long and about 3 mm. in diameter be twisted together at one end as in Fig. 196, and let a Bunsen flame be applied to the twisted ends. Let a match be slid slowly from the cool end of each wire toward the hot end, until the heat from the wire ignites it. The copper will be found to be the best conductor and the German silver the poorest.

In the following table some common substances are arranged in the order of their heat conductivities. The measurements have been made by a method not differing in principle from that just described. For convenience, silver is taken as 100.

Silver	100	Tin	15	Mercury	1.35
Copper	74	Iron	12	Ice	.21
Gold	53	Lead	8.5	Glass	.046
Brass	27	German silver	6.3	Hard rubber	.024

257. Conduction in liquids and gases. Let a small piece of ice be held by means of a glass rod in the bottom of a test tube full of ice water. Let the upper part of the tube be heated with a Bunsen burner as in Fig. 197. The upper part of the water may be boiled for some time without melting the ice. Water is evidently, then, a very poor conductor of heat. The same thing may be shown more strikingly as follows: The bulb of an air thermometer is placed only a few millimeters beneath the surface of water contained in a large funnel arranged as in Fig. 198. If, now, a spoonful of ether is poured on the water and set on fire, the index of the air thermometer will show scarcely any change, in spite of the fact that the air thermometer is a very sensitive indicator of changes in temperature.

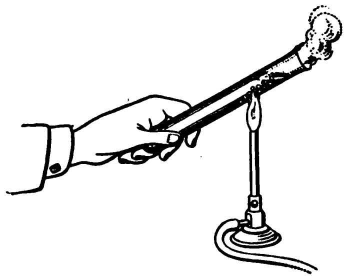


FIG. 197. Water a nonconductor

Careful measurements of the conductivity of water show that it is only about $\frac{1}{1200}$ of that of silver. The conductivity of gases is even smaller, not amounting on the average to more than $\frac{1}{25}$ that of water.

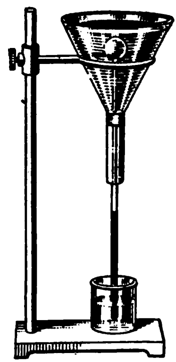


FIG. 198. Burning ether on the water does not affect the air thermometer

258. Conductivity and sensation. It is a fact of common observation that on a cold day in winter a piece of metal feels much colder to the hand than a piece of wood, notwithstanding the fact that the temperature of the wood must be the same as that of the metal. On the other hand, if the same two bodies had been lying in the hot sun in midsummer, the wood might be handled without discomfort, but the metal would be uncomfortably hot. The explanation of these phenomena is found in the fact that the iron, being a much better conductor than the wood, removes heat from

the hand much more rapidly in winter, and imparts heat to the hand much more rapidly in summer, than does the wood. In general, the better a conductor the hotter it will feel to a hand colder than itself, and the colder to a hand hotter than itself. Thus in a cold room oilcloth, a fairly good conductor, feels much colder to the touch than a carpet, a comparatively poor conductor. For the same reason linen clothing feels cooler to the touch in winter than woolen goods.

259. The rôle of air in nonconductors. Feathers, fur, felt, etc., make very warm coverings, because they are very poor conductors of heat and thus prevent the escape of heat from the body. Their poor conductivity is due in large measure to the fact that they are full of minute spaces containing air, and gases are the best nonconductors of heat. It is for this reason that freshly fallen snow is such an efficient protection to vegetation. Farmers always fear for their fruit trees and vines when there is a severe cold snap in winter, unless there is a coating of snow on the ground to prevent a deep freezing.

260. The Davy safety lamp. Let a piece of wire gauze be held above an open gas jet, and a match applied above the gauze. The flame will be found to burn above the gauze as in Fig. 199, (1), but it will not pass through to the lower side. If it is ignited below the gauze, the flame will not pass through to the upper side but will burn as shown in Fig. 200, (2).

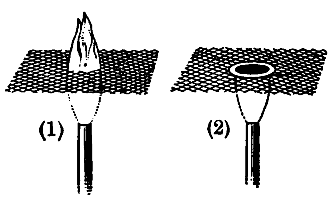


FIG. 199. A flame will not pass through wire gauze

The explanation is found in the fact that the gauze conducts the heat away from the flame so rapidly that the gas on the other side is not raised to the temperature of ignition. Safety lamps used by miners are completely incased in gauze, so that if the mine is full of inflammable gases, they are not ignited by the lamp outside of the gauze.

QUESTIONS AND PROBLEMS

1. Why is the outer pail of an ice-cream freezer made of thick wood and the inner can of thin metal?

2. Why do firemen wear flannel shirts in summer to keep cool and in winter to keep warm?

3. Why do we wrap up ice cream in thick woolen blankets in summer to keep it from melting?

4. If ice in a refrigerator is wrapped up in blankets, what is the effect on the ice? on the refrigerator?

5. If a piece of paper is wrapped tightly around a metal rod and held for an instant in a Bunsen flame, it will not be scorched. If held in a flame when wrapped around a wooden rod, it will be scorched at once. Explain.

6. If one touches the pan containing a loaf of bread in a hot oven, he receives a much more severe burn than if he touches the bread itself, although the two are at the same temperature. Explain.

7. Why are plants often covered with paper on a night when frost is expected?

8. Why will a moistened finger or the tongue freeze instantly to a piece of iron on a cold winter's day, but not to a piece of wood?

9. Does clothing ever afford us heat in winter? How, then, does it keep us warm?

CONVECTION

261. Convection in liquids. Although the conducting power of liquids is so small, as was shown in the experiment of § 257, they are yet able, under certain circumstances, to transmit heat much more effectively than solids. Thus, if the ice in the experiment of Fig. 197 had been placed at the top and the flame at the bottom, the ice would have been melted very quickly. This shows that heat is transferred with enormously greater readiness from the bottom of the tube toward the top than from the top toward the bottom. The mechanism of this heat transference will be evident from the following experiment:

Let a round-bottomed flask (Fig. 200) be half filled with water and a few crystals of magenta dropped into it. Then let the bottom of the flask be heated with a Bunsen burner. The magenta will reveal the fact

that the heat sets up currents the direction of which is upward in the region immediately above the flame but downward at the sides of the vessel. It will not be long before the whole of the water is uniformly colored. This shows how thorough is the mixing accomplished by the heating.

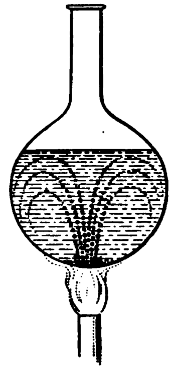


Fig. 200. Convection currents

The explanation of the phenomenon is as follows: The water nearest the flame became heated and expanded. It was thus rendered less dense than the surrounding water, and was therefore forced to the top by the pressure transmitted from the colder and therefore denser water at the sides which then came in to take its place.

It is obvious that this method of heat transfer is applicable only to fluids. The essential difference between it and conduction is that the heat is not transferred from molecule to molecule throughout the whole mass, but is rather transferred by the bodily movement of comparatively large masses of the heated liquid from one point to another. This method of heat transference is known as *convection*.

262. Winds and ocean currents. Winds are convection currents in the atmosphere caused by unequal heating of the earth by the sun. Let us consider, for example, the land and sea breezes so familiar to all dwellers near the coasts of large bodies of water. During the daytime the land is heated more rapidly than the sea, because the specific heat of water is much greater than that of earth. Hence the hot air over the land expands and is forced up by the colder and denser air over the sea which moves in to take its place. This constitutes the sea breeze which blows during the daytime, usually reaching its maximum strength in the late afternoon. At night the earth cools more rapidly than the sea and hence the direction of the wind is reversed. The effect of these breezes is seldom felt more than twenty-five miles from shore.

Ocean currents are caused partly by the unequal heating of the sea and partly by the direction of the prevailing winds. In general both winds and currents are so modified by the configuration of the continents that it is only over broad expanses of the ocean that the direction of either can be predicted from simple considerations.

Radiation

263. A third method of heat transference. There are certain phenomena in connection with the transfer of heat for which conduction and convection are wholly unable to account. For example, if one sits in front of a hot grate fire, the heat which he feels cannot come from the fire by convection, because the currents of air are moving toward the fire rather than away from it. It cannot be due to conduction, because the conductivity of air is extremely small and the colder currents of air moving toward the fire would more than neutralize any transfer outward due to conduction. There must therefore be some way in which heat travels across the intervening space other than by conduction or convection.

It is still more evident that there must be a third method of heat transfer when we consider the heat which comes to us from the sun. Conduction and convection take place only through the agency of matter; but we know that the space between the earth and the sun is not filled with ordinary matter, or else the earth would be retarded in its motion through space. *Radiation* is the name given to this third method by which heat travels from one place to another, and which is illustrated in the passing of heat from a grate fire to a body in front of it, or from the sun to the earth.

264. The nature of radiation. The nature of radiation will be discussed more fully in Chapter XXI. It will be sufficient here to call attention to the following differences between conduction, convection, and radiation.

First, while conduction and convection are comparatively slow processes, the transfer of heat by radiation takes place with the enormous speed with which light travels, namely 186,000 miles per second. That the two speeds are the same is evident from the fact that at the time of an eclipse of the sun the shutting off of heat from the earth is observed to take place at the same time as the shutting off of light.

Second, radiant heat travels in straight lines, while conducted or convected heat may follow the most circuitous routes. The proof of this statement is found in the familiar fact that radiation may be cut off by means of a screen placed directly between a source and the body to be protected.

Third, radiant heat may pass through a medium without heating it. This is shown by the fact that the upper regions of the atmosphere are very cold, even in the hottest days in summer, or that a hothouse may be much warmer than the glass through which the sun's rays enter it.

The Heating and Ventilating of Buildings

265. The principle of ventilation. The heating and ventilating of buildings are accomplished chiefly through the agency of convection.

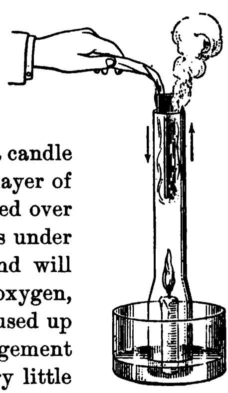


Fig. 201. Convection currents in air

To illustrate the principle of ventilation, let a candle be lighted and placed in a vessel containing a layer of water (Fig. 201). When a lamp chimney is placed over the candle so that the bottom of the chimney is under the water, the flame will slowly die down and will finally be extinguished. This is because the oxygen, which is essential to combustion, is gradually used up and no fresh supply is possible with the arrangement described. If the chimney is raised even a very little above the water, the dying flame will at once brighten. Why? If a metal or cardboard partition is inserted in the chimney, as in Fig. 201, the flame will burn

continuously, even when the bottom of the chimney is under water. The reason will be clear if a piece of burning touch paper (blotting paper soaked in a solution of potassium nitrate and dried) is held over the chimney. The smoke will show the direction of the air currents. If the chimney is a large one, in order that the first part of the above experiment may succeed, it may be necessary to use two candles; for too small a heated area permits the formation of downward currents at the sides.

266. Ventilation of houses. In order to secure satisfactory ventilation it is estimated that a room should be supplied with 2000 cubic feet of fresh air per hour for each occupant (a gas burner is equivalent in oxygen consumption to four persons). A current of air moving with a speed great enough to be just perceptible has a velocity of about 3 feet per second. Hence the area of opening required for each person when fresh air is entering at this speed is about 25 or 30 square inches. The manner of supplying this requisite amount of fresh air in dwelling houses depends upon the method of heating employed.

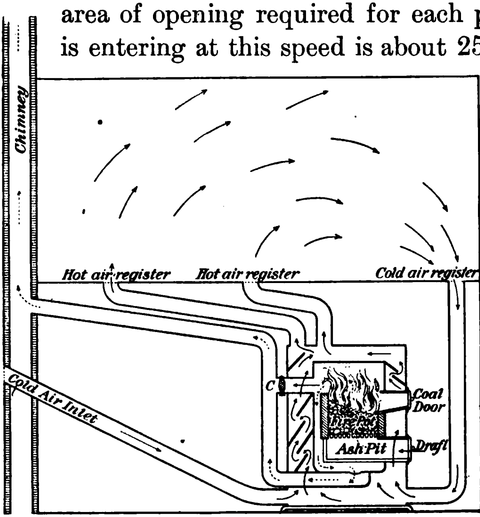


FIG. 202. Hot-air heating

If a house is heated by stoves or fireplaces, no special provision for ventilation is needed. The foul air is drawn up the chimney with the smoke, and the fresh air which replaces it finds entrance through cracks about the doors and windows and through the walls.

267. Hot-air heating. In houses heated by hot-air furnaces an air duct ought always to be supplied for the entrance of fresh cold air, in the manner shown in Fig. 202 (see "cold-air inlet"). This cold air

from out of doors is heated by passing in a circuitous way, as shown by the arrows, over the outer jacket of iron which covers the fire box. It is then delivered to the rooms. Here a part of it escapes through windows and doors, and the rest returns through the cold-air register to be reheated, after being mixed with a fresh supply from out of doors.

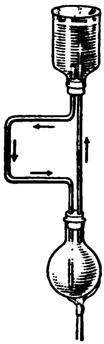


FIG. 203. Principle of hot-water heating

The course of the air which feeds the fire is shown by the dotted arrows. When the fire is first started, in order to gain a strong draft the damper C is opened so that the smoke may pass directly up the chimney. After the fire is under way the damper C is closed so that the smoke and hot gases from the furnace must pass, as indicated by the arrows, over a roundabout path, in the course of which they give up the major part of their heat to the steel walls of the jacket, which in turn pass it on to the air which is on its way to the living rooms.

268. Hot-water heating. To illustrate the principle of hot-water heating let the arrangement shown in Fig. 203 be set up, the upper vessel being filled with colored water, and then let a flame be applied to the lower vessel. The colored water will show that the current moves in the direction of the arrows.

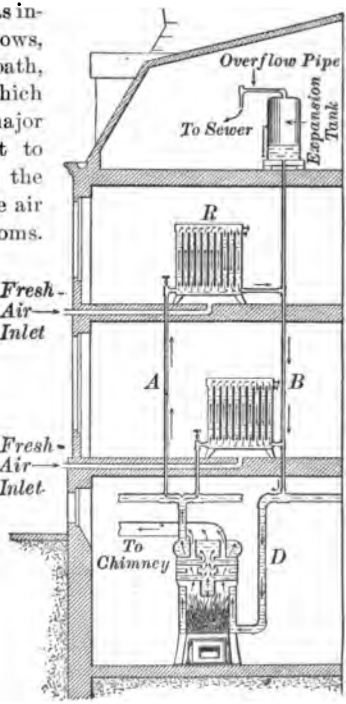


FIG. 204. Hot-water heater

The actual arrangement of boiler and radiators in one system of hot-water heating is shown in Fig. 204. The water heated in the furnace rises directly through the pipe A to a radiator R, and returns again to the bottom of the furnace through the pipes B and D. The circulation is maintained because the column of water in A is hotter and therefore lighter than the water in the return pipe B.

In the most common system of hot-water or steam heating, the so-called *direct-radiation* system, no provision whatever is made for ventilation. The occupants must depend entirely on open windows for their supply of fresh air. In the so-called *direct-indirect* system, shown in Fig. 204, fresh air is introduced through the radiator itself. The *indirect* system differs from this only in that steam or hot-water coils, instead of being in the rooms, are suspended from the ceiling of the basement in wooden boxes (Fig. 205). The arrows indicate the direction which the air currents take as they pass from out of doors, through the heating coils, and finally through the register into the room.

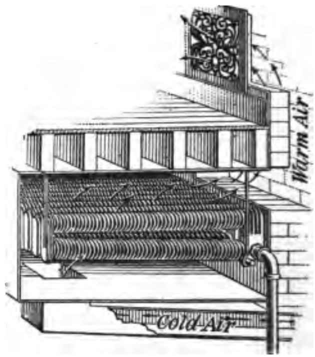


FIG. 205. Indirect system

QUESTIONS AND PROBLEMS

1. If 2 metric tons of coal are burned per month in your house and if your furnace allows one third of the heat to go up the chimney, how many calories do you use per day? (Take 1 g. as yielding 6000 calories.)

2. Explain the underlying principles of the fireless cooker.

3. Why is a hollow wall filled with sawdust a better nonconductor of heat than the same wall filled with air alone?

4. In a system of hot-water heating why does the return pipe always connect at the bottom of the boiler, while the outgoing pipe connects with the top?

5. Which is thermally more efficient, a cook stove or a grate? Why?

6. When a room is heated by a fireplace, which of the three methods of heat transference plays the most important rôle?

7. Which methods of heat transfer are most important in systems of direct and of indirect radiation?

8. Why do you blow on your hands to warm them in winter and fan yourself for coolness in summer?

9. If you open a door between a warm and a cold room, in what direction will a candle flame be blown which is placed at the top of the door? Explain.

10. Why is felt a better conductor of heat when it is very firmly packed than when loosely packed?

CHAPTER XI

MAGNETISM *

General Properties of Magnets

269. Magnets. It has been known for many centuries that some specimens of the ore known as magnetite (Fe_3O_4) have the property of attracting small bits of iron and steel. This ore probably received its name from the fact that it is especially abundant in the province of Magnesia, in Thessaly, although the Latin writer Pliny says that the word "magnet" is derived from the name of the Greek shepherd Magnes, who, on the top of Mount Ida, observed the attraction of a large stone for his iron crook. Pieces of this ore which exhibit this attractive property are known as *natural magnets.*

It was also known to the ancients that artificial magnets may be made by stroking pieces of steel with natural magnets, but it was not until about the twelfth century that the discovery was made that *a suspended magnet will assume a north-and-south position.* Because of this latter property natural ~~magnets~~ became known as lodestones (leading stones), and magnets, either artificial or natural, began to be used for determining directions. The first mention of the use of the compass in Europe is in 1190. It is thought to have been introduced from China.

Magnets are now made either by stroking bars of steel in one direction with a magnet, or by passing electric currents

* This chapter should either be accompanied or preceded by laboratory experiments on magnetic fields and on the molecular nature of magnetism. See, for example, Experiments 25 and 26 of the authors' manual.

†

about the bars in a manner to be described later. The form shown in Fig. 206 is called a *bar magnet*, that shown in Fig. 207 a *horseshoe magnet.* The latter form is the more common.

FIG. 206. A bar magnet

If a magnet is dipped into iron filings, the filings will be seen to cling in tufts near the ends but scarcely at all near the middle (Fig. 208). These places near the ends of a magnet at which its strength seems to be concentrated are called the *poles* of the magnet. The end of a freely swinging magnet which points to the north is designated as the north-seeking, or simply the *north pole* (N); and the other end as the south-seeking, or the *south pole* (S). *The direction in which a compass needle points is called the magnetic meridian.*

FIG. 207. A horseshoe magnet

270. The laws of magnetic attraction and repulsion. In the experiment with the iron filings no particular difference was observed between the action of the two poles. That there is a difference, however, may be shown by experimenting with two magnets, either of which may be suspended (see Fig. 209). If two *N* poles are brought near one another, they are found to repel each other. The *S* poles likewise are found to repel each other. But the *N* pole of one magnet is found to be attracted by the *S* pole of another. The results of these experiments may be summarized in a general law: *Magnet poles of like kind repel each other, while poles of unlike kind attract.*

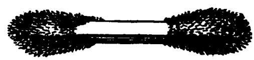


FIG. 208. Iron filings clinging to bar magnet

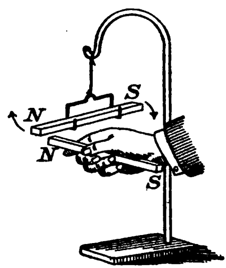


FIG. 209. Magnetic attractions and repulsions

The force which any two poles exert upon each other has been found, like the force of gravitation, to vary inversely as the square of the distance between them.

A unit pole is defined as a pole which when placed at a distance of 1 *centimeter from an exactly similar pole repels it with a force of* 1 *dyne.*

271. Magnetic materials. Iron and steel are the only substances which exhibit magnetic properties to any marked degree. Nickel and cobalt are also attracted appreciably by strong magnets. Bismuth, antimony, and a number of other substances are actually repelled instead of attracted, but the effect is very small. It has recently been found possible to make quite strongly magnetic alloys out of certain nonmagnetic materials. For example, a mixture of 65% copper, 27% manganese, and 8% aluminum is quite strongly magnetic. These are called Heusler alloys. For practical purposes, however, iron and steel may be considered as the only magnetic materials.

272. Magnetic induction. If a small unmagnetized nail is suspended from one end of a bar magnet, it is found that a second nail may be suspended from this first nail, which itself acts like a magnet, a third from the second, etc., as shown in Fig. 210. But if the bar magnet is carefully pulled away from the first nail, the others will instantly fall away from each other, thus showing that the nails were strong magnets only so long as they were in contact with the bar magnet. Any piece of soft iron may be thus magnetized *temporarily* by holding it in contact with a permanent magnet. Indeed, it is not necessary that there be actual contact, for if a nail is simply brought near to the permanent magnet it is found to become a magnet. This may be proved by presenting some iron filings to one end of a nail held near a magnet in the manner shown in Fig. 211. Even inserting a plate of glass, or of copper, or of any other material except iron between S and N will not

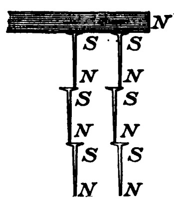


FIG. 210. Magnetism induced by contact

change appreciably the number of filings which cling to the end of S', a fact which shows that *nonmagnetic materials are transparent to magnetic forces.* But as soon as the permanent magnet is removed most of the filings will fall. *Magnetism produced in this way by the mere presence of adjacent magnets, with or without contact, is called induced magnetism.* If the induced magnetism of the nail in Fig. 211 is tested with a compass needle, it is found that the *remote* induced pole is of the same kind as the inducing pole, while the *near* pole is of unlike kind. This is the general law of magnetic induction.

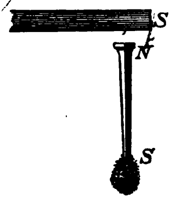


FIG. 211. Magnetism induced without contact

Magnetic induction explains the fact that a magnet attracts an unmagnetized piece of iron, for it first magnetizes it by induction, so that the near pole is unlike the inducing pole, and the remote pole like the inducing pole; and then, since the two unlike poles are closer together than the like poles, the attraction overbalances the repulsion and the iron is drawn toward the magnet. Magnetic induction also explains the formation of the tufts of iron filings shown in Fig. 208, each little filing becoming a temporary magnet such that the end which points toward the inducing pole is unlike this pole, and the end which points away from it is like this pole. The bushlike appearance is due to the repulsive action which the outside free poles exert upon each other.

273. Retentivity and permeability. A piece of soft iron will very easily become a strong temporary magnet, but when removed from the influence of the magnet it loses practically all of its magnetism. On the other hand, a piece of steel will not be so strongly magnetized as the soft iron, but it will retain a much larger fraction of its magnetism after it is removed from the influence of the permanent magnet. This power of resisting either magnetization or demagnetization is called

retentivity. Thus steel has a much greater retentivity than wrought iron, and, in general, the harder the steel the greater its retentivity.

A substance which has the property of becoming strongly magnetic under the influence of a permanent magnet, whether it has a high retentivity or not, is said to possess *permeability* in large degree. Thus iron is much more permeable than nickel.

274. Magnetic lines of force. If we could separate the *N* and *S* poles of a small magnet so as to get an independent *N* pole, and were to place this *N* pole near the *N* pole of a bar magnet, it would move over to the *S* pole along some curved path similar to that shown in Fig. 212. The reason it would move in a curved path is that it would be simultaneously repelled by the *N* pole of the bar magnet, and attracted by its *S* pole, and the relative strengths of these two forces would continually change, as the relative distances of the moving pole from these two poles changed.

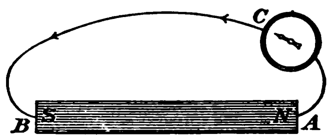


FIG. 212. A line of force set up by the magnet *AB*

To verify this conclusion let a strongly magnetized sewing needle be floated in a small cork in a shallow dish of water, and let a bar or horseshoe magnet be placed just above or just beneath the dish (see Fig. 213). The cork and needle will then move as would an independent pole, since the remote pole of the needle is so much farther from the magnet than the near pole that its influence on the motion is very small. The cork will actually be found to move in a curved path from *N* to *S*.

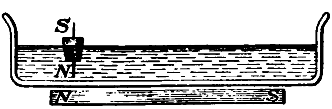


FIG. 213. Showing direction of a motion of an isolated pole near magnet

Any path which an independent *N* pole would take in going from *N* to *S* is called a *line of force*. The simplest way of finding the direction of this path at any point near a magnet is to hold a short compass needle at the point considered. The compass needle sets itself along

the line in which its poles would move if independent, that is, along the line of force which passes through the given point (see C, Fig. 212).

275. Fields of force. The region about a magnet in which its magnetic forces can be detected is called its *field of force.* The easiest way of gaining an idea of the way in which the lines of force are arranged in the magnetic field about any magnet is to sift iron filings upon a piece of paper placed immediately over the magnet. Each little filing becomes a temporary magnet by induction, and therefore, like the compass needle, sets itself in the direction of the line of force at

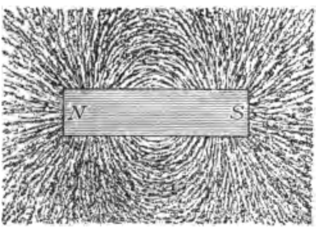


FIG. 214. Arrangement of iron filings about a bar magnet

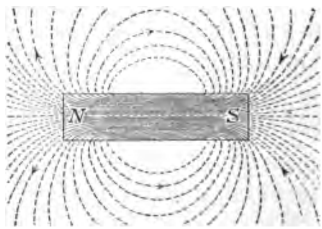


FIG. 215. Ideal diagram of field of a bar magnet

the point where it is. Fig. 214 shows how the filings arrange themselves about a bar magnet. Fig. 215 is the corresponding ideal diagram, showing the lines of force emerging from the N pole and passing about in curved paths to the S pole. It is customary to imagine these lines as returning through the magnet from S to N in the manner shown, so that each line is thought of as a closed curve. This convention was introduced by Faraday, and has been found of great assistance in correlating the facts of magnetism.

A magnetic field of unit strength is defined as a field in which a unit magnet pole experiences 1 *dyne of force.* It is customary to represent graphically such a field by drawing one line per

square centimeter through a surface such as $ABCD$ (Fig. 216) taken at right angles to the lines of force. If a unit N pole between N and S (Fig. 216) were pushed toward S with a force of 1000 dynes, the strength of the field would be 1000 units and it would be represented by 1000 lines per square centimeter.

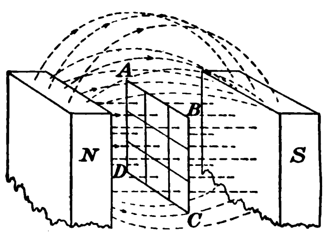


FIG. 216. The strength of a magnetic field is represented by the number of lines of force per square centimeter

276. Molecular nature of magnetism. If a small test tube full of iron filings be stroked from end to end with a magnet, it will be found to have become itself a magnet; but it will lose its magnetism as soon as the filings are shaken up. If a magnetized knitting needle is heated red-hot, it will be found to have lost its magnetism completely. Again, if such a needle is jarred, or hammered, or twisted, the strength of its poles, as measured by their ability to pick up tacks or iron filings, will be found to be greatly diminished.

These facts point to the conclusion that magnetism has something to do with the arrangement of the molecules, since causes which violently disturb the molecules of a magnet weaken its magnetism. Again, if a magnetized needle is broken, each part will be found to be a complete magnet; that is, two new poles will appear at the point of breaking, a new N pole on the part which has the original S pole, and a new S pole on the part which has the original N pole. The subdivision may be continued indefinitely, but always with the same result, as indicated in Fig. 217. This suggests that the molecules of a magnetized bar may themselves be little magnets arranged in rows with their opposite poles in contact.

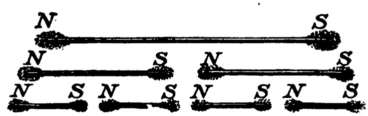


FIG. 217. Effect of breaking a magnet

If an unmagnetized piece of hard steel is pounded vigorously while it lies between the poles of a magnet, or if it is heated to redness and then allowed to cool in this position, it will be found to have become magnetized. This suggests that the molecules of the steel are magnets even when the bar as a whole is not magnetized, and that magnetization may consist in causing them to arrange themselves in rows, end to end, just as the magnetization of the tube of iron filings mentioned above was due to a special arrangement of the filings.

277. Theory of magnetism. In an unmagnetized bar of iron or steel it is probable then that the molecules themselves are tiny magnets which are arranged either haphazard, or in little closed groups or chains, as in Fig. 218, so that, on the whole, opposite poles neutralize each other throughout the bar. But when the bar is brought near a magnet, the molecules are swung around by the outside magnetic force into some such arrangement as that shown in Fig. 219, in which the opposite poles completely neutralize each other only in the middle of the bar. According to this view, heating and jarring weaken the magnet because they tend to shake the molecules out of alignment. On the other hand, heating and jarring facilitate magnetization when the bar is between the poles of a magnet because they assist the magnetizing force in breaking up the molecular groups and chains and getting the molecules into alignment. Soft iron has higher permeability than hard steel because the molecules of the former substance are much easier to swing into alignment

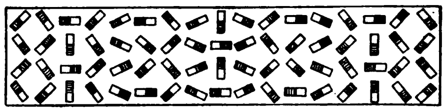


FIG. 218. Arrangement of molecules in an unmagnetized iron bar

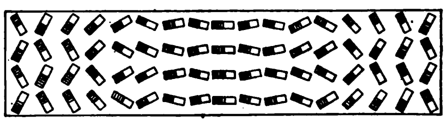


FIG. 219. Arrangement of molecules in a magnetized iron bar

than those of the latter substance. Steel has a very much greater retentivity than soft iron because its molecules are not so easily moved out of position once they have been aligned.

278. Saturation. Strong evidence for the correctness of the above view is found in the fact that a piece of iron or steel cannot be magnetized beyond a certain limit, no matter how strong is the magnetizing force. This limit probably corresponds to the condition in which the axes of all the molecules are brought into parallelism, as in Fig. 220. The magnet is then said to be *saturated*, since it is as strong as it is possible to make it.

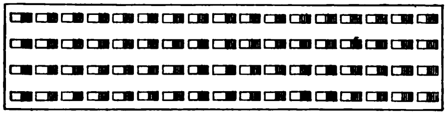


FIG. 220. Arrangement of molecules in a saturated magnet

TERRESTRIAL MAGNETISM

279. The earth's magnetism. The fact that a compass needle always points north and south, or approximately so, indicates that the earth itself is a great magnet, having an S pole near the geographical north pole, and an N pole near the geographical south pole; for the magnetic pole of the earth which is near the geographical north pole must of course be unlike the pole of a suspended magnet which points toward it, and the pole of the suspended magnet which points toward the north is the one which by convention it has been decided to call the N pole. The magnetic pole of the earth which is near the north geographical pole was found in 1831 by Sir James Ross in Boothia Felix, Canada, latitude 70° 30′ N., longitude 95° W. It was located again in 1905 by Captain Amundsen (the discoverer of the geographical south pole, 1912) at a point a little farther west. Its approximate location is 70° 5′ N. and 96° 46′ W. It is probable that it shifts its position slowly.

280. Declination. The earliest users of the compass were aware that it did not point exactly north; but it was Columbus who, on his first voyage to America, made the discovery, much to the alarm of his sailors, that the direction of the compass needle changes as one moves about over the earth's surface. The chief reason for this variation is found in the fact that the magnetic poles do not coincide with the geographical poles; but there are also other causes, such as the existence of large deposits of iron ore, which produce local effects upon the needle. The number of degrees by which, at a given point on the earth, the needle varies from a true north-and-south line is called its *declination* at that point. Lines drawn over the earth through points of equal declination are called *isogonic lines*.

281. Dip of the compass needle. Let an unmagnetized knitting needle a (Fig. 221) be thrust through a cork, and let a second needle b be passed through the cork at right angles to a, and as close to it as possible. Let a pin c be adjusted until the system is in neutral equilibrium about b as an axis, when a is pointing east and west. Then let a be carefully magnetized by stroking one end of it from the middle out with the N pole of a strong magnet, and the other end from the middle out with the S pole of the same magnet. When now the needle is replaced on its supports and turned into a north-and-south position, its N pole will be found to dip so as to cause the needle to make an angle of 60° or 70° with the horizontal.

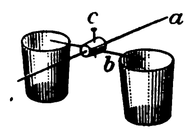


FIG. 221. Arrangement for showing dip

The experiment shows that in this latitude the earth's magnetic lines make a large angle with the horizontal. This angle between the earth's surface and the direction of the magnetic lines is called the *dip*, or *inclination*, of the needle. At Washington it is 71° 5′ and at Chicago 72° 50′. At the magnetic pole it is of course 90°, and at the so-called *magnetic equator*, which is an irregular curved line near the geographical equator, the dip is 0°.

282. The earth's inductive action. That the earth acts like a great magnet may be very strikingly shown in the following way:

Let a steel rod, for example a tripod rod, be held parallel to the earth's magnetic lines (the north end slanting down at an angle of about 70° or 75°) and struck a few sharp blows with a hammer. The rod will be found to have become a magnet with its upper end an *S* pole, like the north pole of the earth, and its lower end an *N* pole. If the rod is reversed and tapped again with the hammer, its magnetism will be reversed. If held in an east-and-west position and tapped, it will become demagnetized, as will be shown by the fact that either end of it will attract either end of a compass needle.

QUESTIONS AND PROBLEMS

1. If a bar magnet is floated on a piece of cork, will it tend to float toward the north? Why?

2. Will a bar magnet pull a floating compass needle toward it? Compare the answer to this question with that to the preceding one.

3. Why should the needle used in the experiment of § 281 be placed east and west, when adjusting for neutral equilibrium, before it is magnetized?

4. The dipping needle is suspended from one arm of a steel-free balance and carefully weighed. It is then magnetized. Will its apparent weight increase?

5. Explain, on the basis of induced magnetization, the process by which a magnet attracts a piece of soft iron.

6. When a piece of soft iron is made a temporary magnet by bringing it near the *N* pole of a bar magnet, will the end of the iron nearest the magnet be an *N* or an *S* pole?

7. Devise an experiment which will show that a piece of iron attracts a magnet just as truly as the magnet attracts the iron.

8. How would an ordinary compass needle act if placed over one of the earth's magnetic poles? How would a dipping needle act at these points?

9. Do the facts of induction suggest to you any reason why a horseshoe magnet retains its magnetism better when a bar of soft iron (a keeper, or armature) is placed across its poles than when it is not so treated? (See Fig. 219.)

10. With what force will an *N* magnetic pole of strength 6 attract, at a distance of 5 cm., an *S* pole of strength 1? of strength 9?

CHAPTER XII

STATIC ELECTRICITY

General Facts of Electrification

283. Electrification by friction. If a piece of hard rubber or a stick of sealing wax is rubbed with flannel or cat's fur and then brought near some dry pith balls, bits of paper, or other light bodies, these bodies are found to jump toward the rod. This sort of attraction, so familiar to us from the behavior of our hair in winter when we comb it with a rubber comb, was observed as early as 600 B.C., when Thales of Greece commented upon the fact that rubbed amber draws to itself threads and other light objects. It was not, however, until 1600 A.D. that Dr. William Gilbert, physician to Queen Elizabeth, and sometimes called the father of the modern science of electricity and magnetism, discovered that the effect could be produced by rubbing together a great variety of other substances besides amber and silk, such, for example, as glass and silk, sealing wax and flannel, hard rubber and cat's fur, etc.

Gilbert named the effect which was produced upon these various substances by friction, *electrification*, after the Greek name *electron*, meaning "amber." *Thus a body which, like rubbed amber, has been endowed with the property of attracting light bodies is said to have been electrified, or to have been given a charge of electricity.* In this statement nothing whatever is said about the nature of electricity. We simply define an electrically charged body as one which has been put into the condition in which it acts toward light bodies like the rubbed amber or the rubbed sealing wax. To this day we do not know

William Gilbert (1540–1603)

English physician and physicist; first Englishman to appreciate fully the value of experimental observations; first to discover through careful experimentation that the compass points to the north, not because of some influence of the stars, but because the earth is itself a great magnet; first to use the word "electricity"; first to discover that electrification can be produced by rubbing a great many different kinds of substances; author of the epoch-making book entitled "De Magnete, etc.," published in London in 1600

with certainty what the nature of electricity is, but we are fairly familiar with the laws which govern its action. It is to these laws that attention will be mainly devoted in the following sections.

284. Positive and negative electricity. Let a pith ball suspended by a silk thread, as in Fig. 222, be touched to a glass rod which has been rubbed with silk and thus been put into the condition in which it is strongly repelled by this rod. Next let a stick of sealing wax or an ebonite rod which has been rubbed with cat's fur or flannel be brought near the charged ball. It will be found that it is not repelled, but, on the contrary, is very strongly attracted. Similarly, if the pith ball has touched the sealing wax so that it is repelled by it, it is found to be strongly attracted by the glass rod. Again, two pith balls both of which have been in contact with the glass rod are found to repel each other, while pith balls one of which has been in contact with the glass rod and the other with the sealing wax attract each other.

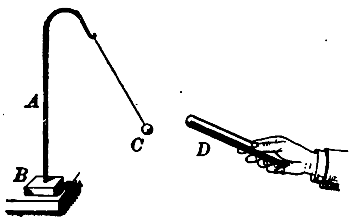


FIG. 222. Pith-ball electroscope

Evidently, then, the electrifications which are imparted to glass by rubbing it with silk and to sealing wax by rubbing it with flannel are opposite in the sense that an electrified body that is attracted by one is repelled by the other. We say, therefore, that there are two kinds of electrification, and we arbitrarily call one *positive* and the other *negative.* Thus a *positively electrified body* is one which acts with respect to other electrified bodies like a *glass rod which has been rubbed with silk*, and a *negatively electrified body* is one which acts like a piece of *sealing wax which has been rubbed with flannel.* These facts and definitions may then be stated in the following general law: *Electrical charges of like kind repel each other, while charges of unlike kind attract each other.* The forces of attraction or repulsion are found, like those of gravitation and magnetism, *to decrease as the square of the distance increases.*

285. Measurement of electrical quantities. The fact of attraction and repulsion is taken as the basis for the definition and measurement of so-called *quantities* of electricity. Thus a small charged body is said to contain 1 unit of electricity when it will repel an exactly equal and similar charge placed 1 centimeter away with a force of 1 dyne. The number of units of electricity on any charged body is then measured by the force which it exerts upon a unit charge placed at a given distance from it; for example, a charge which at a distance of 10 centimeters repels a unit charge with a force of 1 dyne contains 100 units of electricity, for this means that at a distance of 1 centimeter it would repel the unit charge with a force of 100 dynes (see § 284).

286. Conductors and nonconductors. Let an electroscope *E* (Fig. 223), consisting of a pair of gold leaves *a* and *b*, suspended from an insulated metal rod *r*, and protected from air currents by a case *J*, be connected with the metal ball *B* by means of a wire. Let an ebonite rod be now electrified and rubbed over *B*. The immediate divergence of the gold leaves will show that a portion of the electric charge placed upon *B* has been carried by the wire to the gold leaves, where it causes them to diverge in accordance with the law that bodies charged with the same kind of electricity repel each other.

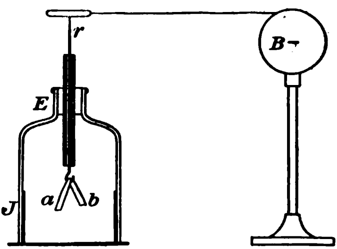


FIG. 223. Illustrating conduction

Let the experiment be repeated when *E* and *B* are connected with a thread of silk or a long rod of wood instead of the metal wire. No divergence of the leaves will be observed. If a moistened thread connects *E* and *B*, the leaves will be seen to diverge slowly when the ball *B* is charged, showing that a charge is carried slowly by the moist thread.

These experiments make it clear that while electric charges pass with perfect readiness from one point to another in a wire, they are quite unable to pass along dry silk or wood, and pass with difficulty along moist silk. We are therefore accustomed to divide substances into two classes, *conductors* and *nonconductors*, or *insulators*, according to their ability to transmit electrical charges from point to point. Thus metals and solutions

of salts and acids in water are all conductors of electricity, while glass, porcelain, rubber, mica, shellac, wood, silk, vaseline, turpentine, paraffin, and oils generally are insulators. No hard-and-fast line, however, can be drawn between conductors and nonconductors, since all so-called insulators conduct to some slight extent, while the so-called conductors differ greatly among themselves in the facility with which they transmit charges.

The fact of conduction brings out sharply one of the most essential distinctions between electricity and magnetism. Magnetic poles exist only in iron and steel, while electrical charges may be communicated to any body whatever, provided it is insulated. These charges pass over conductors, and can be transferred by contact from one body to any other, while magnetic poles remain fixed in position, and are wholly uninfluenced by contact with other bodies, unless these bodies themselves are magnets.

287. Electrostatic induction. Let the ebonite rod be electrified by friction and slowly brought toward the knob of the gold-leaf electroscope (Fig. 224). The leaves will be seen to diverge, even though the rod does not approach to within a foot of the electroscope.

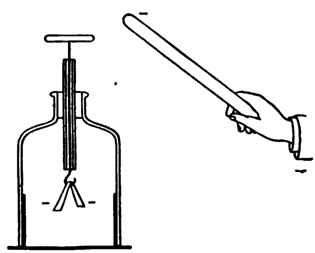


FIG. 224. Illustrating induction

This makes it clear that the mere *influence* which an electric charge exerts upon a conductor placed in its neighborhood is able to produce electrification in that conductor. This method of producing electrification is called *electrostatic induction.*

As soon as the charged rod is removed the leaves will be seen to collapse completely. This shows that this form of electrification is only a temporary phenomenon which is due simply to the presence of the charged body in the neighborhood.

288. Nature of electrification produced by induction. Let a metal ball A (Fig. 225) be strongly charged by rubbing it with a charged rod, and let it then be brought near an insulated* metal body B, which is provided with pith balls or strips of paper a, b, c, as shown. The divergence of a and c will show that the ends of B have received electrical charges because of the presence of A, while the failure of b to diverge will show that the middle of B is uncharged. Further, the rod which charged A will be found to repel c, but to attract a.

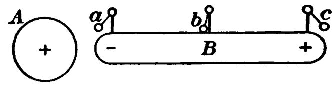


FIG. 225. Nature of induced charges

We conclude, therefore, that *when a conductor is brought near a charged body, the end away from the inducing charge is electrified with the same kind of electricity as that on the inducing body, while the end toward the inducing body receives electricity of opposite kind.*

289. Two-fluid theory of electricity. We can describe the facts of induction conveniently by assuming that in every conductor there exists an equal number of positively and negatively charged corpuscles which are very much smaller than atoms, and which are able to move about freely within the conductor. When no electrified body is near the conductor B, it appears to have no charge at all, because all the little positive charges within it counteract the effects upon outside bodies of all the little negative charges. But as soon as an electrical charge is brought near B, it drives as far away as possible the corpuscles which carry charges of sign like its own, while it attracts the corpuscles of unlike sign. B therefore becomes electrified like A at its remote end and unlike A at its near end. As soon as the inducing charge is removed, B immediately becomes neutral again because the little positive and negative corpuscles come together under the influence

* **Sulphur is practically a perfect insulator in all weathers, wet or dry. Metal conductors of almost any shape resting upon pieces of sulphur will serve the purposes of this experiment in summer or winter.**

of their mutual attractions. This picture of the mechanism of electrification by induction is a modern modification of the so-called *two-fluid theory* of electricity, which conceived of all conductors as containing equal amounts of two weightless electrical fluids called positive electricity and negative electricity. Although it is now quite improbable that this theory represents the actual conditions within a conductor, yet we are able to say with perfect positiveness that *the electrical behavior of a conductor is exactly what it would be if it did contain equal amounts of positive and negative electrical fluids,* or equal numbers of minute positive and negative corpuscles which are free to move through the conductor under the influence of outside electrical forces. Furthermore, since the real nature of electricity has been altogether unknown, it has gradually become a universally recognized convention to speak of the positive electricity within a conductor as being repelled to the remote end, and the negative electricity as attracted to the near end by an outside positive charge, and vice versa. This does not imply acceptance of the two-fluid theory. It is merely a way of describing the fact that the remote end does acquire a charge like that of the inducing body, and the near end a charge unlike that of the inducing body.

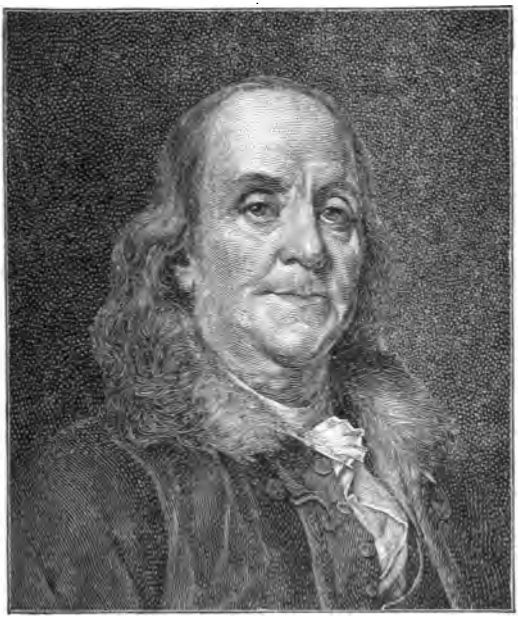
290. The electron theory. A slightly different theory, called the one-fluid theory, was originally suggested by Benjamin Franklin, and in the following modified form is now pretty generally held by physicists. The atoms of all substances are now known to contain as constituents both positive and negative electricity, the latter existing in the form of minute corpuscles or electrons, each of which has a mass of about $\frac{1}{1760}$ that of the hydrogen atom. These electrons are probably grouped in some way about the positive electricity as a nucleus. The sum of the negative charges of these electrons is supposed to be just equal to the positive charge of the nucleus, so that in its normal condition the whole atom is neutral or uncharged.

But in conductors electrons are continually getting loose from the atoms and reëntering other atoms, so that at any given instant there are always in every conductor a number of free negative electrons and a corresponding number of atoms which have lost electrons and which are therefore positively charged. Such a conductor would, as a whole, show no charge of either positive or negative electricity. But as soon as a body charged, for example negatively, is brought near such a conductor, the negatively charged electrons stream away to the remote end, leaving behind them the positively charged atoms, which are not free to move from their positions. On the other hand, if a positively charged body is brought near the conductor, the negative electrons are attracted and the remote end is left with the immovable plus atoms.

The only advantage of this theory over that suggested in § 289, in which the existence of both positive and negative corpuscles was assumed, is that there is much direct experimental evidence for the existence of such negatively charged corpuscles or electrons of about $\frac{1}{1760}$ the mass of the hydrogen atom (see Chapter XXI), but no direct evidence as yet for the existence of positively charged bodies smaller than atoms.

The charge of one electron is called the elementary electrical charge. Its value has recently (1913) been accurately measured. There are 2.095 billion of them in one of the units defined in § 285. *Every electrical charge consists of an exact number of these ultimate electrical atoms scattered over the surface of the charged body.*

291. Charging by induction. Let two metal balls or two eggshells A and B, which have been gilded or covered with tin foil, be suspended by silk threads and touched together, as in Fig. 226. Let a positively charged body C be brought near them. As described above, A and B will at once exhibit evidences of electrification; that is, A will repel a positively charged pith ball, while B will attract it. If C is removed while A and B are still in contact, the separated charges reunite and A



BENJAMIN FRANKLIN (1706–1790)

Celebrated American statesman, philosopher, and scientist; born at Boston, the sixteenth child of poor parents; printer and publisher by occupation; pursued scientific studies in electricity as a diversion rather than as a profession; first proved that the two coats of a Leyden jar are oppositely charged; introduced the terms positive and negative electricity; proved the identity of lightning and frictional electricity by flying a kite in a thunderstorm and drawing sparks from the insulated lower end of the kite string; invented the lightning rod; originated the one-fluid theory of electricity which regarded a positive charge as indicating an excess, a negative charge a deficiency, in a certain normal amount of an all-pervading electrical fluid

and B cease to exhibit electrification. But if A and B are separated from each other while C is in place, A will be found to be permanently positively charged and B negatively charged. This may be proved either by the attractions and repulsions which they show for charged rods brought near them, or by the effects which they produce upon a charged electroscope brought into their vicinity, the leaves of the latter falling together when it is brought near one and spreading farther apart when brought near the other.

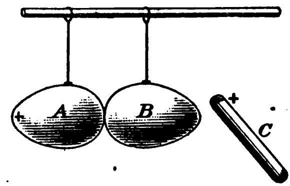


FIG. 226. Obtaining a plus and a minus charge by induction

We see, therefore, that *if we cut in two, or separate into two parts, a conductor while it is under the influence of an electric charge, we obtain two permanently charged bodies, the remoter part having a charge of the same sign as that of the inducing charge, and the near part having a charge of unlike sign.*

Let the conductor B (Fig. 227) be touched by the finger while a charged rod C is near it. Then let the finger be removed and after it the rod C. If now a negatively charged pith ball is brought near B, it will be repelled, showing that B has become negatively charged. In this experiment the body of the experimenter corresponds to the egg A of the preceding experiment, and removing the finger from B corresponds to separating the two eggshells. Let the last experiment be repeated with only this modification, that B is touched at b rather than at a. When B is again tested with the pith ball it will still be found to have a negative charge, exactly as when the finger was touched at a.

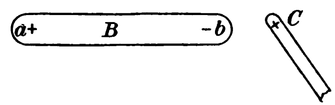


FIG. 227. A body charged by induction has a charge of sign opposite to that of the inducing charge

We conclude, therefore, that no matter where the body B is touched, *the sign of the charge left upon it is always opposite to that of the inducing charge.* This is because the negative electricity, that is, the electrons, can under no circumstances escape from b so long as C is present, for they are "bound" by the attraction of the positive charge on C. Indeed, the final

negative charge on B is due merely to the fact that the positive charge on C pulls electrons into B from the finger, no matter where B is touched. In the same way, if C had been negative it would have pushed electrons off from B through the finger and have thus left B positively charged.

292. Charging the electroscope by induction. Let an ebonite rod which has been rubbed with cat's skin be brought near the knob of the electroscope (Fig. 224). The leaves at once diverge. Let the knob be touched with the finger while the rod is held in place. The leaves will fall together. Let the finger be removed and then the rod. The leaves will fly apart again.

The electroscope has been charged by induction, and since the charge on the ebonite rod was negative, the charge on the electroscope must be positive. If this conclusion is tested by bringing the ebonite rod near the electroscope, the leaves will fall together as the rod approaches the knob. How does this prove that the charge on the electroscope is positive?

293. Plus and minus electricities always appear simultaneously and in equal amounts. Let an ebonite rod be completely discharged by passing it quickly through a Bunsen flame. Let a flannel cap having a silk thread attached be slipped over the rod, as in Fig. 228, and twisted rapidly around a number of times. When rod and cap together are held near a charged electroscope, no effect will be observed; but if the cap is pulled off, it will be found to be positively charged, while the rod will be found to have a negative charge.

FIG. 228. Plus and minus electricities always developed in equal amounts

Since the two together produce no effect, the experiment shows that the plus and minus charges were equal in amount. This experiment confirms the view already brought forward in connection with induction, that electrification always consists in a separation of plus and minus charges which already exist in equal amounts within the bodies in which the electrification is developed.

QUESTIONS AND PROBLEMS

1. Charge a gold-leaf electroscope by induction from a glass rod. Warm a piece of paper and stroke it on the clothing. Hold it over the charged electroscope. If the divergence of the gold leaves is increased, is the charge on the paper + or − ? If the divergence of the gold leaves is decreased, what is the sign of the charge on the paper?

2. Given a gold-leaf electroscope, a glass rod, and a piece of silk, how, in general, would you proceed to test the sign of the electrification of an unknown charge?

3. If pith balls, or any light figures, are placed between two plates (Fig. 229), one of which is connected to earth and the other to one knob of an electrical machine in operation, the figures will bound back and forth between the two plates as long as the machine is operated. Explain.

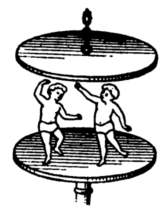


FIG. 229

4. If you are given a positively charged insulated sphere, how could you charge two other spheres, one positively and the other negatively, without diminishing the charge on the first sphere?

5. If you bring a positively charged glass rod near the knob of an electroscope and then touch the knob, why do you not remove the negative electricity which is on the knob?

6. In charging an electroscope by induction, why must the finger be removed before the removal of the charged body?

7. If you hold a brass rod in the hand and rub it with silk, the rod will show no sign of electrification; but if you hold the brass rod with a piece of sheet rubber and then rub it with silk, you will find it electrified. Explain.

8. Why is repulsion between an unknown body and an electrified pith ball a surer sign that the unknown body is electrified than is attraction?

9. State as many differences as you can between the phenomena of magnetism and those of electricity.

DISTRIBUTION OF ELECTRIC CHARGE UPON CONDUCTORS

294. Electric charges reside only upon the outside surface of conductors. Let a deep tin cup (Fig. 230) be placed upon an insulating stand and charged as strongly as possible either from an ebonite rod or from an electrical machine. If now a smooth metal ball suspended by a silk thread is touched to the *outside* of the charged cup, and then brought near the knob of a charged electroscope, it will show a strong charge; but if it is touched to the *inside* of the cup, it will show no charge at all.

These experiments show that *an electric charge resides entirely on the outside surface of a conductor.* This is a result which might have been inferred from the fact that all the little electrical charges of which the total charge is made up repel each other and therefore move through the conductor until they are, on the average, as far apart as possible.

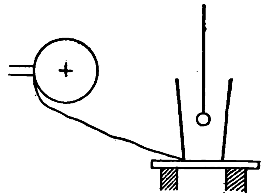


FIG. 230. Proof that charge resides on surface

295. Density of charge greatest where curvature of surface is greatest. Since all of the parts of an electric charge tend, because of their mutual repulsions, to get as far apart as possible, we should infer that if a charge of either sign is placed upon an oblong conductor like that of Fig. 231, (1), it will distribute itself so that the electrification at the ends will be stronger than that at the middle.

To test this inference let a proof plane (a flat metal disk, for example a cent, provided with an insulating handle) be touched to one end of such a charged body, the charge conveyed to a gold-leaf electroscope, and the amount of separation of the leaves noted. Then let the experiment be repeated when the proof plane touches the middle of the body. The separation of the leaves in the latter case will be found to be very much less than in the former. If we should test the distribution on a pear-shaped body [Fig. 231, (2)] in the same way, we should find the density of electrification considerably greater on the small end than on the large one. By density of electrification is meant the quantity of electricity on unit area of the surface.

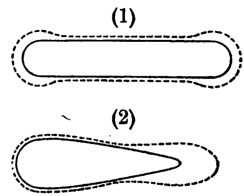


FIG. 231. Distribution of charge over oblong bodies

296. Discharging effect of points. The above experiments indicate that if one end of a pear-shaped body is made more and more pointed, then when the body is charged the electric density on this end will become greater and greater.

The following experiment will show what happens when the conductor is provided with a sharp point.

Let a very sharp needle be attached to any smooth insulated metal body provided with paper or pith-ball indicators, as in Fig. 225, p. 222. If the body is now charged either with a rubbed rod or with an electric machine, as soon as the supply of electricity is stopped the paper indicators will immediately fall, showing that the body is losing its charge. To show that this is certainly due to the effect of the point, remove the needle and repeat. The indicators will fall very slowly, if at all.

The experiment shows that the electrical density upon the point is so great that the charge escapes from it into the air. This is because the intense charge on the point causes many of the adjacent molecules of the air to lose an electron. This leaves these molecules positively charged. The free electrons attach themselves to neutral molecules, thus charging them negatively. One set of these electrically charged molecules (called *ions*) is attracted to the point and the other repelled away from it. The former set move to the conductor, give up their charges to it, and thus neutralize the charge upon it.

The effect of points may be shown equally well by charging the gold-leaf electroscope and holding a needle in the hand within a few inches of the knob. The leaves will fall together rapidly. In this case the needle point becomes electrified by induction and discharges to the knob electricity of the opposite kind to that on the knob, thus neutralizing its charge. An entertaining variation of the last experiment is to attach a tassel of tissue paper to an insulated conductor and electrify it strongly. The paper streamers under their mutual repulsions will stand out in all directions, but as soon as a needle point is held in the hand near them, they will at once fall together (Fig. 232), since they are discharged as described above.

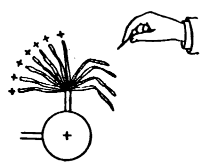


FIG. 232. Discharging effect of points

297. The electric whirl. Let an electric whirl (Fig. 233) be balanced upon a pin point and attached to one knob of an electric machine. As soon as the machine is started, the whirl will rotate rapidly in the direction of the arrows.

The explanation is as follows: The air close to each point is *ionized*, as explained in § 296. The ions of sign unlike that of the charge on the point are drawn to the point and discharged. The other set of ions is repelled. But since this repulsion is mutual, the point is pushed back with the same force with which these ions are pushed forward; hence the rotation. The repelled ions in their turn drag the air with them in their forward motions, and thus produce the "electric wind," which may be detected easily by the hand or by a candle flame (Fig. 234).

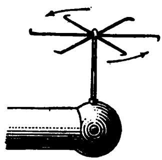


FIG. 233. The electric whirl

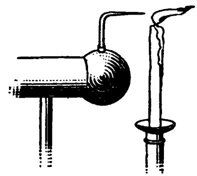


FIG. 234. The electric wind

298. Lightning and lightning rods. It was in 1752 that Franklin, during a thunderstorm, sent up his historic kite. This kite was provided with a pointed wire at the top. As soon as the hempen kite string had become wet he succeeded in drawing ordinary electric sparks from a key attached to the lower end. This experiment demonstrated for the first time that thunderclouds carry ordinary electrical charges which may be drawn from them by points, just as the charge was drawn from the tassel in the experiment of § 296. It also showed that lightning is nothing but a huge electric spark. Franklin applied this discovery in the invention of the lightning rod. The way in which the rod discharges the cloud and protects the building is as follows: As the charged cloud approaches the building it induces an opposite charge in the rod. This induced charge escapes rapidly and quietly from the sharp point in the manner explained above and thus neutralizes the charge of the cloud.

To illustrate, let a metal plate C (Fig. 235) be supported above a metal ball E, and let C and E be attached to the two knobs of an electrical machine. When the machine is started sparks will pass from C to E,

but if a point p is connected to E, the sparking will cease; that is, the point will protect E from the discharges, even though the distance Cp be considerably greater than CE.

The lower end of a lightning rod should be buried deep enough so that it will always be surrounded by moist earth, since dry earth is a poor conductor. It will be seen, therefore, that lightning rods protect buildings not because they conduct the lightning to earth, but because they prevent the formation of powerful charges in the neighborhood of the buildings on which they are placed.

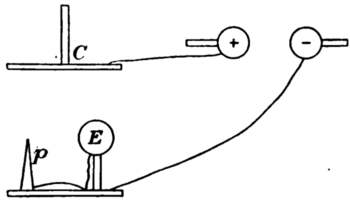


FIG. 235. Illustrating the action of a lightning rod

299. Electric screens. That the charge on the outside of a conductor always distributes itself in such a way that there is no electric force within the conductor was first proved experimentally by Faraday. He covered a large box with tin foil and went inside with the most delicate electroscopes obtainable. He found that the outside of the box could be charged so strongly that long sparks were flying from it without any electrical effects being observable anywhere inside the box.

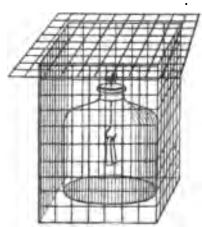


FIG. 236. Electroscope protected by a wire cage

To repeat the experiment in modified form, let an electroscope be placed beneath a bird cage or wire netting, as in Fig. 236. Let charged rods or other powerfully charged bodies be brought near the electroscope outside the cage. The leaves will be found to remain undisturbed.

Hence, if we wish to protect an electrical instrument from outside electrical disturbances, we have only to surround it with a metal covering.*

* A laboratory exercise on static electrical effects should follow the discussion of this section. See, for example, Experiment 27 of the authors' manual.

Potential and Capacity

300. Potential difference. There is a very instructive analogy between the use of the word "potential" in electricity and "pressure" in hydrostatics. For example, if water will flow from tank A to tank B through the connecting pipe R (Fig. 237), we infer that the hydrostatic pressure at a must be greater than that at b, and we attribute the flow directly to this difference in pressure. In exactly the same way, if, when two bodies A and B (Fig. 238) are connected by a conducting wire r, a charge of + electricity is found to pass from A to B, that is, if electrons are found to pass from B to A, we say that the electrical potential is higher at A than at B, and we assign this *difference of potential* as the cause of the flow.* Thus, just as water tends to flow from points of higher hydrostatic pressure to points of lower hydrostatic pressure, so electricity tends to flow from points of higher electrical pressure or potential to points of lower electrical pressure or potential.

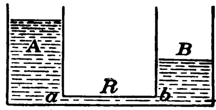


FIG. 237. Illustrating hydrostatic pressure

Again, if water is not continuously supplied to one of the tanks A or B of Fig. 237, we know that the pressures at a and b must soon become the same. Similarly, if no electricity is supplied to the bodies A and B of Fig. 238, their potentials very quickly become the same. In other words, *all points on a system of connected conductors in which the electricity is in a stationary or static condition are at the same*

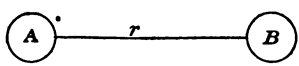


FIG. 238. Illustrating electrical pressure

* Franklin thought that it was the positive electricity which moved through a conductor, while he conceived the negative as inseparably associated with the atoms. Hence it became a universally recognized convention to regard electricity as moving through a conductor in the direction in which a + charge would have to move to produce the observed effect. It is not desirable to attempt to change this convention now even though the electron theory has exactly inverted the rôles of the + and − charges.

potential. This result follows at once from the fact of mobility of electric charges through conductors.

But if water is continuously poured into A and removed from B (Fig. 237), the pressure at a will remain permanently above the pressure at b, and a continuous flow of water will take place through R. So if A (Fig. 238) is connected with an electrical machine and B to earth, a permanent potential difference will exist between A and B, and a continuous current of electricity will flow through r. Difference in potential is commonly denoted simply by the letters P.D. (Potential Difference).

301. Some methods of measuring potentials. The simplest and most direct way of measuring the potential difference between two bodies is to connect one to the knob, the other to the conducting case,* of an electroscope. The amount of separation of the gold leaves is a measure of the P.D. between the bodies. The unit in which P.D. is usually expressed is called the *volt.* It will be accurately defined in § 331. It will be sufficient here to say that it is approximately equal to the electrical pressure between the ends of a strip of copper and a strip of zinc immersed in dilute sulphuric acid (see Fig. 247).

Since the earth is on the whole a good conductor, its potential is everywhere the same (§ 300); hence it makes a convenient standard of reference in potential measurements. To find the potential of a body relative to that of the earth, we connect the outer case of the electroscope to the earth by means of a wire, and connect the body to the knob. If the electroscope is calibrated in volts, its reading gives the P.D. between the body and the earth. Such calibrated electroscopes are called *electrostatic voltmeters.* They are the simplest and in

* If the case is of glass it should always be made conducting by pasting tin-foil strips on the inside of the jar opposite the leaves and extending these strips over the edge of the jar and down on the outside to the conducting support on which the electroscope rests. The object of this is to maintain the walls always at the potential of the earth.

many respects the most satisfactory forms of voltmeters to be had. Their use, both in laboratories and in electrical power plants, is rapidly increasing. They can be made to measure a P.D. as small as $\frac{1}{1000}$ volt and as large as 200,000 volts. Fig. 239 shows one of the simpler forms. The outer case is of metal and is connected to earth at the point *a*. The body whose potential is sought is connected to the knob *b*. This is in metallic contact with the light aluminium vane *c*, which takes the place of the gold leaf.

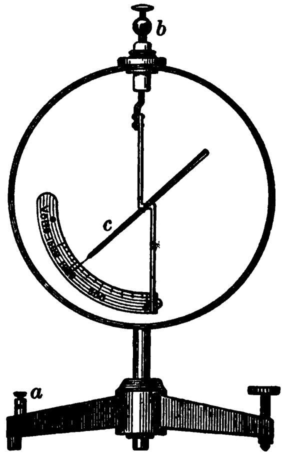


FIG. 239. Electrostatic voltmeter

A very convenient way of measuring a *large* P.D. without a voltmeter is to measure the length of the spark which will pass between the two bodies whose P.D. is sought. The P.D. is roughly proportional to spark length, each centimeter of spark length representing a P.D. of about 30,000 volts, if the electrodes are large compared to their distance apart.

302. Condensers. Let a metal plate *A* be mounted on an insulating base and connected with an electroscope, as in Fig. 240. Let a second plate *B* be similarly mounted and connected to the earth by a conducting wire. Let *A* be charged and the deflection of the gold leaves noted. If, now, we push *B* toward *A*, we shall observe that as it comes near, the leaves begin to fall together, showing that the potential of *A* is diminished by the presence of *B*, although the quantity of electricity on *A* has remained unchanged. If we convey additional − charges to *A* with the

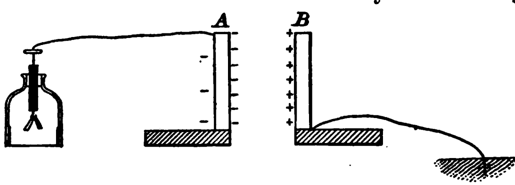


FIG. 240. The principle of the condenser

Count Alessandro Volta (1745–1827)

Great Italian physicist, professor at Como and at Pavia; inventor of the electroscope, the electrophorus, the condenser, and the voltaic pile (a form of galvanic cell); first measured the potential differences arising from the contact of dissimilar substances; ennobled by Napoleon for his scientific services; the volt, the practical unit of potential difference, is named in his honor

aid of a proof plane, we shall find that many times the original amount of electricity may now be put on A before the leaves return to their original divergence; that is, before the body regains its original potential.

We say, therefore, that the *capacity* of A for holding electricity has been very greatly increased by bringing near it another conductor which is connected to earth. It is evident from this statement that *we measure the capacity of a body by the amount of electricity which must be put upon it to raise it to a given potential.* The explanation of the increase in capacity in this case is obvious. As soon as B was brought near to A it became charged, by induction, with electricity of opposite sign to A, the electricity of like sign to A being driven off to earth through the connecting wire. The attraction between these opposite charges on A and B drew the electricity on A to the face nearest to B and removed it from the more remote parts of A, so that it became possible to put a very much larger charge on A before the tendency of the electricity on A to pass over to the electroscope became as great as it was at first; that is, before the potential of A rose to its initial value. In such a condition the electricity on A is said to be "bound" by the opposite electricity on B.

An arrangement of this sort consisting of two conductors separated by a nonconductor is called a condenser. If the conducting plates are very close together and one of them grounded, the capacity of the system may be thousands of times as great as that of one of the plates alone.

303. The Leyden jar. The most common form of condenser is a glass jar coated part way to the top inside and outside with tin foil (Fig. 241). The inside coating is connected by a chain to the knob, while the outside coating is connected to earth. Condensers of this sort first came into use in Leyden, Holland, in 1745. Hence they are now called *Leyden jars.*

To charge a Leyden jar the outer coating is held in the hand while the knob is brought into contact with one terminal of an electrical

machine, for example the negative. As fast as electrons pass to the knob they spread to the inner coat of the jar, where they repel electrons from the outer coat to the earth, thus leaving it positively charged. If the inner and outer coatings are now connected by a discharging rod, as in Fig. 241, a powerful spark will be produced. Let a charged jar be placed on a glass plate so as to insulate the outer coat. Let the knob be touched with the finger. No appreciable discharge will be noticed. Let the outer coat be in turn touched with the finger. Again no appreciable discharge will appear. But if the inner and outer coatings are connected with the discharger, a powerful spark will pass.

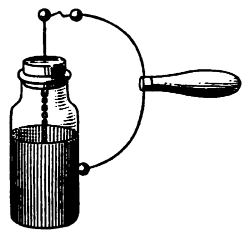


Fig. 241. The Leyden jar

The experiment shows that it is impossible to discharge one side of the jar alone, for practically all of the charge is *bound* by the opposite charge on the other coat. The full discharge can therefore occur only when the inner and outer coats are connected.

Electrical Generators

304. The electrophorus. The electrophorus is a simple electrical generator which illustrates well the principle underlying the action of all electrostatic machines. All such machines generate electricity primarily by induction, not by friction. *B* (Fig. 242) is a hard rubber plate which is first charged by rubbing it with fur or flannel. *A* is a metal plate provided with an insulating handle. When the plate *A* is placed upon *B*, touched with the finger, and then removed, it is found possible to draw a spark from it, which in dry weather may be a quarter of an inch or more in length. The process may be repeated an indefinite number of times without producing any diminution in the size of the spark which may be drawn from *A*.

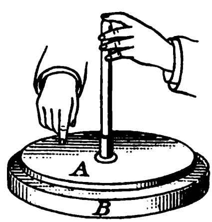


Fig. 242. The electrophorus

If the sign of the charge on A is tested by means of an electroscope, it will be found to be positive. This proves that A has been charged by induction, not by contact with B, for it is to be remembered that the latter is charged negatively. The reason for this is that even when A rests upon B it is in reality separated from it, at all but a very few points, by an insulating layer of air; and, since B is a nonconductor, its charge cannot pass off appreciably through these few points of contact. It simply repels negative electricity to the top side of the metal plate A, and thus charges positively the lower side. The negative passes off to earth when the plate is touched with the finger. Hence, when the finger is removed and A lifted, it possesses a strong positive charge.

305. The Toepler-Holtz electrical machine. The ordinary static machine is nothing but a continuously acting electrophorus. Fig. 243, (1), represents the so-called Toepler-Holtz type of such a machine. Upon

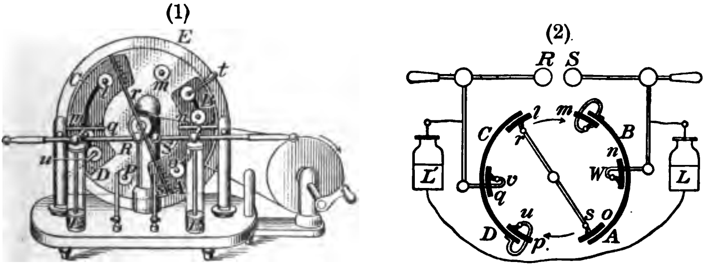


FIG. 243. Toepler-Holtz induction machine

the back of the stationary plate E are pasted paper sectors, beneath which are strips of tin foil AB and CD, called *inductors*. In front of E is a revolving glass plate carrying disks l, m, n, o, p, and q, called *carriers*. To the inductors AB and CD are fastened metal arms t and u, which bring C and D into electrical contact with the disks l, m, n, o, p, and q, when these disks pass beneath the tinsel brushes carried by t and u. A stationary metallic rod rs carries at its ends stationary brushes as well as sharp-pointed, metallic combs. The two knobs R and S have their capacity increased by the Leyden jars L and L', the outer coatings of which are connected beneath the base of the machine.

306. Action of the Toepler-Holtz machine. The action of the machine described above is best understood from the diagram of Fig. 243, (2). Suppose that a small + charge is originally placed on the inductor *CD*. Induction takes place in the metallic system consisting of the disks *l* and *o* and the rod *rs*, *l* becoming negatively charged and *o* positively charged. As the plate carrying *l, m, n, o, p, q* rotates in the direction of the arrow the negative charge on *l* is carried over to the position *m*, where a part of it passes over to the inductor *AB*, thus charging it negatively. When *l* reaches the position *n*, the remainder of its charge, being repelled by the negative which is now on *AB*, passes over into the Leyden jar *L*. When *l* reaches the position *o*, it again becomes charged by induction, this time positively, and more strongly than at first, since now the negative on *AB*, as well as the positive on *CD*, is acting inductively upon the rod *rs*. When *l* reaches the position *u*, its now strong positive charge pulls negative from *CD*, thus increasing the positive charge upon this inductor. In the position *v* negative is pulled out of *L′*, thus leaving it positively charged and discharging *l*. This completes the cycle for *l*. Thus, as the rotation continues, *AB* and *CD* acquire stronger and stronger charges, the inductive action upon *rs* becomes more and more intense, and positive and negative charges are continuously imparted to *L′* and *L* until a discharge takes place between the knobs *R* and *S*.

There is usually sufficient charge on one of the inductors to start the machine, but in damp weather it will often be found necessary to apply a charge to one of the inductors before the machine will start.

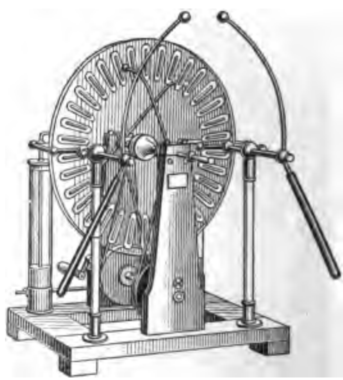


FIG. 244. The Wimshurst induction machine

307. The Wimshurst electrical machine. The essential difference between the Toepler-Holtz (Fig. 243) and the Wimshurst electrical machine (Fig. 244) is that the latter has two plates revolving in opposite directions, and that these plates carry a large number of tin-foil strips which act alternately as inductors and as carriers, thus dispensing with the necessity of separate inductors. The action of the machine may be understood readily from Fig. 245. Suppose that a small negative charge is placed on *a*. This, acting inductively on the rod *rs*, charges

a' positively. When a' in the course of the rotation reaches the position b', it acts inductively upon the rod $s'r'$ and thus charges the disk b negatively. It will be seen that henceforth all the disks in the inner circle receive + charges as they pass the brush r, and that all the disks in the outer circle, that is, on the back plate, receive − charges as they pass the brush s'. Similarly, on the lower half of the plates all the disks on the inner circle receive − charges as they pass the brush s, and all the disks on the outer circle receive + charges as they pass the brush r'.

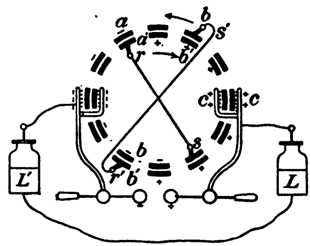


FIG. 245. Principle of Wimshurst machine

When the positive charges on the inner disks come opposite the combs c, they are transferred to the + knob of the machine or to the Leyden jar connected with it. The same process is occurring on the other side, where − charges are being taken off. When a spark passes, the Leyden jars and the connecting system of conductors are restored to their initial conditions and the process begins again.

QUESTIONS AND PROBLEMS

1. With a stick of sealing wax and a piece of flannel, in what two ways could you give a positive charge to an insulated body?

2. Will a solid sphere hold a larger charge of electricity than a hollow one of the same diameter?

3. When a negatively electrified cloud passes over a house provided with a lightning rod, the rod discharges positive electricity into the cloud. Explain.

4. Why is the capacity of a conductor greater when another conductor connected to the earth is near it than when it stands alone?

5. A Leyden jar is placed on a glass plate and 10 units of electricity placed on the inner coating. The knob is then connected to a gold-leaf electroscope. Will the leaves of the electroscope stand farther apart now or after the outside coating has been connected to the earth?

6. Why cannot a Leyden jar be appreciably charged if the outer coat is insulated?

7. Why is it not necessary to connect to earth the outer coatings of the Leyden jars on an electrical machine to charge them fully, provided they are connected to one another?

CHAPTER XIII

ELECTRICITY IN MOTION *

Detection and Measurement of Electric Currents

308. Electricity in motion produces a magnetic effect. Let a powerfully charged Leyden jar be discharged through a coil which surrounds an unmagnetized knitting needle, insulated by a glass tube, in the manner shown in Fig. 246. After the discharge the needle will be found to be distinctly magnetized. If the sign of the charge on the jar is reversed, the poles will in general be reversed.

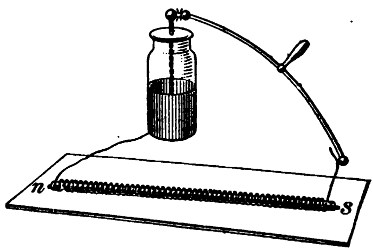


Fig. 246. Magnetizing effect of spark on knitting needle

The experiment shows that there is a definite connection between electricity and magnetism. Just what this connection is we do not yet know with certainty, but we do know that magnetic effects are always observable near the path of a moving electrical charge, while no such effects can ever be observed near a charge at rest.

To prove that a charge at rest does not produce a magnetic effect, let a charged body be brought near a compass needle. It will attract either end of the needle with equal readiness. While the needle is deflected, insert between it and the charge a sheet of zinc, aluminium, brass, or copper. This will act as an electric screen (see § 299, p. 231) and will therefore cut off all effect of the charge. The compass needle will at once swing back to its north-and-south position.

* This chapter should be accompanied or, better, *preceded* by laboratory experiments on the simple cell and on the magnetic effects of a current. See, for example, Experiments 28, 29, and 30 of the authors' manual.

Let the compass needle be deflected by a bar magnet and let the screen be inserted again. The sheet of metal does not cut off the magnetic forces in the slightest degree.

The fact that an electric charge exerts no magnetic force is shown, then, both by the fact that it attracts either end of the compass needle with equal readiness, and also by the fact that the screen cuts off its action completely, while the same screen does not have any effect in cutting off the magnetic force.

An electrical charge in motion is called an electric current, and its presence is most commonly detected by the magnetic effect which it produces.

309. The galvanic cell. When a Leyden jar is discharged, but a very small quantity of electricity passes through the connecting wires, since the current lasts for but a small fraction of a second. If we could keep a current flowing continuously through the wire, we should expect the magnetic effect to be much more pronounced. It was in 1786 that Galvani, an Italian anatomist at the University of Bologna, accidentally discovered that there is a chemical method for producing such a continuous current. His discovery was not understood, however, until Volta, while endeavoring to throw light upon it, in 1800 invented an arrangement which is now known sometimes as the *voltaic* and sometimes as the *galvanic* cell. This consists, in its simplest form, of a strip of copper and a strip of zinc immersed in dilute sulphuric acid (Fig. 247).

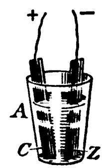


FIG. 247. Simple voltaic cell

Let the terminals of such a cell be connected for a few seconds to the ends of the coil of Fig. 246 when an unmagnetized needle lies within the glass tube. The needle will be found to have become magnetized much more strongly than before. Again, let the wire which connects the terminals of the cell be held above a magnetic needle, as in Fig. 248; the needle will be strongly deflected.

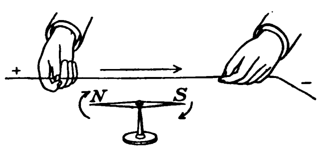


FIG. 248. Oersted's experiment

Evidently, then, the wire which connects the terminals of a galvanic cell carries a current of electricity. Historically the second of these experiments, performed by the Danish physicist Oersted in 1820, preceded the discovery of the magnetizing effects of currents upon needles. It created a great deal of excitement at the time, because it was the first clue which had been found to a relationship between electricity and magnetism.

310. Plates of a galvanic cell are electrically charged. Since an electric current flows through a wire as soon as it is touched to the zinc and copper strips of a galvanic cell, we at once infer that the terminals of such a cell are electrically charged before they are connected. That this is indeed the case may be shown as follows:

Let a metal plate *A* (Fig. 249), covered with shellac on its lower side and provided with an insulating handle, be placed upon a similar plate *B* which is in contact with the knob of an electroscope. Let the copper plate of a galvanic cell be connected with *A* and the zinc plate with *B*, as in Fig. 249. Then let the connecting wires be removed and the plate *A* lifted away from *B*. The opposite electrical charges which were bound by their mutual attractions to the adjacent faces of *A* and *B*, so long as these faces were separated only by the thin coat of shellac, are freed as soon as *A* is lifted, and hence part of the charge on *B* passes to the leaves of the electroscope. These leaves will indeed be seen to diverge. If an ebonite rod which has been rubbed with flannel or cat's fur is brought near the electroscope, the leaves will diverge still farther, thus showing that the zinc plate of the galvanic cell is negatively charged.* If the experiment is repeated with the copper plate in contact with *B* and the zinc in contact with *A*, the leaves will be found to be positively charged.

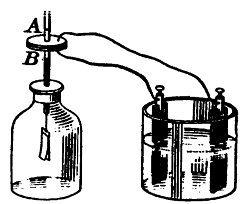


FIG. 249. Showing charges on plates of a voltaic cell

* If the deflection of the gold leaves is too small for purposes of demonstration, let a battery of from five to ten cells be used instead of the single cell. However, if the plates *A* and *B* are three or four inches in diameter, and if their surfaces are very flat, a single cell is sufficient.

Hans Christian Oersted
(1777–1851)

The discoverer of the connection between electricity and magnetism was a Dane and a professor at the University of Copenhagen. His famous experiment made in 1820 stimulated the researches which led to the modern industrial developments of electricity

Joseph Henry (1797–1878)

Born in Albany, New York; taught physics and mathematics in Albany Academy and Princeton College. He invented the electromagnet (1828), discovered the oscillatory nature of the electric spark (1842) by magnetizing needles in the manner described on page 241, and made the first experiments in self-induction (1832). He was the first secretary of the Smithsonian Institution, and the organizer of the Weather Bureau

The terminals of a galvanic cell therefore carry positive and negative charges just as do the terminals of an electrical machine in operation. The + charge is always found upon the copper and the − charge upon the zinc. The source of these charges is the chemical action which takes place within the cell. When these terminals are connected by a conductor a current flows through the latter just as in the case of the electrical machine, and it is the universal custom to consider that it flows from positive to negative (see § 300 and footnote), that is, from copper to zinc.

311. Comparison of a galvanic cell and static machine. If one of the terminals of a galvanic cell is touched directly to the knob of a gold-leaf electroscope, without the use of the condenser plates *A* and *B* of Fig. 249, no divergence of the leaves will be detected; but if one knob of the static machine in operation were so touched, the leaves would probably be torn apart by the violence of the divergence. Since we have seen in § 301 that the divergence of the gold leaves is a measure of the potential of the body to which they are connected, we learn from this experiment that the chemical actions in the galvanic cell are able to produce between its terminals but a very small potential difference in comparison with that produced by the static machine between its terminals. As a matter of fact the potential difference between the terminals of the cell is about one volt, while that between the knobs of the electrical machine may be as much as 200,000 volts.

But if the knobs of the static machine are connected to the ends of the wire of Fig. 248, and the machine operated, the current sent through the wire will not be large enough to produce any appreciable effect upon the needle. Since under these same circumstances the galvanic cell produced a very large effect upon the needle, we learn that although the cell develops a very small P.D. between its terminals, it nevertheless sends through the connecting wire very much more electricity per second

than the static machine is able to send. This is because the chemical action of the cell is able to recharge the plates to their small P.D. practically as fast as they are discharged through the wire, whereas the static machine requires a relatively long time to recharge its terminals to their high P.D. after they have been once discharged.

312. Shape of the magnetic field about a current. If we place the wire which connects the plates of a galvanic cell in a vertical position [Fig. 250, (1)] and explore with a compass needle the shape of the

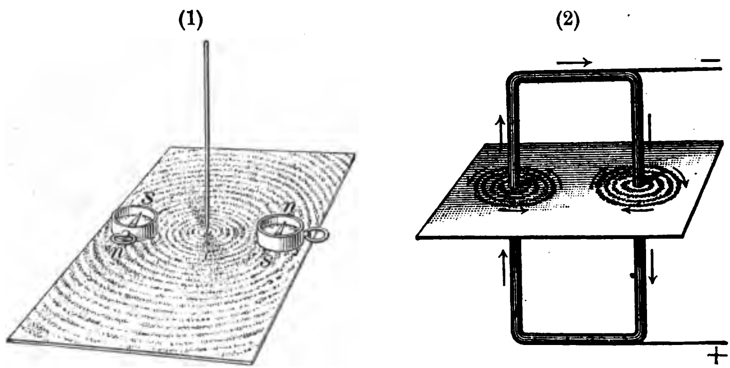


FIG. 250. Magnetic field about a current

magnetic field about the current, we find that the magnetic lines are concentric circles lying in a plane perpendicular to the wire and having the wire as their common center. If we reverse the direction of the current, we find that the direction in which the compass needle points reverses also. If the current is very strong (say 40 amperes), this shape of the field can be shown by scattering iron filings on a plate through which the current passes, in the manner shown in Fig. 250, (1). If the current is weak the experiment should be performed as indicated in Fig. 250, (2).

The relation between the direction in which the current flows and the direction in which the N pole of the needle points (this is, by definition, the direction of the magnetic field) is given in the following convenient rule: *If the right*

hand grasps the wire as in Fig. 251, so that the thumb points in the direction in which the current is flowing, then the magnetic lines encircle the wire in the same direction as do the fingers of the hand.

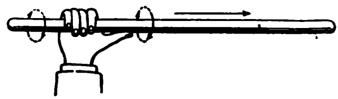


FIG. 251. The right-hand rule

313. The measurement of electrical currents. Electrical currents are, in general, measured by the strength of the magnetic effect which they are able to produce under specific conditions. Thus, if the wire carrying a current is wound into circular form as in Fig. 252, the right-hand rule shows us that the shape of the magnetic field at the center of the coil is similar to that shown in the figure. If, then, the coil is placed in a north-and-south plane and a compass needle is placed at the center, the passage of the current through the coil tends to deflect the needle so as to make it point east and west. The amount of deflection under these conditions is taken as the measure of current strength. The unit of current is called the *ampere* and is in fact approximately the same as the current which, flowing through a circular coil of three turns and 10 centimeters radius, set in a north-and-south plane, will produce a deflection of 45 degrees at Washington in a small compass needle placed at its center. The legal definition of the ampere is, however, based on the chemical effect of a current. It will be given in § 339.

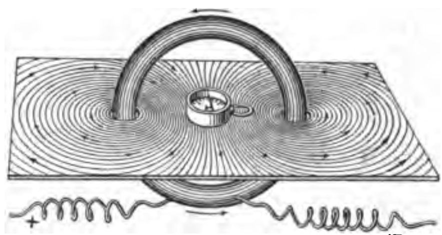


FIG. 252. Magnetic field about a circular coil carrying a current

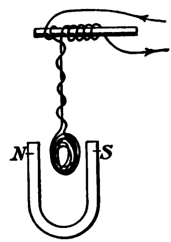


FIG. 253. Simple suspended-coil galvanometer

Nearly all current-measuring instruments consist essentially either of a small compass needle at the center of a fixed coil as in Fig. 252, or of a movable coil suspended between the poles of a fixed magnet in the manner illustrated roughly in Fig. 253. The passage of the current through the coil produces a deflection, in the first case, of the magnetic needle with reference to the fixed coil, and in the second case, of the coil with reference to the fixed magnet. If the instrument has been calibrated to give the strength of the current directly in amperes, it is called an *ammeter*, otherwise a *galvanometer* (Fig. 254).

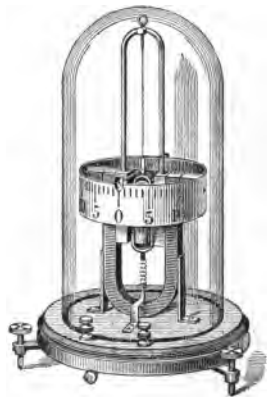


FIG. 254. A lecture-table galvanometer

QUESTIONS AND PROBLEMS

1. How could you test whether or not the strength of an electric current is the same in all parts of a circuit? Try it.

2. Under what conditions will an electric charge produce a magnetic effect?

3. In what direction will the north pole of a magnetic needle be deflected if it is held above a current flowing from north to south?

4. A man stands beneath a north-and-south trolley line and finds that a magnetic needle in his hand has its north pole deflected toward the east. What is the direction of the current flowing in the wire?

5. A loop of wire lying on the table carries a current which flows around it in clockwise direction. Would a north magnetic pole at the center of the loop tend to move up or down?

6. When a compass needle is placed, as in Fig. 252, at the middle of a coil of wire which lies in a north-and-south plane, the deflection produced in the needle by a current sent through the coil is approximately proportional to the strength of the current, provided the deflection is small — not more, for example, than 20° or 25°; but when the deflection becomes large — say 60° or 70° — it increases very much more slowly than does the current which produces it. Can you see any reason why this should be so?

ANDRÉ MARIE AMPÈRE (1775–1836)

French physicist and mathematician; son of one of the victims of the guillotine in 1793; professor at the Polytechnic School in Paris and later at the College of France; began his experiments on electromagnetism in 1820, very soon after Oersted's discovery; published his great memoir on the magnetic effects of currents in 1823; first stated the rule for the relation between the direction of a current in a wire and the direction of the magnetic field about it. The ampere, the practical unit of current, is named in his honor

Electromotive Force and Resistance

314. Electromotive force and its measurement.* The potential difference which a galvanic cell or any other generator of electricity is able to maintain between its terminals when these terminals are not connected by a wire, that is, the total electrical pressure which the generator is capable of exerting, is commonly called its *electromotive force*, usually abbreviated to E.M.F. *The E.M.F. of an electrical generator may then be defined as its capacity for producing electrical pressure, or P.D.* This P.D. might be measured, as in § 301, by the deflection produced in an electroscope when one terminal was connected to the case of the electroscope and the other terminal to the knob. Potential differences are in fact measured in this way in all so-called electrostatic voltmeters.

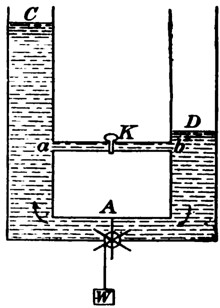


FIG. 255. Hydrostatic analogy of the action of a galvanic cell

The more common type of potential-difference measurer consists, however, of an instrument made like a galvanometer (Fig. 254), save that the coil of wire is made of very many turns of extremely fine wire, so that it carries a very small current. The amount of current which it does carry, however, is proportional to the difference in electrical pressure existing between its ends when these are touched to the two points whose P.D. is sought. The principle underlying this type of voltmeter will be better understood from a consideration of the following water analogy. If the stopcock *K* (Fig. 255) in the pipe connecting the water tanks *C* and *D* is closed, and if the water wheel *A* is set in motion by applying a weight *W*, the wheel will turn until it creates such a difference in the

* This subject should be preceded or accompanied by laboratory work on E.M.F. See, for example, Experiment 31 of the authors' manual.

†

water levels between C and D that the back pressure against the left face of the wheel stops it and brings the weight W to rest. In precisely the same way the chemical action within the galvanic cell whose terminals are not joined (Fig. 256) develops positive and negative charges upon these terminals; that is, creates a P.D. between them, until the back electrical pressure through the cell due to this P.D. is sufficient to put a stop to further chemical action. The seat of the E.M.F. is at the surfaces of contact of the metals with the acid, where the chemical actions take place. The E.M.F. of the cell has its water analogy in the wheel A, which creates the difference in water level between C and D.

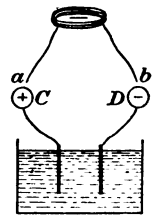


FIG. 256. Measurement of P.D. between the terminals of a galvanic cell

Now, if the water reservoirs (Fig. 255) are put in communication by opening the stopcock K, the difference in level between C and D will begin to fall, and the wheel will begin to build it up again. But if the carrying capacity of the pipe ab is small in comparison with the capacity of the wheel to remove water from D and to supply it to C, then the difference of level which permanently exists between C and D when K is open will not be appreciably smaller than when it is closed. In this case the current which flows through AB may obviously be taken as a measure of the difference in pressure which the pump is able to maintain between C and D when K is closed.

In precisely the same way, if the terminals C and D of the cell (Fig. 256) are connected by attaching to them the terminals a and b of any conductor, they at once begin to discharge through this conductor, and their P.D. therefore begins to fall. But if the chemical action in the cell is able to recharge C and D very rapidly in comparison with the ability of the wire to discharge them, then the P.D. between C and D will not be appreciably lowered by the presence of the connecting

conductor. In this case the current which flows through the conducting coil, and therefore the deflection of the needle at its center, may be taken as a measure of the electrical pressure developed by the cell; that is, of the P.D. between its unconnected terminals.

The common voltmeter is, then, exactly like an ammeter, save that it offers so high a resistance to the passage of electricity through it that it does not appreciably reduce the P.D. between the points to which it is connected.

To determine experimentally whether or not touching the ends of any particular galvanometer to the terminals of a cell does appreciably lower their P.D., let the galvanometer in question * be connected directly to the terminals of the cell and the deflection noted; then let the ends of a second coil of wire which has exactly the same carrying capacity as the galvanometer coil be also touched to the terminals, the galvanometer coil being still in circuit. If the second coil has sufficient carrying capacity to appreciably discharge the terminals, the deflection of the needle of the galvanometer will be instantly diminished when the ends of the second coil are brought into contact with them. If no such diminution is observed, we may know that the second coil does not discharge the terminals of the cell fast enough to appreciably lower their P.D., and hence that the introduction of the first coil, which was of equal carrying capacity, also did not appreciably lower the P.D. between the terminals. To show that a coil of greater carrying capacity will at once lower the P.D. between *C* and *D* as soon as it is touched across them, let a coil of thicker wire be so touched. The deflection of the needle will be diminished instantly.

FIG. 257. Lecture-table voltmeter

315. The electromotive forces of galvanic cells. Let a voltmeter of any sort be connected to the terminals of a simple galvanic cell, like that of Fig. 247. Then let the distance between the plates and the

* A vertical lecture-table voltmeter (Fig. 257) and a similar ammeter are desirable for this and some of the following experiments, but homemade high- and low-resistance galvanometers, like those described in the authors' manual, are thoroughly satisfactory, save for the fact that one student must take the readings for the class.

amount of their immersion be changed through wide limits. It will be found that the deflection produced is altogether independent of the shape or size of the plates or their distance apart. But if the nature of the plates is changed, the deflection changes. Thus, while copper and zinc in dilute sulphuric acid have an E.M.F. of one volt, carbon and zinc show an E.M.F. of at least 1.5 volts, while carbon and copper will show an E.M.F. of very much less than a volt. Similarly, by changing the nature of the liquid in which the plates are immersed, we can produce changes in the deflection of the voltmeter.

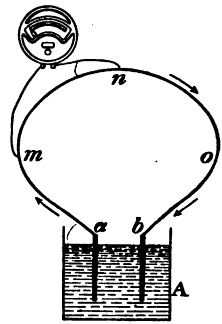


FIG. 258. Showing method of connecting voltmeter to find P.D. between any two points m and n on an electrical circuit

We learn therefore that *the E.M.F. of a galvanic cell depends simply upon the materials of which the cell is composed and not at all upon the shape, size, or distance apart of the plates.*

316. Fall of potential along a conductor carrying a current. Not only does a P.D. exist between the terminals of a cell on open circuit, but also between any two points on a conductor through which a current is passing. For example, in the electrical circuit shown in Fig. 258 the potential at the point a is higher than that at m, that at m higher than that at n, etc., just as in the water circuit shown in Fig. 259, the hydrostatic pressure at a is greater than that at m, that at m greater than that at n, etc. The fall in the water pressure between m and n (Fig. 259) is measured by the water head $n's$. If we wish to measure the fall in electrical potential between m and n (Fig. 258), we touch the

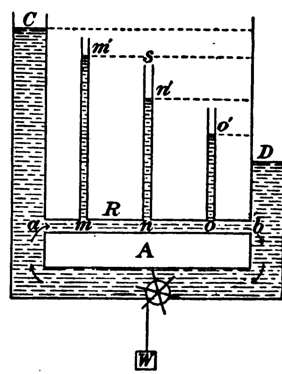


FIG. 259. Hydrostatic analogy of fall of potential in an electrical circuit

terminals of a voltmeter to these points in the manner shown in the figure. Its reading gives us at once the P.D. between *m* and *n* in volts, provided always that its own current-carrying capacity is so small that it does not appreciably lower the P.D. between the points *m* and *n* by being touched across them; that is, provided the current which flows through it is negligible in comparison with that which flows through the conductor which already joins the points *m* and *n*.

317. Electrical resistance.* Let the circuit of a galvanic cell be connected through a lecture-table ammeter, or any low-resistance galvanometer, and, for example, 20 feet of No. 30 copper wire, and let the deflection of the needle be noted. Then let the copper wire be replaced by an equal length of No. 30 German-silver wire. The deflection will be found to be a very small fraction of what it was at first.

A cell, therefore, which is capable of developing a certain fixed electrical pressure is able to force very much more current through a given wire of copper than through an exactly similar wire of German silver. We say, therefore, that German silver offers a higher *resistance* to the passage of electricity than does copper. Similarly, every particular substance has its own characteristic power of transmitting electrical currents. Since silver is the best conductor known, resistances of different substances are commonly referred to it as a standard, and the ratio between the resistance of a given wire of any substance and the resistance of an exactly similar silver wire is called the *specific resistance* of that substance. The specific resistances of some of the commoner metals in terms of silver are given below:

Silver . . .	1.00	Soft iron . .	6.00	German silver .	14–20
Copper . . .	1.11	Nickel . . .	4.67	Hard steel . .	13.5
Aluminum .	1.87	Platinum . .	7.20	Mercury . . .	63.1

* This subject should be accompanied and followed by laboratory experiments on Ohm's law, on the comparison of wire resistances, and on the measurement of internal resistances. See, for example, Experiments 32, 33, and 34 of the authors' manual.

The resistance of any conductor is directly proportional to its length and inversely proportional to the area of its cross section.

The unit of resistance is the *ohm*, so called in honor of the great German physicist, Georg Ohm (1789–1854). A length of 9.35 feet of No. 30 copper wire, or 6.2 inches of No. 30 German-silver wire, has a resistance of about one ohm. *The legal definition of the ohm is a resistance equal to that of a column of mercury* 106.3 *centimeters long and* 1 *square millimeter in cross section, at* 0° *C.*

318. Resistance and temperature. Let the circuit of a galvanic cell be closed through a very low-resistance galvanometer and about 10 feet of No. 30 iron wire wrapped about a strip of asbestos. Let the deflection of the galvanometer be observed as the wire is heated in a Bunsen flame. As the temperature rises higher and higher the current will be found to fall continually.

The experiment shows that *the resistance of iron increases with rising temperature.* This is a general law which holds for all metals. In the case of liquid conductors, on the other hand, the resistance usually decreases with increasing temperature. Carbon and a few other solids show a similar behavior, the filament in an incandescent electric lamp having only about half the resistance when hot which it has when cold.

319. Ohm's law. In 1826 Ohm announced the discovery that *the currents furnished by different galvanic cells, or combinations of cells, are always directly proportional to the E.M.F.'s existing in the circuits in which the currents flow, and inversely proportional to the total resistances of these circuits;* that is, if C represents the current in amperes, E the E.M.F. in volts, and R the resistance of the circuit in ohms, then Ohm's law as applied to the complete circuit is:

$$C = \frac{E}{R}; \text{ that is, current} = \frac{\text{electromotive force}}{\text{resistance}}. \qquad (1)$$

GEORG SIMON OHM (1787–1854)

German physicist and discoverer of the famous law in physics which bears his name. He was born and educated at Erlangen. It was in 1826, while he was teaching mathematics at a gymnasium in Cologne, that he published his famous paper on the experimental proof of his law. At the time of his death he was professor of experimental physics in the university at Munich

As applied to any portion of an electrical circuit, Ohm's law is

$$C = \frac{PD}{r}; \text{ that is, current} = \frac{\text{potential difference}}{\text{resistance}}, \qquad (2)$$

where P.D. represents the difference of potential in volts between any two points in the circuit, and r the resistance in ohms of the conductor connecting these two points. This is one of the most important laws in physics.

Both of the above statements of Ohm's law are included in the equation:

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}. \qquad (3)$$

320. Internal resistance of a galvanic cell. Let the zinc and copper plates of a simple galvanic cell be connected to an ammeter, and the distance between the plates then increased. The deflection of the needle will be found to decrease; or, if the amount of immersion is decreased, the current also will decrease.

Now, since the E.M.F. of a cell was shown in § 315 to be wholly independent of the area of the plates immersed or of the distance between them, it will be seen from Ohm's law that the change in the current in these cases must be due to some change in the total resistance of the circuit. Since the wire which constitutes the outside portion of the circuit has remained the same, we must conclude that *the liquid within the cell, as well as the external wire, offers resistance to the passage of the current.* This internal resistance of the liquid is directly proportional to the distance between the plates, and inversely proportional to the area of the immersed portion of the plates. If, then, we represent the external resistance of the circuit of a galvanic cell by R_e and the internal by R_i, Ohm's law as applied to the entire circuit takes the form

$$C = \frac{E}{R_e + R_i}. \qquad (4)$$

Thus, if a simple cell has an internal resistance of 2 ohms and an E.M.F. of 1 volt, the current which will flow through the circuit when its terminals are connected by 9.35 ft. of No. 30 copper wire (1 ohm) is $\frac{1}{1+2} = .33$ ampere.

321. Measurement of internal resistance. A simple and direct method of finding a length of wire which has a resistance equivalent to the internal resistance of a cell is to connect the cell first to an ammeter or any galvanometer of negligible resistance* and then to introduce enough German-silver wire into the circuit to reduce the galvanometer reading to half its original value. The internal resistance of the cell is then equal to that of the German-silver wire. Why? A still easier method in case both an ammeter and a voltmeter are available is to divide the E.M.F. of the cell as given by the voltmeter by the current which the cell is able to send through the ammeter when connected directly to its terminals; for in this case R_e of equation (4) is 0; therefore $R_i = \frac{E}{C}$. This gives the internal resistance directly in ohms.

322. Measurement of any resistance by ammeter-voltmeter method. The simplest way of measuring the resistance of a wire, or in general of any conductor, is to connect it into the circuit of a galvanic cell in the manner shown in Fig. 260. The ammeter A is inserted to measure the current, and the voltmeter V to measure the P.D. between the ends a and b of the wire r, the resistance of which is sought. The resistance of r in ohms is obtained at once from the ammeter and voltmeter readings with the aid of the law $C = \frac{\text{P.D.}}{r}$, from which it follows that $r = \frac{\text{P.D.}}{C}$. Thus, if the voltmeter indicates a P.D. of .4 volt and the ammeter a current of .5 ampere, the resistance of r is $\frac{.4}{.5} = .8$ ohm.†

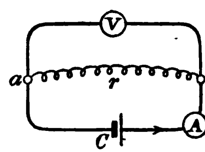


FIG. 260. Ammeter-voltmeter method of measuring resistance

* A lecture-table ammeter is best, but see note on page 249.

† The Wheatstone's bridge method of measuring resistance is recommended for laboratory study. See, for example, Experiment 33 of the authors' manual.

323. Joint resistance of conductors connected in series and in parallel. When resistances are connected as in Fig. 261, so that the same current flows through each of them in succession, they are said to be connected *in series*. The total resistance of a number of conductors so connected is the sum of the several resistances. Thus, in the case shown in the figure, the total resistance between a and b is 10 ohms.

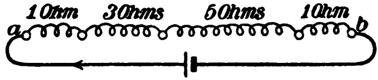


FIG. 261. Series connections

When n exactly similar conductors are joined in the manner shown in Fig. 262, that is, *in parallel*, the total resistance between a and b is $1/n$ of the resistance of one of them; for, obviously, with a given P.D. between the points a and b, four conductors will carry four times as much current as one, and n conductors will carry n times as much current as one. Therefore the resistance, which is inversely proportional to the carrying capacity (see § 319), is $1/n$ as much as that of one.

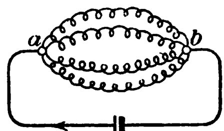


FIG. 262. Parallel connections

Even if the resistances are not all alike, since the total current C between a and b is obviously the sum of the currents in the branches, that is, since $C = C_1 + C_2 + C_3 + C_4$, we have at once the joint resistance R is given by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}$.

324. Shunts. A wire connected in parallel with another wire is said to be a *shunt* to that wire. Thus the conductor X (Fig. 263) is said to be shunted across the resistance R. Under such conditions the currents carried by R and X will be inversely proportional to their resistances, so that, if X is 1 ohm and R 10 ohms, R will carry $\frac{1}{10}$ as much current as X, or $\frac{1}{11}$ of the whole current.

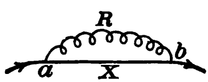


FIG. 263. A shunt

QUESTIONS AND PROBLEMS

1. Two wires of the same material but of diameters in the ratio 1 to 2 are connected in series in the same electrical circuit. The fall of potential in the smaller wire is 2 volts per foot of length. What is it in the larger wire?

2. If the potential difference between the terminals of a cell on open circuit is to be measured by means of a galvanometer, why must the galvanometer have a high resistance?

3. How long a piece of No. 30 copper wire will have the same resistance as a meter of No. 30 German-silver wire?

4. The resistance of a certain piece of German-silver wire is 1 ohm. What will be the resistance of another piece of the same length but of twice the diameter?

5. How much current will flow between two points whose P.D. is 2 volts, if they are connected by a wire having a resistance of 12 ohms?

6. What P.D. exists between the ends of a wire whose resistance is 100 ohms, when the wire is carrying a current of .3 ampere?

7. If a voltmeter attached across the terminals of an incandescent lamp shows a P.D. of 110 volts, while an ammeter connected in series with the lamp (see Fig. 260) indicates a current of .5 ampere, what is the resistance of the incandescent filament?

8. Ten pieces of wire, each having a resistance of 5 ohms, are connected in parallel (see Fig. 262). If the junction *a* is connected to one terminal of a Daniell cell and *b* to the other, what is the total current which will flow through the circuit when the E.M.F. of the cell is 1 volt and its resistance 2 ohms?

9. A voltmeter which has a resistance of 2000 ohms is shunted across the terminals *A* and *B* of a wire which has a resistance of 1 ohm. What fraction of the total current flowing from *A* to *B* will be carried by the voltmeter?

10. In a given circuit the P.D. across the terminals of a resistance of 19 ohms is found to be 3 volts. What is the P.D. across the terminals of a 3-ohm wire in the same circuit?

PRIMARY CELLS

325. Study of the action of a simple cell. If the simple cell already described, that is, zinc and copper strips in dilute sulphuric acid, is carefully observed, it will be seen that, so long as the plates are not connected by a conductor, fine bubbles of gas are slowly formed at the zinc plate, but none at the copper plate. As soon, however, as the two strips are put into metallic connection, bubbles appear in great numbers

about the copper plate (Fig. 264), and at the same time a current manifests itself in the connecting wire. These are bubbles of hydrogen. Their appearance on the zinc may be prevented either by using a plate of chemically pure zinc or by amalgamating impure zinc; that is, by coating it over with a thin film of mercury. But the bubbles on the copper cannot be thus disposed of. They are an invariable accompaniment of the current in the circuit. If the current is allowed to run for a considerable time, it will be found that the zinc wastes away, even though it has been amalgamated, but that the copper plate does not undergo any change.

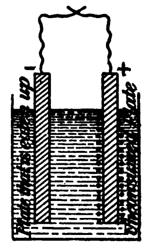


FIG. 264. Chemical actions in the voltaic cell

We learn, therefore, that the electrical current in the simple cell is accompanied with the eating up of the zinc plate by the liquid, and by the evolution of hydrogen bubbles at the copper plate. In every type of galvanic cell actions similar to these two are always found; that is, *one of the plates is always eaten up, and upon the other plate some element is deposited.* The plate which is eaten is always the one which is found to be negatively charged when tested as in § 310, so that in all galvanic cells, when the terminals are connected through a wire, the negative electricity flows through this wire from the eaten plate to the uneaten plate. This means, in accordance with the convention mentioned in § 300, that *the direction of the current through the external circuit is always from the uneaten to the eaten plate.*

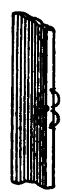


FIG. 265. Local action

326. Local action and amalgamation. The cause of the appearance of the hydrogen bubbles at the surface of impure zinc when dipped in dilute sulphuric acid is that little electrical circuits are set up between the zinc and the small impurities in it, — carbon or iron particles, — in the manner indicated in Fig. 265. If the zinc is pure, these little local currents cannot, of course, be set up, and consequently no

hydrogen bubbles appear. Amalgamation stops this so-called *local action*, because the mercury dissolves the zinc, while it does not dissolve the carbon, iron, or other impurities. The zinc-mercury amalgam formed is a homogeneous substance which spreads over the whole surface and covers up the impurities. It is important, therefore, to amalgamate the zinc in a battery in order to prevent the consumption of the zinc when the cell is on open circuit. The zinc is under all circumstances eaten up when the current is flowing, amalgamation serving only to prevent its consumption when the circuit is open.

327. Theory of the action of a simple cell. A simple cell may be made of any two dissimilar metals immersed in a solution of any acid or salt. For simplicity let us examine the action of a cell composed of plates of zinc and copper immersed in a dilute solution of hydrochloric acid. The chemical formula for hydrochloric acid is HCl. This means that each molecule of the acid consists of one atom of hydrogen combined with one atom of chlorine. In accordance with the theory now in vogue among physicists and chemists, when hydrochloric acid is mixed with water so as to form a dilute solution, the HCl molecules split up into two electrically charged parts, called *ions*, the hydrogen ion carrying a positive charge and the chlorine ion an equal negative charge (Fig. 266). This phenomenon is known as *dissociation*. The solution as a whole is neutral; that is, it is uncharged, because it contains just as many positive as negative ions.

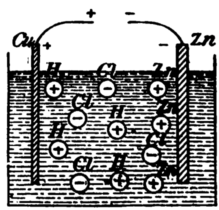


FIG. 266. Showing dissociation of hydrochloric acid molecules in water

When a zinc plate is placed in such a solution the acid attacks it and pulls zinc atoms into solution. Now whenever a metal dissolves in an acid, its atoms, for some unknown

reason, go into solution bearing little positive charges. *The corresponding negative charges must be left on the zinc plate* in precisely the same way in which a negative charge is left on silk when positive electrification is produced on a glass rod by rubbing it with the silk. It is in this way, then, that we account for the negative charge which we found upon the zinc plate in the experiment which was performed in connection with § 310.

The passage of positively charged zinc ions into solution gives a positive charge to the solution about the zinc plate, so that the hydrogen ions tend to be repelled toward the copper plate. When these repelled hydrogen ions reach the copper plate, some of them give up their charges to it and then collect as bubbles of hydrogen gas. It is in this way that we account for the positive charge which we found on the copper plate in the experiment of § 310.

If the zinc and copper plates are not connected by an outside conductor, this passage of positively charged zinc ions into solution continues but a very short time, for the zinc soon becomes so strongly charged negatively that it pulls back on the + zinc ions with as much force as the acid is pulling them into solution. In precisely the same way the copper plate soon ceases to take up any more positive electricity from the hydrogen ions, since it soon acquires a large enough + charge to repel them from itself with a force equal to that with which they are being driven out of solution by the positively charged zinc ions. It is in this way that we account for the fact that on open circuit no chemical action goes on in the simple galvanic cell, the zinc and copper plates simply becoming charged to a definite difference of potential which is called the E.M.F. of the cell.

When, however, the copper and zinc plates are connected by a wire, a current at once flows from the copper to the zinc and the plates thus begin to lose their charges. This allows

the acid to pull more zinc into solution at the zinc plate, and allows more hydrogen to go out of solution at the copper plate. These processes, therefore, go on continuously so long as the plates are connected. Hence a continuous current flows through the connecting wire until the zinc is all eaten up or the hydrogen ions have all been driven out of the solution; that is, until either the plate or the acid has become exhausted.

328. Polarization. If the simple galvanic cell described be connected to a lecture-table ammeter through two or three feet of No. 30 German-silver wire, the deflection of the needle will be found to decrease slowly; but if the hydrogen is removed from the copper plate (this can be done completely only by removing and thoroughly drying the plate), the deflection will be found to return to its first value.

The experiment shows clearly that the observed falling off in current was due to the collection of hydrogen about the copper plate. This phenomenon of the weakening of the current from a galvanic cell is called the *polarization* of the cell.

329. Causes of polarization. The presence of the hydrogen bubbles on the positive plate causes a diminution in the strength of the current for two reasons: first, since hydrogen is a nonconductor, by collecting on the plate it diminishes the effective area of the plate and therefore increases the internal resistance of the cell; second, the collection of the hydrogen about the copper plate lowers the E.M.F. of the cell, because it virtually substitutes a hydrogen plate for the copper plate, and we have already seen (§ 315) that a change in any of the materials of which a cell is composed changes its E.M.F. That there is a real fall in E.M.F. as well as a rise in internal resistance when a cell polarizes may be directly proved in the following way:

Let the deflection of a lecture-table voltmeter whose terminals are attached to the freshly cleaned plates of a simple cell be noted. Then let the cell's terminals be short-circuited through a coarse wire for half a minute. As soon as the wire is removed the E.M.F., indicated by the

voltmeter, will be found to be much lower than at first. It will, however, gradually creep back toward its old value as the hydrogen disappears from the plate, but a thorough cleaning and drying of the plate will be necessary to restore completely the original E.M.F.

The different forms of galvanic cells in common use differ chiefly in different devices employed either for disposing of the hydrogen bubbles or for preventing their formation. The most common types of such cells are described in the following sections.

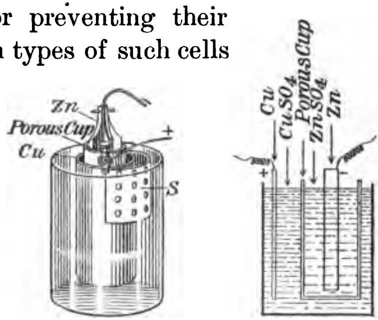


FIG. 267. The Daniell cell

330. The Daniell cell. The Daniell cell consists of a zinc plate immersed in zinc sulphate and a copper plate immersed in copper sulphate, the two liquids being kept apart either by means of an unglazed earthen cup, as in the type shown in Fig. 267,* or else by gravity, as in the type shown in Fig. 268. This last type, commonly called the gravity, or crowfoot, type, is used almost exclusively on telegraph lines. The copper sulphate, being the heavier of the two liquids, remains at the bottom about the copper plate, while the zinc sulphate remains at the top about the zinc plate.


FIG. 268. The gravity cell

In this cell polarization is almost entirely avoided, for the reason that no opportunity is given for the formation of hydrogen bubbles. For just as the hydrochloric acid solution described in § 327 consists of positive hydrogen ions and negative chlorine ions in water, so the zinc sulphate ($ZnSO_4$) solution consists of positive zinc ions and negative SO_4 ions, and the

* To set up, fill the battery jar with a saturated solution of copper sulphate. Fill the porous cup with water and add a handful of zinc sulphate crystals.

†

copper sulphate solution of positive copper ions and negative SO_4 ions. Now the zinc of the zinc plate goes into solution in the zinc sulphate in precisely the same way that it goes into solution in the hydrochloric acid of the simple cell described in § 327. This gives a positive charge to the solution about the zinc plate, and causes a movement of the positive ions between the two plates from the zinc toward the copper, and of negative ions in the opposite direction, both the Zn and the SO_4 ions being able to pass through the porous cup. Since the positive ions about the copper plate consist of atoms of copper, it will be seen that the material which is driven out of solution at the copper plate, instead of being hydrogen, as in the simple cell, is metallic copper. Since, then, the element which is deposited on the copper plate is the same as that of which it already consists, it is clear that neither the E.M.F. nor the resistance of the cell can be changed because of this deposit; that is, the cause of the polarization of the simple cell has been removed.

The great advantage of the Daniell cell lies in the relatively high degree of constancy in its E.M.F. (1.08 volts). It has a comparatively high internal resistance (one to six ohms) and is therefore incapable of producing very large currents, about one ampere at most. It will furnish a very constant current, however, for a great length of time, in fact, until all of the copper is driven out of the copper sulphate solution. In order to keep a constant supply of the copper ions in the solution, copper sulphate crystals are kept in the compartment *S* of the cell of Fig. 267, or in the bottom of the gravity cell. These dissolve as fast as the solution loses its strength through the deposition of copper on the copper plate.

The Daniell is a so-called "closed-circuit" cell; that is, its circuit should be left closed (through a resistance of thirty or forty ohms) whenever the cell is not in use. If it is left on open circuit, the copper sulphate diffuses through the porous

cup and a brownish muddy deposit of copper or copper oxide is formed upon the zinc. Pure copper is also deposited in the pores of the porous cup. Both of these actions damage the cell. When the circuit is closed, however, since the electrical forces always keep the copper ions moving toward the copper plate, these damaging effects are to a large extent avoided.

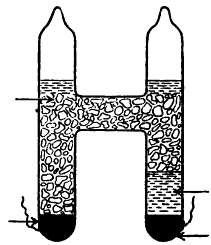


FIG. 269. The Weston normal cell

331. The Weston normal cell; the volt. This cell consists of a positive electrode of mercury in a paste of mercurous sulphate, and a negative electrode of cadmium amalgam in a saturated solution of cadmium sulphate (Fig. 269). It is so easily and exactly reproducible and has an E.M.F. of such extraordinary constancy that it has been taken by international agreement as the standard in terms of which all E.M.F.'s and P.D.'s are rated.

Thus the *E.M.F of a Weston normal cell at* 20° *C. is taken as* 1.0183 *volts. The legal definition of the volt is then an electrical pressure equal to* $\frac{1}{1.0183}$ *of that produced by a Weston normal cell.*

332. The Leclanché cell. The Leclanché cell (Fig. 270) consists of a zinc rod in a solution of ammonium chloride (150 grams to a liter of water), and a carbon plate placed inside of a porous cup which is packed full of manganese dioxide and powdered graphite or carbon. As in the simple cell, the zinc dissolves in the liquid and hydrogen is liberated at the carbon, or positive, plate. Here it is slowly attacked by the manganese dioxide. This chemical action is, however, not quick enough to prevent rapid polarization when large currents are taken from the cell. The cell slowly recovers when allowed to stand for a while on open circuit. The E.M.F. of a Leclanché cell is about 1.5 volts, and its initial internal resistance is somewhat less than an ohm. It therefore furnishes a momentary current of from one to three amperes.

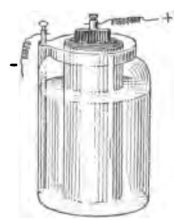


FIG. 270
The Leclanché cell

The immense advantage of this type of cell lies in the fact that the zinc is not at all eaten by the ammonium chloride when the circuit is open, and that therefore, unlike the Daniell cell, it can be left for an indefinite time on open circuit without deterioration. Leclanché cells are used almost exclusively where momentary currents only are needed, as, for example, on doorbell circuits. The cell requires no attention for years at a time, other than the occasional addition of water to replace loss by evaporation, and the occasional addition of ammonium chloride (NH_4Cl) to keep positive NH_4 and negative Cl ions in the solution.

333. The dry cell. The dry cell is only a modified form of the Leclanché cell. It is not really *dry*, since the zinc and carbon plates are embedded in moist paste which consists usually of one part of crystals of ammonium chloride, three parts of plaster of Paris, one part of zinc oxide, one part of zinc chloride, and two parts of water. The plaster of Paris is used to give the paste rigidity. As in the Leclanché cell, it is the action of the ammonium chloride upon the zinc which produces the current.

334. Combinations of cells. There are two ways in which cells may be combined: first, *in series*; and second, *in parallel*. When they are connected in series the zinc of one cell is joined to the copper of the second, the zinc of the second to the copper of the third, etc., the copper of the first and the zinc of the last being joined to the ends of the external resistance (see Fig. 271). The E.M.F. of such a combination is the sum of the E.M.F.'s of the single cells. The internal resistance of the combination is also the sum of the internal resistances of the single cells. Hence, if the external resistances are very small, the current furnished by the combination will be no larger than that furnished by a single cell, since the total resistance of the circuit has been increased in the same ratio as the total E.M.F. But if the external resistance is large, the current produced by the combination will be very much greater than that produced by a single cell. Just how much greater can always be determined by applying Ohm's law, for if there are n cells in series, and E is the E.M.F. of each cell, the total E.M.F.

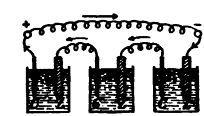


FIG. 271. Cells connected in series

of the circuit is nE. Hence, if R_e is the external resistance and R_i the internal resistance of a single cell, then Ohm's law gives

$$C = \frac{nE}{R_e + nR_i}.$$

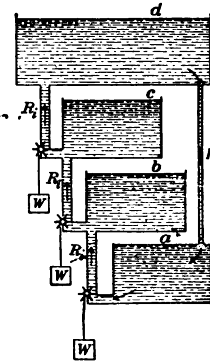


FIG. 272. Water analogy of cells in series

If the n cells are connected in parallel, that is, if all the coppers are connected together and all the zincs, as in Fig. 273, the E.M.F. of the combination is only the E.M.F. of a single cell, while the internal resistance is $1/n$ of that of a single cell, since connecting the cells in this way is simply equivalent to multiplying the area of the plates n times. The current furnished by such a combination will be given by the formula

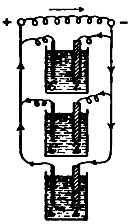


FIG. 273. Cells in parallel

$$C = \frac{E}{R_e + \frac{R_i}{n}}.$$

If, therefore, R_e is negligibly small, as in the case of a heavy copper wire, the current flowing through it will be n times as great as that which could be made to flow through it by a single cell. Figs. 272 and 274 show by means of the water analogy why the E.M.F. of cells in series is the sum of the several E.M.F.'s and why the E.M.F. of cells in parallel is no greater than that of a single cell. These considerations show that the rules which should govern the combination of cells are as follows: *Connect in series when R_e is large in comparison with R_i; connect in parallel when R_i is large in comparison with R_e.*

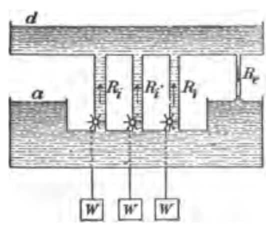


FIG. 274. Water analogy of cells in parallel

QUESTIONS AND PROBLEMS

1. A Daniell cell is found to send a current of .5 ampere through an ammeter of negligible resistance. What is its internal resistance?

2. Why is a Leclanché cell better than a Daniell cell for ringing doorbells?

3. If the internal resistance of a Daniell cell of the gravity type is 4 ohms, and its E.M.F. 1.08 volts, how much current will 40 cells in series send through a telegraph line having a resistance of 500 ohms? What current will one such cell send through the same circuit? What current will 40 cells joined in parallel send through the same circuit?

4. What current will the 40 cells in parallel send through an ammeter which has a resistance of .1 ohm? What current would the 40 cells in series send through the same ammeter? What current would a single cell send through the same ammeter?

5. Can you prove from a consideration of Ohm's law that when wires of different resistances are inserted in series in a circuit, the P.D.'s between the ends of the various wires are proportional to the resistances of these wires?

6. Under what conditions will a small cell give practically the same current as a large one of the same type?

7. Why must a galvanometer which is to be used for measuring voltages have a high resistance?

8. Why is it desirable that a galvanometer which is to be used for measuring currents have as small a resistance as possible?

9. A 50-volt lamp, resistance 110 ohms, and a 110-volt lamp, resistance 220 ohms, are connected in parallel. What is their joint resistance?

10. With the aid of Figs. 272 and 274 discuss the water analogies of the rules on the bottom of page 265.

CHAPTER XIV

CHEMICAL, MAGNETIC, AND HEATING EFFECTS OF THE ELECTRIC CURRENT

CHEMICAL EFFECTS; ELECTROLYSIS *

335. Electrolysis. Let two platinum electrodes be dipped into a solution of dilute sulphuric acid, and let the terminals of a battery producing an E.M.F. of 10 volts or more be applied to these *electrodes.* Oxygen gas is found to be given off at the electrode at which the current enters the solution, called the *anode,* while hydrogen is given off at the electrode at which the current leaves the solution, called the *cathode.* These gases may be collected in test tubes in the manner shown in Fig. 275.

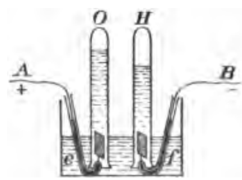


FIG. 275. The electrolysis of water

The modern theory of this phenomenon is as follows: Sulphuric acid (H_2SO_4), when it dissolves in water, breaks up into positively charged hydrogen ions and negatively charged SO_4 ions. As soon as an electrical field is established in the solution by connecting the electrodes to the positive and negative terminals of a battery, the hydrogen ions begin to migrate toward the cathode, and there, after giving up their charges, unite to form molecules of hydrogen gas. On the other hand, the negative SO_4 ions migrate to the positive electrode (that is, the anode), where they give up their charges to it, and then act upon the water (H_2O), thus forming H_2SO_4 and liberating oxygen.

* This subject should be accompanied or followed by a laboratory experiment on electrolysis and the principle of the storage battery. See, for example, Experiment 35 of the authors' manual.

If the volumes of hydrogen and of oxygen are measured, the hydrogen is found to occupy in every case just twice the volume occupied by the oxygen. This is, indeed, one of the reasons for believing that water consists of two atoms of hydrogen and one of oxygen.

336. Electroplating. If the solution, instead of being sulphuric acid, had been one of copper sulphate ($CuSO_4$), the results would have been precisely the same in every respect, except that, since the hydrogen ions in the solution are now replaced by copper ions, the substance deposited on the cathode is pure copper instead of hydrogen. This is the principle involved in electroplating of all kinds. In commercial work the positive plate, that is, the plate at which the current enters the bath, is always made from the same metal as that which is to be deposited from the solution, for in this case the SO_4 or other negative ions dissolve this plate as fast as the metal ions are deposited upon the other. The strength of the solution, therefore, remains unchanged. In effect, the metal is simply taken from one plate and deposited on the other. Fig. 276 represents a silver-plating bath. The bars joined to the anode *A* are of pure silver. The spoons to be plated are connected to the cathode *K*. The solution consists of 500 grams of potassium cyanide and 250 grams of silver cyanide in 10 liters of water.

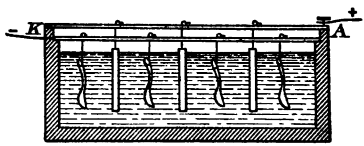


FIG. 276. Electroplating bath

337. Electrotyping. In the process of electrotyping the page is first set up in the form of common type. A mold is then taken in wax or gutta-percha. This mold is then coated with powdered graphite to render it a conductor, after which it is ready to be suspended as the cathode in a copper-plating bath, the anode being a plate of pure copper and the liquid a solution of copper sulphate. When a sheet of copper as thick as a visiting card has been deposited on the mold, the latter is

removed and the wax replaced by a type-metal backing, to give rigidity to the copper films. From such a plate as many as a hundred thousand impressions may be made. Practically all books which run through large editions are printed from such electrotypes.

338. Refining of metals. If the solution consists of pure copper sulphate, it is not necessary that the anode be of chemically pure copper in order to obtain a pure copper deposit on the cathode. *Electrolytic* copper, which is the purest copper on the market, is obtained as follows: The unrefined copper is used as an anode. As it is eaten up the impurities contained in it fall as a residue to the bottom of the tank and pure copper is deposited on the cathode by the current. This method is also extensively used in the refining of metals other than copper.

339. Legal units of current and quantity. In 1834 Faraday found that a given current of electricity flowing for a given time always deposits the same amount of a given element from a solution, whatever be the nature of the solution which contains the element. For example, one ampere always deposits in an hour 4.025 grams of silver, whether the electrolyte is silver nitrate, silver cyanide, or any other silver compound. Similarly, an ampere will deposit in an hour 1.181 grams of copper, 1.203 grams of zinc, etc. Faraday further found that the amount of metal deposited in a given cell depended solely on the product of the current strength by the time; that is, on the *quantity* of electricity which had passed through the cell. These facts are made the basis of the legal definitions of current and quantity, thus:

The unit of quantity, called the coulomb, is the quantity of electricity required to deposit .001118 *gram of silver.*

The unit of current, the ampere, is the current which will deposit .001118 *gram of silver in one second.*

340. Storage batteries. Let two 6 by 8 inch lead plates be screwed to a half-inch strip of some insulating material, as in Fig. 277, and immersed in a solution consisting of one part of sulphuric acid to

ten parts of water. Let a current from two storage or three dry cells in series, C, be sent through this arrangement, an ammeter A or any low-resistance galvanometer being inserted in the circuit. As the current flows, hydrogen bubbles will be seen to rise from the cathode (the plate at which the current leaves the solution), while the positive plate, or anode, will begin to turn dark brown. At the same time the reading of the ammeter will be found to decrease rapidly. The brown coating is a compound of lead and oxygen, called lead peroxide (PbO_2), which is formed by the action upon the plate of the oxygen which is liberated precisely as in the experiment on the electrolysis of water (§ 335). Let now the batteries be removed from the circuit by opening the key K_1, and let an electric bell B be inserted in their place by closing the key K_2. The bell will ring and the ammeter A will indicate a current flowing in a direction opposite to that of the original current. This current will decrease rapidly as the energy which was stored in the cell by the original current is expended in ringing the bell.

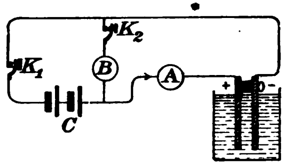


FIG. 277. The principle of the storage battery

This experiment illustrates the principle of the *storage battery*. Properly speaking, there has been no storage of *electricity*, but only a storage of *chemical energy*.

Two similar lead plates have been changed by the action of the current into two dissimilar plates, one of lead and one of lead peroxide. In other words, an ordinary galvanic cell has been formed; for any two dissimilar metals in an electrolyte constitute a primary galvanic cell. In this case the lead peroxide plate corresponds to the copper of an ordinary cell, and the lead plate to the zinc. This cell tends to create a current opposite in direction to that of the charging current; that is, its E.M.F. pushes back against the E.M.F. of the charging cells. It was for this reason that the ammeter reading fell. When the charging current is removed this cell acts exactly like a *primary* galvanic cell and furnishes a current until the thin coating of peroxide is used up. The only important difference between a commercial storage cell (Fig. 278) and the

one which we have here used is that the former is provided in the making with a much thicker coat of the "active material" (lead peroxide on the positive plate and a porous, spongy lead on the negative) than can be formed by a single charging such as we used. This material is pressed into interstices in the plates, as shown in Fig. 278. The E.M.F. of the storage cell is about 2 volts. Since the plates are always very close together and may be given any desired size, the internal resistance is usually small, so that the currents furnished may be very large.

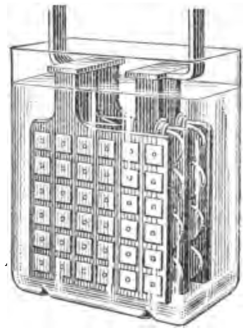


FIG. 278. Lead-plate storage cell

The usual efficiency of the storage cell is about 75%; that is, only about three fourths as much electrical energy can be obtained from it as is put into it.

QUESTIONS AND PROBLEMS

1. The coulomb (§ 339) is 3 billion times as large as the electrostatic unit of quantity defined in § 285. How many electrons pass per second past a given point on a lamp filament which is carrying 1 ampere of current (see § 290)?

2. If the terminals of a battery are immersed in a glass of acidulated water, how can you tell from the rate of evolution of the gases at the two electrodes which is positive and which negative?

3. How long will it take a current of 1 ampere to deposit 1 g. of silver from a solution of silver nitrate?

4. If the same current used in Problem 3 were led through a solution containing a zinc salt, how much zinc would be deposited in the same time?

5. In calibrating an ammeter, the current which produces a certain deflection is found to deposit $\frac{1}{2}$ g. of silver in 50 min. What is the strength of the current?

6. Why is it possible to get a much larger current from a storage cell than from a Daniell cell?

7. A certain storage cell having an E.M.F. of 2 volts is found to furnish a current of 20 amperes through an ammeter whose resistance is .05 ohm. Find the internal resistance of the cell.

Magnetic Properties of Coils

341. Loop of wire carrying a current equivalent to a magnet disk. Let a single loop of wire be suspended from a thread in the manner shown in Fig. 279, so that its ends dip into two mercury cups. Then let the current from three or four dry cells be sent through the loop. The latter will be found to slowly set itself so that the face of the loop from which the magnetic lines emerge, as given by the right-hand rule (see § 312 and also Fig. 280), is toward the north. Let a bar magnet be brought near the loop. The latter will be found to behave toward the magnet in all respects as though it were a flat magnetic disk whose boundary is the wire, the face which turns toward the north being an N pole and the other an S pole.

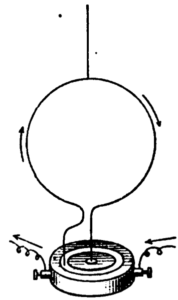


Fig. 279. A loop equivalent to a flat magnetic disk

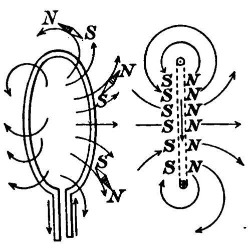


Fig. 280. North pole of disk is face from which magnetic lines emerge; south face is face into which they enter

The experiment shows what position a loop bearing a current will always tend to assume in a magnetic field. For since a magnet always tends to set itself so that the line connecting its poles is parallel to the direction of the magnetic lines of the field in which it is placed, a loop must set itself so that a line connecting its magnetic poles is parallel to the lines of the magnetic field, that is, so that *the plane of the loop is perpendicular to the field* (see Fig. 281); or, to state the same thing in slightly different form, *if a loop of wire, free to turn, is carrying a current in a magnetic field, the loop will set itself so as to include as many as possible of the lines of force of the field.*

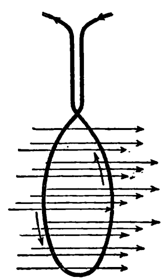


Fig. 281. Position assumed by a loop carrying a current in a magnetic field

342. Helix carrying a current is equivalent to a bar magnet. Let a wire bearing a current be wound in the form of a helix and held near a suspended magnet, as in Fig. 282. It will be found to act in every respect like a magnet, with an N pole at one end and an S pole at the other.

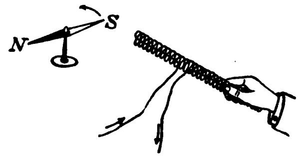


FIG. 282. Magnetic effect of a helix

This result might have been predicted from the fact that a single loop is equivalent to a flat-disk magnet. For when a series of such disks is placed side by side, as in the helix, the result must be the same as placing a series of disk magnets in a row, the N pole of one being directly in contact with the S pole of the next, etc. These poles would therefore all neutralize each other except at the two ends. We therefore get a magnetic field of the shape shown in Fig. 283, the direction of the arrows representing as usual the direction in which an N pole tends to move.

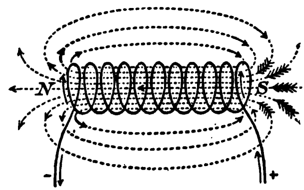


FIG. 283. Magnetic field of helix

The right-hand rule as given in § 312 is sufficient in every case to determine which is the N and which the S pole of a helix; that is, from which end the lines of magnetic force emerge from the helix and at which end they enter it. But it is found convenient, in the consideration of coils, to restate the right-hand rule in a slightly different way, thus: *If the coil is grasped in the right hand in such a way that the fingers point in the direction in which the current is flowing in the wires, the thumb will point in the direction of the north pole of the helix* (see Fig. 284). Similarly, if the sign of the poles is known, but the direction of the current unknown, it may

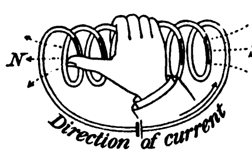


FIG. 284. Rule for poles of helix

be determined as follows: *If the right hand is placed against the coil with the thumb pointing in the direction of the lines of force* (that is, toward the north pole of the helix), *the fingers will pass around the coil in the direction in which the current is flowing.*

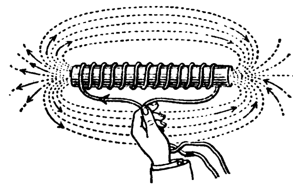


FIG. 285. The bar electromagnet

343. The electromagnet. Let a core of soft iron be inserted in the helix (Fig. 285). The poles will be found to be enormously stronger than before. This is because the core is magnetized by induction from the field of the helix in precisely the same way in which it would be magnetized by induction if placed in the field of a permanent magnet. The new field strength about the coil is now the sum of the fields due to the core and that due to the coil. If the current is broken, the core will at once lose the greater part of its magnetism. If the current is reversed, the polarity of the core will be reversed. Such a coil with a soft-iron core is called an *electromagnet.*

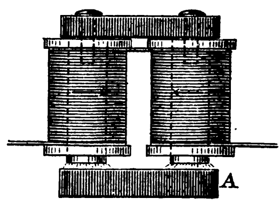


FIG. 286. The horseshoe electromagnet

The strength of an electromagnet can be very greatly increased by giving it such form that the magnetic lines can remain in iron throughout their entire length instead of emerging into air, as they do in Fig. 285. For this reason electromagnets are usually built in the horseshoe form and provided with an armature *A* (Fig. 286), through which a complete iron path for the lines of force is established, as shown in Fig. 287. The strength of such a magnet depends chiefly upon the number of *ampere turns* which encircle it, the expression "ampere turns" denoting the product of the number of turns of wire about the magnet by the number of amperes

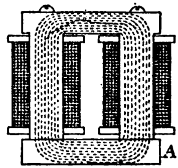


FIG. 287. Magnetic circuit of an electromagnet

flowing in each turn. Thus a current of $\frac{1}{100}$ ampere flowing 1000 times around a core will make an electromagnet of precisely the same strength as a current of 1 ampere flowing 10 times about the core.

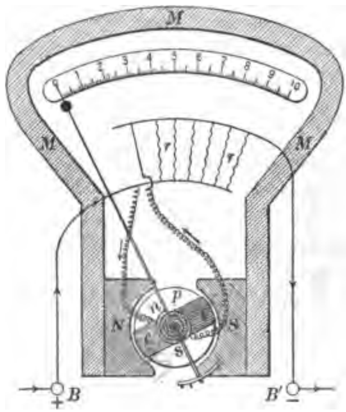


FIG. 288. Construction of a commercial ammeter

344. Commercial ammeters and voltmeters. Fig. 288 shows the construction of the usual form of commercial ammeter. The coil *c* is pivoted on jewel bearings and is held at its zero position by a spiral spring *p*. When a current flows through the instrument, were it not for the spring *p* the coil would turn through about 120°, or until its *n* pole came opposite the *S* pole of the magnet (see Fig. 281). This zero position of the coil is chosen because it enables the scale divisions to be nearly equal. The shunt coils *r* are of practically negligible resistance.

The voltmeter differs from the ammeter only in that the coils *r* are in series with *c* and are of high resistance. The same instrument may have its range changed or may even be used interchangeably as an ammeter or a voltmeter by suitably changing the coils *r*.

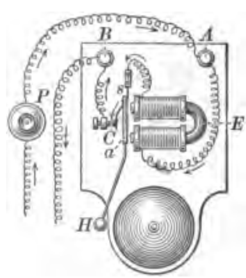


FIG. 289. The electric bell

345. The electric bell. The electric bell (Fig. 289) is one of the simplest applications of the electromagnet. When the button *P* (Figs. 289 and 290) is pressed, the electric circuit of the battery is closed, and a current flows in at *A*, through the coils of the magnet, over the closed contact *C*, and out again at *B*. But no sooner is this current established than the electromagnet *E* pulls over the armature *a*, and in so doing breaks the contact at *C*. This stops

the current and demagnetizes the magnet E. The armature is then thrown back against C by the elasticity of the spring s which supports it. No sooner is the contact made at C than the current again begins to flow and the former operation is repeated. Thus the circuit is automatically made and broken at C, and the hammer H is in consequence set into rapid vibration against the rim of the bell.

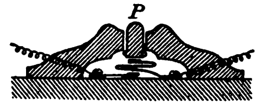


FIG. 290. Cross section of the electric push button

346. The telegraph. The electric telegraph is another simple application of the electromagnet. The principle is illustrated in Fig. 291. As soon as the key K at Chicago, for example, is closed, the current flows over the line to, we will say, New York. There it passes through the electromagnet m, and thence back to Chicago through the earth. The armature b is held down by the electromagnet m as long as the key K is kept closed. As soon as the circuit is broken at K the armature is pulled up by the spring d. By means of a clockwork device the tape c is drawn along at a uniform rate beneath the pencil or pen carried by the armature b. A very short time of closing of K produces a dot upon the tape, a longer time a dash. As the Morse, or telegraphic, alphabet consists of certain combinations of dots and dashes, any desired message may be sent from Chicago and recorded in New York. In modern practice the message is

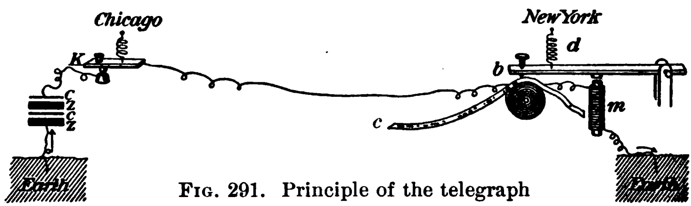


FIG. 291. Principle of the telegraph

not ordinarily recorded on a tape, for operators have learned to read messages by ear, a very short interval between two clicks being interpreted as a dot, a longer interval as a dash.

The first commercial telegraph line was built by S. F. B. Morse between Baltimore and Washington. It was opened on May 24, 1844, with the now famous message, "What hath God wrought!"

347. The relay and sounder. On account of the great resistance of long lines, the current which passes through the electromagnet is so weak that the armature of this magnet must be made very light in order to respond to the action of the current. The clicks of such an

SAMUEL F. B. MORSE (1791–1872)

The inventor of the electromagnetic recording telegraph and of the dot-and-dash alphabet known by his name, was born at Charlestown, Massachusetts, graduated at Yale College in 1810, invented the commercial telegraph in 1832, and struggled for twelve years in great poverty to perfect it and secure its proper presentation to the public. The first public exhibition of the completed instrument was made in 1837 at the College of the City of New York, signals being sent through 1700 feet of copper wire. It was with the aid of a $30,000 grant from Congress that the first commercial line was constructed in 1844 between Washington and Baltimore

armature are not sufficiently loud to be read easily by an operator. Hence at each station there is introduced a local circuit, which contains a local battery, and a second and heavier electromagnet, which is called a sounder. The electromagnet on the main line is then called the relay (see Figs. 292, 293, and 294). The sounder has a very heavy armature (A, Fig. 293), which is so arranged that it clicks both when it is drawn down by its electromagnet against the stop S and when it is pushed up again by its spring, on breaking the current, against the stop t. The interval which elapses between these two clicks indicates to the operator whether a dot or dash is sent. The current in the main line simply serves to close and open the circuit in the local battery which operates the sounder (see Fig. 294). The electromagnets of the relay and the sounder differ in that the former consists of many thousand turns of fine wire, usually having a resistance of about 150 ohms, while the latter consists of a few hundred turns of coarse wire having ordinarily a resistance of about 4 ohms.

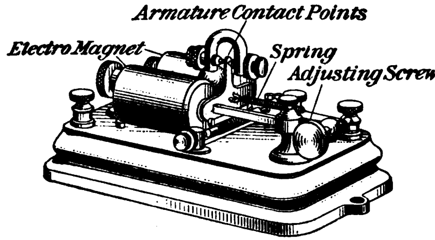


FIG. 292. The relay

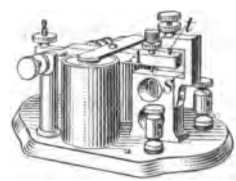


FIG. 293. The sounder

348. Plan of a telegraphic system. The actual arrangement of the various parts of a telegraphic system is shown in Fig. 294. When an operator at Chicago wishes to send a message to New York, he first opens the switch which is connected to his key, and which is always kept closed except when he is sending a message. He then begins to operate his key, thus controlling the clicks of both his own sounder and that at New York. When the Chicago switch is closed and the one at New York open, the New York operator is able to

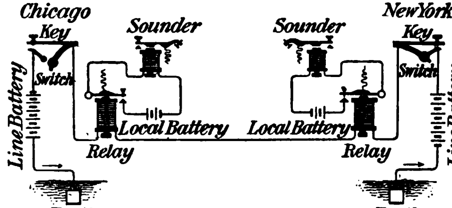


FIG. 294. Telegraphic system

send a message back over the same line. In practice a message is not usually sent as far as from Chicago to New York over a single line, save in the case of transoceanic cables. Instead it is automatically transferred at, say, Cleveland to a second line, which carries it on to Buffalo, where it is again transferred to a third line, which carries it on to New York. The transfer is made in precisely the same way as the transfer from the main circuit to the sounder circuit. If, for example, the sounder circuit at Cleveland is lengthened so as to extend to Buffalo, and if the sounder itself is replaced by a relay (called in this case a repeater), and the local battery by a line battery, then the sounder circuit has been transformed into a repeater circuit and all the conditions are met for an automatic transfer of the message at Cleveland.

QUESTIONS AND PROBLEMS

1. Why are iron wires used on telegraph lines but copper wires on trolley systems?

2. The plane of a suspended loop of wire is east and west. A current is sent through it, passing from east to west on the upper side. What will happen to the loop if it is perfectly free to turn?

3. When a strong current is sent through a suspended-coil galvanometer, what position will the coil assume?

4. If one looks down on the ends of a U-shaped electromagnet, does the current encircle the two coils in the same or in opposite directions? Does it run clockwise or counterclockwise about the N pole?

5. Draw a diagram, showing how an electric bell works.

6. Draw a diagram, showing how the relay and sounder operate in a telegraphic circuit.

7. Ordinary No. 9 telegraph wire has a resistance of 20 ohms to the mile. What current will 100 Daniell cells, each of E.M.F. of 1 volt, send through 100 miles of such wire, if the relays have a resistance of 150 ohms each and the cells an internal resistance of 4 ohms each?

8. If the relays of the preceding problem had each 10,000 turns of wire in their coils, how many ampere turns were effective in magnetizing their electromagnets?

9. If on the above telegraph line sounders having a resistance of 3 ohms each and 500 turns were to be put in the place of the relays, how many ampere turns would be effective in magnetizing their cores? Why, then, must a relay be a high-resistance electromagnet?

10. If the earth's magnetism is due to a surface charge rotating with the earth, must this charge be positive or negative to produce the sort of magnetic poles which the earth has? (This is actually the present theory of the earth's magnetism.)

Heating Effects of the Electric Current

349. Heat developed in a wire by an electric current. Let the terminals of two or three dry cells in series be touched to a piece of No. 40 iron or German-silver wire and the length of wire between these terminals shortened to $\frac{1}{4}$ inch or less. The wire will be heated to incandescence and probably melted.

The experiment shows that just as in the charging of a storage battery the energy of the electric current was transformed into the energy of chemical separation, so here in the passage of the current through the wire the energy of the electric current is transformed into heat energy.

350. Energy relations of the electric current. In Chapter IX we found that energy expended on a water turbine is equal to the quantity of water passing through it times the difference in level through which the water falls. In just the same way it is found that when a current of electricity passes through a conductor, the energy expended is equal to the quantity of electricity passing times the difference in potential between the ends of the conductor. If the quantity of electricity is expressed in coulombs and the P.D. in volts, the energy is given in joules, and we have

$$\text{Volts} \times \text{coulombs} = \text{joules.} \tag{1}$$

Since the number of coulombs is equal to the number of amperes of current multiplied by the number of seconds,

$$\text{Volts} \times \text{amperes} \times \text{seconds} = \text{joules.} \tag{2}$$

But a watt is defined as a joule per second (see § 192). Hence the energy expended per second by the current, that is, the *power* of the current, is given by

$$\text{Volts} \times \text{amperes} = \text{watts.} \tag{3}$$

351. Calories of heat developed in a wire. The electrical energy expended when a current flows between points of given P.D. may be spent in a variety of ways. For example, it may

be spent in producing chemical separation, as in the charging of a storage cell; it may be spent in doing mechanical work, as is the case when the current flows through an electric motor; or it may be spent wholly in heating the wire, as was the case in the experiment of § 349. It will always be expended in this last way when no chemical or mechanical changes are produced by it. The number of calories of heat produced per second in the wire of the last experiment is found, then, by multiplying the number of joules expended by the current per second by the heat equivalent of the joule in calories, that is, .24 calorie, since, by § 173 and § 213, 1 calorie is 4.2 joules. Therefore when all of the electrical energy of a current is transformed into heat energy, we have

$$\text{Calories per second} = \text{volts} \times \text{amperes} \times .24. \qquad (4)$$

The total number of calories H developed in t seconds will be given by

$$H = \text{P.D.} \times C \times t \times .24. \qquad (5)$$

Thus a current of 10 amperes flowing in a wire whose terminals are at a potential difference of 12 volts will develop in 5 minutes $10 \times 12 \times 300 \times .24 = 8640$ calories.

Since by Ohm's law P.D. $= C \times R$, we have, by substituting CR for P.D. in (5),

$$H = C^2R \times t \times .24; \qquad (6)$$

or *the heat generated in a conductor is proportional to the time, to the resistance, and to the square of the current.* This is known as Joule's law, having been first announced by him as the result of experimental researches.

352. Incandescent lamps. The ordinary incandescent lamp consists of a carbon filament heated to incandescence by an electric current (Fig. 295). Since the carbon would burn instantly in air, the filament is placed in a highly exhausted glass bulb. Even then it disintegrates slowly. The normal life of a 16-candle-power lamp filament is from 1000 to 2000 working

hours. The filament is made by carbonizing a special form of cotton thread. The ends of the carbonized thread are attached to platinum wires which are sealed into the glass walls of the bulb, and which make contact one with the base of the socket and the other with its rim, these being the electrodes through which the current enters and leaves the lamp.

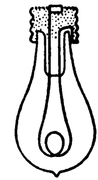


Fig. 295. The incandescent lamp

The ordinary 16-candle-power lamp is most commonly run on a circuit which maintains a potential difference of either 110 or 220 volts between the terminals of the lamp. In the former case the lamp carries about .5 ampere of current, and in the latter case about .25 ampere. It will be seen from these figures that the rate of consumption of energy is about 3.4 watts per candle power.

A customer usually pays for his light by the "watt hour," a watt hour being the energy furnished in one hour by a current whose rate of expenditure of energy is one watt. Thus the rate at which energy is consumed by a 16-candle-power lamp is $110 \times .5 = 55$ watts. One such lamp running for ten hours would therefore consume 550 watt hours of energy. Large quantities of electricity are sold by the *kilowatt hour.*

At the present time tungsten and tantalum filaments are being used very largely for incandescent lamps. They are nearly three times as efficient as the carbon lamps, the "Mazda" form taking but 1.25 watts per candle. This is because they can be operated at much higher temperatures than carbons. They are, however, much more fragile and more expensive.

353. The arc light. When two carbon rods are placed end to end in the circuit of a powerful electric generator, the carbon about the point of contact is heated red-hot. If, then, the ends of the carbon rods are separated one-fourth inch or so, the current will still continue to flow, for a conducting layer of incandescent vapor called an *electric arc* is

produced between the poles. The appearance of the arc is shown in Fig. 296. At the + pole a hollow, or crater, is formed in the carbon, while the – carbon becomes cone shaped, as in the figure. The carbons are consumed at the rate of about an inch an hour, the + carbon wasting away about twice as fast as the – one. The light comes chiefly from the + crater, where the temperature is about 3800° C., the highest attainable by man. All known substances are volatilized in the electric arc.

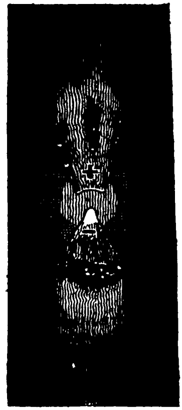


FIG. 296. The arc light

The ordinary arc requires a current of 10 amperes and a P.D. between its terminals of about 50 volts. Such a lamp produces about 500* candle power, and therefore consumes energy at the rate of about 1 watt per candle power. This makes an arc light about 3.5 times as efficient as an incandescent light. The recently invented *flaming arc*, produced between carbons which have a composite core consisting of carbon, lime, magnesia, silica, or other light-giving minerals, sometimes reaches an efficiency as high as .27 watt per candle power.

354. The arc-light automatic feed. Since the two carbons of the arc gradually waste away, they would soon become so far separated that the arc could no longer be maintained were it not for an automatic feeding device which keeps the distance between the carbon tips very nearly constant. Fig. 297 shows the essential features of one form of this device. When no current is flowing through the lamp, gravity holds the carbon tips at *e* together; but as soon as the current is thrown on, it energizes the low-resistance electromagnet *M*, which is in series with the carbons. This draws down the iron plunger *c*, which acts upon the lever *L* and "strikes the arc" at *e*. But the introduction of the resistance of the arc into the circuit *sABMt* raises the P.D. between *s* and *t* and thus causes an appreciable current to flow through the high-resistance magnet *N*, which is shunted across this circuit. This tends to *raise* the plunger *c* and thus to shorten the arc. There is thus one particular lengt

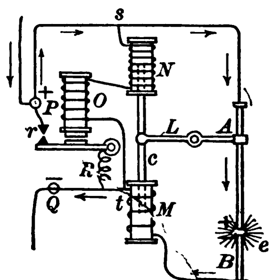


FIG. 297. Feeding device for arc lamp

* This is the so-called "mean spherical" candle power. The candle power the direction of maximum illumination is from 1000 to 1200.

of arc for which equilibrium exists between the effects of the series magnet M and the shunt magnet N. This length the lamp automatically maintains. The magnet O is the so-called "cut-out" inserted so that, if the lamp gets out of order and the arc burns out, a current at once flows through N and O of sufficient strength to close the contact points at r and thus permit the main current to flow on to the next lamp over the path $PrRQ$.

355. The Cooper-Hewitt mercury lamp. The Cooper-Hewitt mercury lamp (Fig. 298) is the most efficient of all electric lights, unless it be the flaming arc. It differs from the arc lamp in that the incandescent body is a long column of mercury vapor instead of an incandescent solid. The lamp consists of an exhausted tube three or four feet long, the positive electrode at the top consisting of a plate of iron, while the negative electrode at the bottom is a small quantity of mercury. Under a sufficient difference of potential between these terminals a long mercury-vapor arc is formed which stretches from terminal to terminal in the tube. This arc emits a very brilliant light, but it is almost entirely wanting in red rays. The efficiency of the lamp is very high, since it requires but .3 watt per candle power. It is rapidly finding important commercial uses, especially in photography. The chief objection to it arises from the fact that, on account of the absence of red rays, the light gives objects an unnatural color.

FIG. 289. The Cooper-Hewitt mercury lamp

QUESTIONS AND PROBLEMS

1. What horse power is required to run an incandescent lamp carrying .5 ampere at 110 volts? How much heat is developed per second?

2. Fig. 299 shows the connections for a lamp L which can be turned on or off at two different points a or b. Explain how it works.

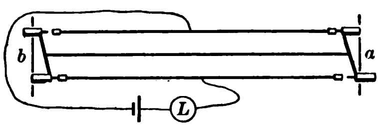


FIG. 299

3. A 220-volt lamp has a resistance, when hot, of about 750 ohms. How many calories will be developed in it in 10 min.?

4. If a storage cell has an E.M.F. of 2 volts, and furnishes a current of 5 amperes, what is its rate of expenditure of energy in watts?

5. How many cells, working as in Problem 4, would be equivalent to 1 H.P.? (See § 191, p. 147.)

A great light dawns.

CHAPTER XV

INDUCED CURRENTS

The Principle of the Dynamo and Motor

356. Current induced by a magnet. Let 400 or 500 turns of No. 22 copper wire be wound into a coil C (Fig. 300) about two and a half inches in diameter. Let this coil be connected into circuit with a lecture-table galvanometer (Fig. 254), or even a simple detector made by suspending in a box, with No. 40 copper wire, a coil of 200 turns of No. 30 copper wire (see Fig. 300). Let the coil C be thrust suddenly over the N pole of a strong horseshoe magnet. The deflection of the pointer p of the galvanometer will indicate a momentary current flowing through the coil. Let the coil be held stationary over the magnet. The pointer will be found to come to rest in its natural position. Now let the coil be removed suddenly from the pole. The pointer will move in a direction opposite to that of its first deflection, showing that a reverse current is now being generated in the coil.

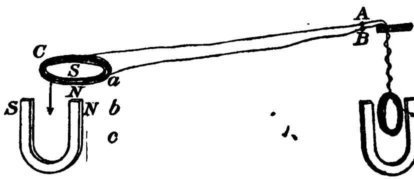


Fig. 300. Induction of electric currents by magnets

We learn, therefore, that *a current of electricity may be induced in a conductor by causing the latter to move through a magnetic field*, while a magnet has no such influence upon a conductor which is at rest with respect to the field. This discovery, one of the most important in the history of science, was announced by the great Faraday in 1831. From it have sprung directly most of the modern industrial developments of electricity.

MICHAEL FARADAY (1791–1867)

Famous English physicist and chemist; one of the most gifted of experimenters; son of a poor blacksmith; apprenticed at the age of thirteen to a London bookbinder, with whom he worked nine years; applied for a position in Sir Humphry Davy's laboratory at the Royal Institution in 1813; became director of this laboratory in 1825; discovered electromagnetic induction in 1831; made the first dynamo; discovered in 1833 the laws of electrolysis, now known as Faraday's laws; the farad, the practical unit of electrical capacity, is named in his honor

357. Direction of induced current. Lenz's law. In order to find the *direction* of the induced current, let a very small P.D. from a galvanic cell be applied to the terminals A and B (Fig. 300), and note the direction in which the pointer moves when the current enters, say at A. This will at once show in what direction the current was flowing in the coil C when it was being thrust over the N pole. By a simple application to C of the right-hand rule (§ 342) we can then tell which was the N and which the S face of the coil when the induced current was flowing through it. In this way it will be found that if the coil was being moved past the N pole of the magnet, the current induced in it was in such a direction as to make the lower face of the coil an N pole during the downward motion and an S pole during the upward motion. In the first case the repulsion of the N pole of the magnet and the N pole of the coil tended to *oppose* the motion of the coil while it was moving from a to b, and the attraction of the N pole of the magnet and the S pole of the coil tended to oppose the motion while it was moving from b to c. In the second case the repulsion of the two N poles tended to oppose the motion between b and c, and the attraction between the N pole of the magnet and the S pole of the coil tended to oppose the upward motion from b to a. *In every case, therefore, the motion is made against an opposing force.*

From these experiments, and others like them, we arrive at the following law: *Whenever a current is induced by the relative motion of a magnetic field and a conductor, the direction of the induced current is always such as to set up a magnetic field which opposes the motion.* This is Lenz's law. This law might have been predicted at once from the principle of the conservation of energy; for this principle tells us that since an electric current possesses energy, such a current can appear only through the expenditure of mechanical work or of some other form of energy.

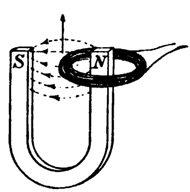


FIG. 301. Currents induced only when conductor *cuts* lines of force

358. Condition necessary for an induced E.M.F. Let the coil be held in the position shown in Fig. 301, and moved back and forth *parallel* to the magnetic field; that is, parallel to the line NS. No current will be induced.

By experiments of this sort it is found that an E.M.F. is induced in a coil only *when the motion takes place in such a way as to change the total number of magnetic lines of force which are inclosed by the coil.* Or, to state this rule in more general form, *an E.M.F. is induced in any element of a conductor when, and only when, that element is moving in such a way as to cut magnetic lines of force.**

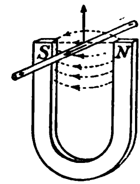


FIG. 302. E.M.F. induced when a straight conductor cuts magnetic lines

It will be noticed that the first statement of the rule is included in the second, for whenever the number of lines of force which pass through a coil changes, some lines of force must cut across the coil from the inside to the outside, or vice versa.

359. The principle of the electric motor. Let a vertical wire *ab* be rigidly attached to a horizontal wire *gh*, and let the latter be supported by a ring or other metallic support, in the manner shown in Fig. 303, so that *ab* is free to oscillate about *gh* as an axis. Let the lower end of *ab* dip into a trough of mercury. When a magnet is held in the position shown and a current from a dry cell is sent down through the wire, the wire will instantly move in the direction indicated by the arrow *f*, namely, at right angles to the direction of the lines of magnetic force. Let the direction of the current in the wire be reversed. The direction of the force acting on the wire will be found to be reversed also.

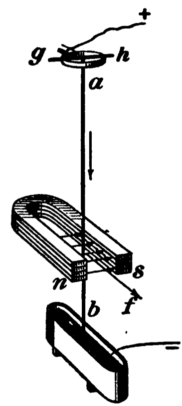


FIG. 303. The principle of the electric motor

We learn, therefore, that *a wire carrying a current in a magnetic field tends to move in*

* If a strong electromagnet is available, these experiments are more instructive if performed, not with a coil, as in Fig. 301, but with a straight rod (Fig. 302) to the ends of which are attached wires leading to a galvanometer. Whenever the rod moves parallel to the lines of magnetic force there will be no deflection, but whenever it moves across the lines the galvanometer needle will move at once.

a direction at right angles both to the direction of the field and the direction of the current. This fact underlies the operation of all electric motors.

360. The motor and dynamo rules. A convenient rule for determining whether the wire *ab* (Fig. 303) will move forward or back in a given case may be obtained as follows: If the field of a magnet alone is represented by Fig. 304, and that due to the current * alone by Fig. 305, then the resultant field

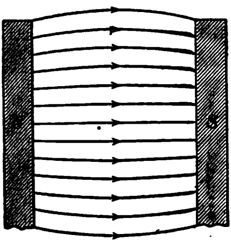


FIG. 304. Field of magnet alone

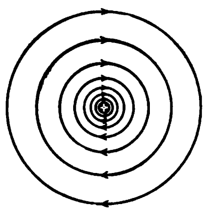


FIG. 305. Field of current alone

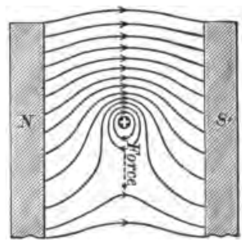


FIG. 306. Field of magnet and current

when the current-bearing wire is placed between the poles of the magnet is that shown in Fig. 306; for the strength of the field above the wire is now the sum of the two separate fields, while the strength below it is their difference. Now Faraday thought of the lines of force as acting like stretched rubber bands. This would mean that the wire in Fig. 306 would be pushed *down.* Whether the lines of force are so conceived or not, *the motor rule* may be stated thus:

A current in a magnetic field tends to move away from the side on which its lines are added to those of the field.

The dynamo rule follows at once from the motor rule and Lenz's law. Thus when a wire is moved through a magnetic field the current induced in it must be in such a direction as

* The cross in the conductor of Fig. 305, representing the tail of a retreating arrow, is to indicate that the current flows away from the reader. A dot, representing the head of an advancing arrow, indicates a current flowing toward the reader.

to oppose the motion; therefore *the induced current will be in such a direction as to increase the number of lines on the side toward which it is moving.**

361. Strength of the induced E.M.F. The strength of an induced E.M.F. is found to depend simply upon *the number of lines of force cut per second* by the conductor, or, in the case of a coil, upon the *rate of change* in the number of lines of force which pass through the coil. The strength of the current which flows is then given by Ohm's law; that is, it is equal to the induced E.M.F. divided by the resistance of the circuit. The number of lines of force which the conductor cuts per second may always be determined if we know the velocity of the conductor and the strength of the magnetic field through which it moves. For it will be remembered that, according to the convention of § 275, a field of unit strength is said to contain one line of force per square centimeter, a field of 1000 units strength 1000 lines per square centimeter, etc. In a conductor which is cutting lines at the rate of 100,000,000 per second there is an induced E.M.F. of 1 volt.† The reason that we used a coil of 500 turns instead of a single turn in the experiment of § 356 was that by thus making the conductor in which the current was to be induced cut the lines of force of the magnet 500 times instead of once, we obtained 500 times as strong an induced E.M.F., and therefore 500 times as strong a current for a given resistance in the circuit.

362. Currents induced in rotating coils. Let a 400- or 500-turn coil of No. 28 copper wire be made small enough to rotate between the poles of a horseshoe magnet, and let it be connected into the circuit of a galvanometer, precisely as in § 356. Starting with the coil in the position of Fig. 307, (1), let it be rotated suddenly clockwise (looking down

*This may be thrown into a convenient rule of thumb as follows: "*Grasp the conductor with the right hand, the fingers extending in the direction of the lines of force to be cut; the thumb will indicate the direction of the induced current.*"

† This may be considered as *the scientific definition of the volt*, convenience alone having dictated the legal definition given in § 331.

from above) through 180°. A strong deflection of the galvanometer will be observed. Let it be rotated through the next 180° back to the starting point. An opposite deflection will be observed.

The arrangement is a *dynamo* in miniature. During the first half of the revolution [see Fig. 307, (2)] the wires on the right side of the loop were cutting the lines of force in one direction, while the wires on the left side were cutting them in the opposite direction. A current was being generated down on the right side of the coil and up on the left side (see dynamo rule). It will be seen that both currents flow around the coil in the same direction. The induced current is strongest when the coil is in the position shown in Fig. 307, (2), because there the lines of force are being cut most rapidly. Just as the coil is moving into or out of the position shown in Fig. 307, (1), it is moving *parallel* to the lines of force, and hence no current is induced, since no lines of force are being cut. As the coil moves through the last 180° of its revolution both sides are cutting the same lines of force as before, but they are cutting them in an opposite direction; hence the current generated during this last half is opposite in direction to that of the first half.*

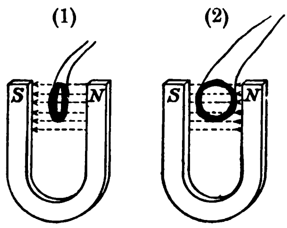


Fig. 307. Direction of currents induced in a coil rotating in a magnetic field

QUESTIONS AND PROBLEMS

1. Under what conditions may an electric current be produced by a magnet?

2. A current is flowing from top to bottom in a vertical wire. In what direction will the wire tend to move on account of the earth's magnetic field?

3. State Lenz's law, and show how it follows from the principle of the conservation of energy.

* A laboratory experiment on the principles of induction should be performed at about this point. See, for example, Experiment 36 of the authors' manual.

4. If the coil of a sensitive galvanometer is set to swinging while the circuit through the coil is open, it will continue to swing for a long time; but if the coil is short-circuited, it will come to rest after a very few oscillations. Why? (The experiment may easily be tried. Remember that currents are induced in the moving coil. Apply Lenz's law.)

5. A coil is thrust over the *S* pole of a magnet. Is the direction of the induced current clockwise or counterclockwise as you look down upon the pole?

6. A ship having an iron mast is sailing east. In what direction is the E.M.F. induced in the mast by the earth's magnetic field? If a wire is brought from the top of the mast to its bottom, no current will flow through the circuit. Why?

7. When a wire is cutting lines of force at the rate of 100,000,000 per second, there is induced in it an E.M.F. of one volt. A certain dynamo armature has 50 coils of 5 loops each and makes 600 revolutions per minute. Each wire cuts 2,000,000 lines of force twice in a revolution. What is the E.M.F. developed?

8. If a coil of wire is rotated about a vertical axis in the earth's field, an alternating current is set up in it. In what position is the coil when the current changes direction?

Dynamos

363. A simple alternating-current dynamo. The simplest form of commercial dynamo consists of a coil of wire so arranged as to rotate continuously between the poles of a powerful electromagnet (Fig. 308).

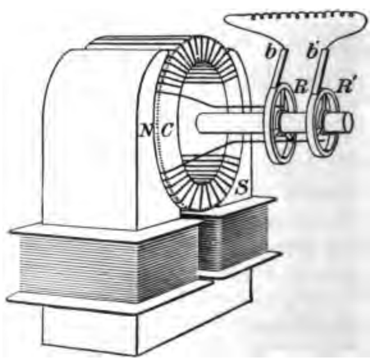


FIG. 308. Ring-wound armature

In order to make the magnetic field in which the conductor is moved as strong as possible, the coil is wound upon an iron core *C*. This greatly increases the total number of lines of magnetic force which pass between *N* and *S*, for the core offers an iron path, as shown in Fig. 309, instead of an air path.

The rotating part, consisting of the coil with its core, is called the *armature*. If the coil is wound in the manner shown in Figs. 308 and 309, the armature is said to be of the *ring* type; if in the manner shown in Figs. 310 and 311, it is said to be of the *drum* type. The latter form of winding is used almost exclusively in modern machines.

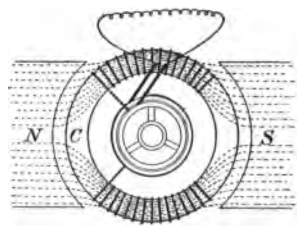


FIG. 309. End view of ring armature

One end of the coil is attached to the insulated metal ring R, which is attached rigidly to the shaft of the armature and therefore rotates with it, while the other end of the coil is attached to a second ring R'. The brushes b and b', which constitute the terminals of the external circuit, are always in contact with these rings.

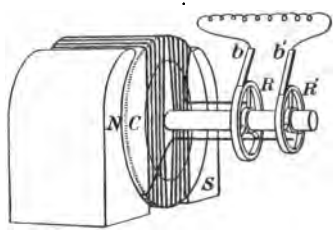


FIG. 310. Drum-wound armature

As the coil rotates an induced alternating current passes through the circuit. This current reverses direction as often as the coil passes through the position shown in Figs. 309 and 311, that is, the position in which the conductors are moving *parallel* to the lines of force; for at this instant the conductors which were moving up begin to move down, and those which were moving down begin to move up. The current reaches its maximum value when the coils are moving through a position 90° farther on, for then the lines of force are being cut most rapidly by the conductors on both sides of the coil.

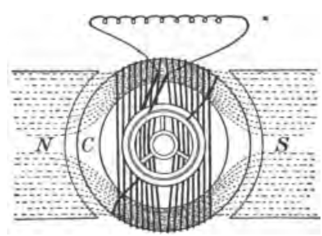


FIG. 311. End view of drum armature

†

364. The multipolar alternator. For most commercial purposes it is found desirable to have 120 or more alternations of current per second. This could not be attained easily with two-pole machines like those sketched in Figs. 308 to 311. Hence commercial alternators are usually built with a large number of poles alternately N and S, arranged around the circumference of a circle in the manner shown in Fig. 312. The dotted lines represent the direction of the lines of force through the iron. It will be seen that the coils which are passing beneath N poles have induced currents set up in them, the direction of which is opposite to that of the currents which are induced in the conductors which are passing beneath the S poles. Since, however, the direction of winding of the armature coils changes between each two poles, all the inductive effects of all the poles are added in the coil and constitute at any instant one single current flowing around the complete circuit in the manner indicated by the arrows in the diagram. This current reverses direction at the instant at which all the coils pass the midway points between the N and S poles. The number of alternations per second is equal to the number of poles multiplied by the number of revolutions per second. The field magnets N and S of such a dynamo are usually excited by a direct current from some other source. Fig. 313 represents a modern commercial alternator.

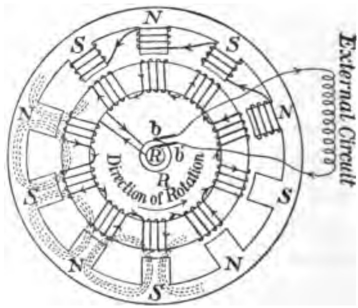


FIG. 312. Diagram of alternating-current dynamo

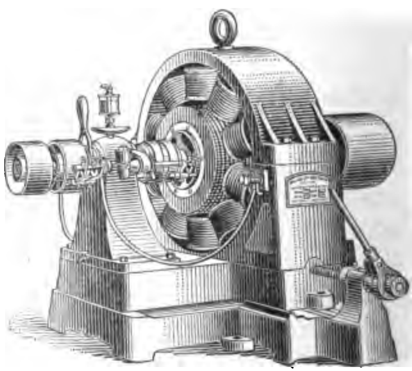


FIG. 313. Alternating-current dynamo

365. The principle of the commutator. By the use of a so-called *commutator* it is possible to transform a current which

is alternating in the coils of the armature to one which always flows in the same direction through the external portion of the circuit. The simplest possible form of such a commutator is shown in Fig. 314. It consists of a single metallic ring which is split into two equal insulated semicircular segments *a* and *c*. One end of the rotating coil is soldered to one of these semicircles, and the other end to the other semicircle. Brushes *b* and *b′* are set in such positions that they lose contact with one semicircle and make contact with the other at the instant at which the current changes direction in the armature. The current therefore always passes out to the external circuit through the same brush. While a current from such a coil and commutator as that shown in the figure would always flow in the same direction through the external circuit, it would be of a pulsating rather than a steady character, for it would rise to a maximum and fall again to zero twice during each complete revolution of the armature. This effect is avoided in the commercial direct-current dynamo by building a commutator of a large number of segments instead of two, and connecting each to a portion of the armature coil in the manner shown in Fig. 315.

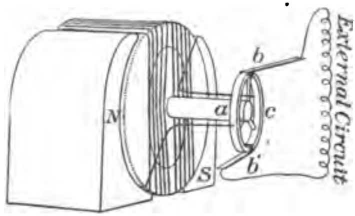


FIG. 314. The simple commutator

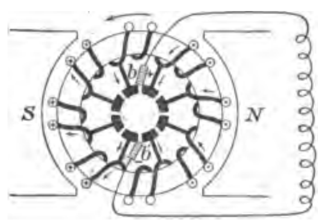


FIG. 315. Two-pole direct-current dynamo with ring armature

366. The ring-armature direct-current dynamo. Fig. 315 is a diagram illustrating the construction of a commercial two-pole direct-current dynamo of the ring-armature type. The figure represents an end view of a core like that shown in Fig. 308. The coil is wound continuously

around the core, each segment being connected to a corresponding segment of the commutator, in the manner shown in the figure. At a given instant currents are being induced in the same direction in all the conductors on the outside of the core on the left half of the armature. The cross on these conductors, representing the tail of a retreating arrow, is to indicate that these currents flow away from the reader. No E.M.F.'s are induced in the conductors on the inner side of the ring, since these conductors cut no lines of force (see Fig. 309); nor are currents induced in the conductors at the top and bottom of the ring where the motion is parallel to the magnetic lines. The addition of all these similarly directed currents in the various convolutions of the continuous coil on the left side of the ring constitutes one single current flowing upward through this coil toward the brush *b* (see arrows). On the right half of the ring, on the other hand, the induced currents are all in the opposite direction, that is, toward the reader, since the conductors are here all moving up instead of down. The dot in the middle of these conductors represents the head of an approaching arrow. The summation of these currents constitutes one single current also flowing upward in the right half of the coil toward the brush *b*. These two currents from the two halves of the ring pass out at *b* through the external circuit and back at *b′*. This condition always exists, no matter how fast the rotation; for it will be seen that as each loop rotates into the position where the direction of its current reverses, it passes a brush and therefore at once becomes a part of the circuit on the other half of the ring, where the currents are all flowing in the opposite direction.

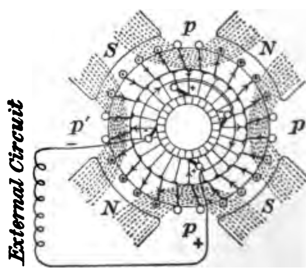


Fig. 316. Four-pole direct-current dynamo, ring-armature type

If the machine is of the four-pole type, like that shown in Fig. 316, the currents flow toward two neutral points, or points of no induction, instead of toward one, as in two-pole machines, and they flow away from two other neutral points (see *p*, *p*, *p′*, *p′*, Fig. 316). Hence there are four brushes, two positive and two negative, as in the figure. Since the two positive and the two negative brushes are connected as shown, both sets of currents flow off to the external circuit on a single wire. The figure with its arrows will explain completely the generation of currents by a four-pole machine.

367. The drum-armature direct-current dynamo. The drum-wound armature, shown in section in Fig. 317, has an advantage over the ring armature in that, while the conductors on the inside of the latter never cut lines of force and are therefore always idle, in the former all of the conductors are cutting lines of force except when they are passing the neutral points. In theory, however, the operation of the drum armature is precisely the same as that of the ring armature. All the conductors on the left side of the line connecting the brushes (see Fig. 317) carry induced currents which flow in one direction, while all the conductors on the right side of this line have opposite currents induced in them. It will be seen, however, in tracing out the connections 1, 1_1, 2, 2_1, 3, 3_1, etc., of Fig. 317 (the dotted lines representing connections at the back of the drum), that the coil is so wound about the drum that the currents in both halves are always flowing toward one brush *b*, from which they are led to the external circuit. Fig. 318 shows a typical modern four-pole generator, and Fig. 319 the corresponding drum-wound armature. Fig. 329 (p. 305) illustrates nicely the method of winding such an armature, each coil beginning on one segment of the commutator and ending on the adjacent segment.

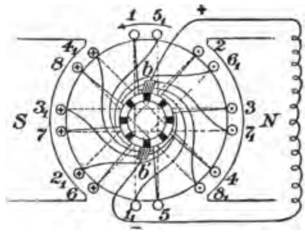


Fig. 317. The direct-current dynamo, drum winding

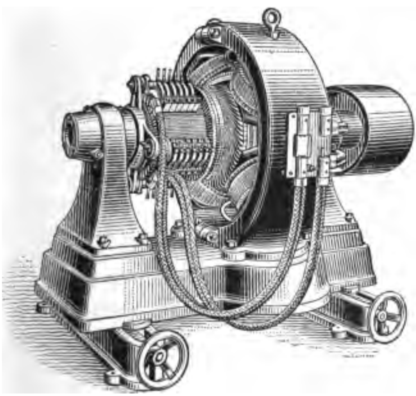


Fig. 318. Holtzer-Cabot four-pole direct-current generator

368. Series-, shunt-, and compound-wound dynamos. In direct-current machines the field magnet *NS* is excited by the current which the

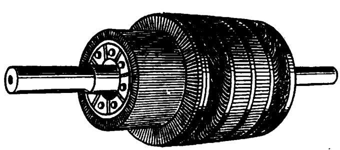


Fig. 319. Holtzer-Cabot armatu-

dynamo itself produces. In the so-called *shunt-wound* machines a small portion of the current is led off from the brushes through a great many turns of fine wire which encircle the core of the magnet, while the rest of the current flows through the external circuit (see Fig. 320). In the so-called *series-wound dynamo* (Fig. 321) the whole of the current is carried through a few turns of coarse wire which encircle the field magnets. These turns are then in series with the external circuit. In the *compound-wound machine* (Fig. 322) there is both a series and a shunt coil. By this arrangement it is possible to maintain a constant potential difference between the brushes, no matter how much the resistance of the external circuit may be varied. Hence, for purposes in which a varying

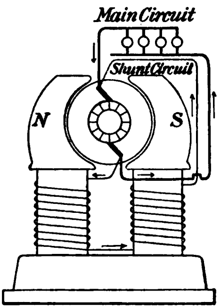


FIG. 320. The shunt-wound dynamo

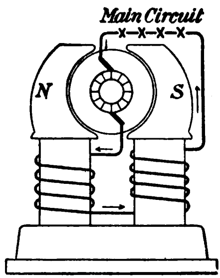


FIG. 321. The series-wound dynamo

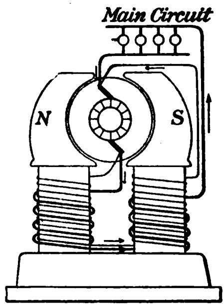


FIG. 322. The compound-wound dynamo

current is demanded, as in incandescent lighting, the operation of street cars, etc., compound-wound dynamos are almost exclusively used.

In all these types of self-exciting machines there is enough residual magnetism left in the iron cores after stopping to start feeble induced currents when started up again. These currents immediately increase the strength of the magnetic field, and so the machine quickly builds up its current until the limit of magnetization is reached.

For incandescent electric lighting it is customary to use a dynamo of the compound type which gives a P.D. between its "mains" of either 110 or 220 volts. The lamps are always arranged in parallel between these mains, as is illustrated in Fig. 322. In arc lighting a series-wound dynamo is usually used, and the lamps are almost invariably arranged in series, as in Fig. 321. About 50 lamps are commonly fed by one machine. This requires a dynamo capable of producing a voltage of 2500 volts, since each lamp requires a pressure of about 50 volts. Since

an arc light usually requires a current of 10 amperes, such a dynamo must furnish 10 amperes at 2500 volts. The power is therefore $10 \times 2500 = 25{,}000$ watts. The dynamo must therefore have an activity of 25 kilowatts, or about 33.5 horse power.

369. The electric motor. In construction the electric motor differs in no essential respect from the dynamo. To analyze the operation as a motor of such a machine as that shown in Fig. 315, suppose a current from an outside source is first sent around the coils of the field magnets and then into the armature at b'. Here it will divide and flow through all the conductors on the left half of the ring in one direction, and through all those on the right half in the opposite direction. Hence, in accordance with the motor rule, all the conductors on the left side are urged upward by the influence of the field, and all those on the right side are urged downward. The armature will therefore begin to rotate, and this rotation will continue so long as the current is sent in at b' and out at b. For as fast as coils pass either b or b', the direction of the current flowing through them changes, and therefore the direction of the force acting on them changes. The left half is therefore always urged up and the right half down. The greater the strength of the current, the greater the force acting to produce rotation.

If the armature is of the drum type (Fig. 317), the conditions are not essentially different. For, as may be seen by following out the windings, the current entering at b' will flow through all the conductors in the left half in one direction and through those on the right half in the opposite direction. The commutator keeps these conditions always fulfilled. The analysis of the operation of a four-pole dynamo (Fig. 316) as a motor is equally simple.

370. Street-car motors. Electric street cars are nearly all operated by direct-current series-wound motors placed under the cars and attached by gears to the axles. Fig. 323 shows a typical four-pole street-car motor. The two upper field poles are raised with the case when the

motor is opened for inspection, as in the figure. The current is generally supplied by compound-wound dynamos which maintain a constant potential of about 500 volts between the trolley, or third rail, and the track which is used as the return circuit. The cars are always operated in parallel, as shown in Fig. 324. In a few instances street cars are operated upon alternating, instead of upon direct-current, circuits. In such cases the motors are essentially the same as direct-current series-wound motors; for since in such a machine the current must reverse in the field

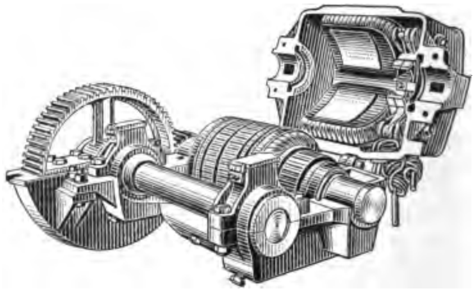


FIG. 323. Railway motor, upper field raised

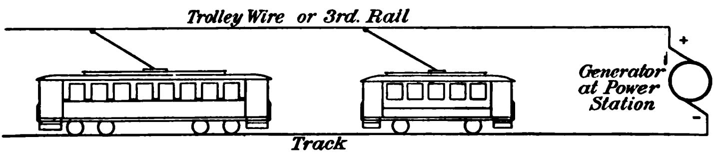


FIG. 324. Street-car circuit

magnets at the same time that it reverses in the armature, it will be seen that the armature is always impelled to rotate in one direction, whether it is supplied with a direct or with an alternating current.

371. Back E.M.F. in motors. When an armature is set into rotation by sending a current from some outside source through it, its coils move through a magnetic field as truly as if the rotation were produced by a steam engine, as is the case in running a dynamo. An induced E.M.F. is therefore set up by this rotation. In other words, while the machine is acting as a motor it is also acting as a dynamo. The direction of the induced E.M.F. due to this dynamo effect will be seen, from Lenz's law or from a consideration of the dynamo and motor rules, to be opposite to the outside P.D., which is causing

current to pass through the motor. The faster the motor rotates, the faster the lines of force are cut, and hence the greater the value of this so-called *back E.M.F.* If the motor were doing no work, the speed of rotation would increase until the back E.M.F. reduced the current to a value simply sufficient to overcome friction. It will be seen, therefore, that in general the faster the motor goes, the less the current which passes through its armature, for this current is always due to the *difference* between the P.D. applied at the brushes — 500 volts in the case of trolley cars — and the back E.M.F. When the motor is starting, the back E.M.F. is zero; and hence, if the full 500 volts were applied to the brushes, the current sent through would be so large as to ruin the armature through overheating. To prevent this each car is furnished with a "starting box," which consists of resistance coils which the motorman throws into series with the motor on starting, and throws out again gradually as the speed increases and the back E.M.F. consequently rises.*

QUESTIONS AND PROBLEMS

1. Explain how an alternating current in the armature is transformed into a unidirectional current in the external circuit.

2. Two successive coils on the armature of a multipolar alternator are cutting lines of force which run in opposite directions. How does it happen that the currents generated flow through the wires in the same direction? (Fig. 312.)

3. A multipolar alternator has 20 poles and rotates 200 times per minute. How many alternations per second will be produced in the circuit?

4. With the aid of the dynamo rule explain why, in Fig. 316, the current in the conductors under the south poles is moving toward the observer, and that in the conductors under the north poles away from the observer. Explain in a similar way the directions of the arrows in Figs. 315 and 317.

5. Explain why the brushes in Fig. 316 touch the commutator in the positions shown rather than at some other points.

*This discussion should be followed by a laboratory experiment on the study of a small electric motor or dynamo. See, for example, Experiment No. 37 of the authors' manual.

6. If a direct-current machine of the same general type as that shown in Fig. 316 had twelve poles, how many brushes would be needed on the commutator?

7. If a current is sent into the armature of Fig. 315 at b', and taken out at b, which way will the armature revolve?

8. When an electric fan is first started, the current through it is much greater than it is after the fan has attained its normal speed. Why?

9. If in the machine of Fig. 316 a current is sent in on the wire marked +, what will be the direction of rotation?

10. Would an armature wound on a wooden core be as effective as one made of the same number of turns wound on an iron core?

11. Will it take more work to rotate a dynamo armature when the circuit is closed than when it is open? Why?

12. Show that if the reverse of Lenz's law were true, a motor once started would run of itself and do work; that is, it would furnish a case of perpetual motion.

13. If a series-wound dynamo is running at a constant speed, what effect will be produced on the strength of the field magnets by diminishing the external resistance and thus increasing the current? What will be the effect on the E.M.F.? (Remember that the whole current goes around the field magnets.)

14. If a shunt-wound dynamo is run at constant speed, what effect will be produced on the strength of the field magnets by reducing the external resistance? What effect will this have on the E.M.F.?

15. In an incandescent-lighting system the lamps are connected in parallel across the mains. Every lamp which is turned on, then, diminishes the external resistance. Explain from a consideration of Problems 13 and 14 why a compound-wound dynamo keeps the P.D. between the mains constant.

16. Explain why a series-wound motor can run either on a direct or an alternating circuit.

17. If the pressure applied at the terminals of a motor is 500 volts, and the back pressure, when running at full speed, is 450 volts, what is the current flowing through the armature, its resistance being 10 ohms?

18. Single dynamos often operate as many as 10,000 incandescent lamps at 110 volts. If these lamps are all arranged in parallel and each requires a current of .5 ampere, what is the total current furnished by the dynamo? What is the activity of the machine in kilowatts and in horse power?

19. How many 110-volt lamps like those of Problem 18 can be lighted by a 12,000-kilowatt generator?

20. Why does it take twice as much work to keep a dynamo running when 1000 lights are on the circuit as when only 500 are turned on?

Principle of the Induction Coil and Transformer

372. Currents induced by varying the strength of a magnetic field. Let about 500 turns of No. 28 copper wire be wound around one end of an iron core, as in Fig. 325, and connected to the circuit of a galvanometer. Let about 500 more turns be wrapped about another portion of the core and connected into the circuit of two dry cells. When the key K is closed the deflection of the galvanometer will indicate that a temporary current has been induced in one direction through the coil s, and when it is opened an equal but opposite deflection will indicate an equal current flowing in the opposite direction.

The experiment illustrates the principle of the induction coil and the transformer. The coil p, which is connected to the source of the current, is called the *primary coil*, and the coil s, in which the currents are induced, is called the *secondary coil*. Causing lines of force to spring into existence inside of s — in other words, magnetizing the space inside of s — has caused an induced current to flow in s; and demagnetizing the space inside of s has also induced a current in s in accordance with the general principle stated in § 358, that *any change in the number of magnetic lines of force which thread through a coil induces a current in the coil.* We may think of the lines as always existing as closed loops (see Fig. 285, p. 274) which collapse upon demagnetization to mere double lines at the axis of the coil. Upon magnetization one of these two lines springs out, cutting the encircling conductors and inducing a current.

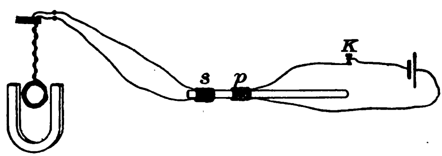


FIG. 325. Induction of current by magnetizing and demagnetizing an iron core

373. Direction of the induced current. Lenz's law, which, it will be remembered, followed from the principle of conservation of energy, enables us to predict at once the direction of the induced currents in the above experiments; and an

observation of the deflections of the galvanometer enables us to verify the correctness of the predictions. Consider first the case in which the primary circuit is *made* and the core thus magnetized. According to Lenz's law, the current induced in the secondary circuit must be in such a direction as to *oppose the change* which is being produced by the primary current, that is, in such a direction as to tend to magnetize the core oppositely to the direction in which it is being magnetized by the primary. This means, of course, that the induced current in the secondary must encircle the core in a direction opposite to the direction in which the primary current encircles it. We learn, therefore, that *on making the current in the primary the current induced in the secondary is opposite in direction to that in the primary.*

When the current in the primary is *broken*, the magnetic field created by the primary tends to die out. Hence, by Lenz's law, the current induced in the secondary must be in such a direction as to tend to oppose this process of demagnetization, that is, in such a direction as to magnetize the core in the same direction in which it is magnetized by the decaying current in the primary. Therefore, *at break the current induced in the secondary is in the same direction as that in the primary.*

374. E.M.F. of the secondary. If half of the 500 turns of the secondary *s* (Fig. 325) are unwrapped, the deflection will be found to be just half as great as before. Since the resistance of the circuit has not been changed, we learn from this that *the E.M.F. of the secondary is proportional to the number of turns of wire upon it* — a result which followed also from § 361. If, then, we wish to develop a very high E.M.F. in the secondary, we have only to make it of a very large number of turns of fine wire.

375. Self-induction. If in the experiment illustrated in Fig. 325 the coil *s* had been made a part of the same circuit as *p*, the E.M.F.'s induced in it by the changes in the magnetism of the core would of course have been just the same as above.

In other words, when a current starts in a coil the magnetic field which it itself produces tends to induce a current opposite in direction to that of the starting current, that is, tends to oppose the starting of the current; and when a current in a coil stops, the collapse of its own magnetic field tends to induce a current in the same direction as that of the stopping current, that is, tends to oppose the stopping of the current. This means merely that *a current in a coil acts as though it had inertia, and opposes any attempt to start or stop it.* This inertia-like effect of a coil upon itself is called *self-induction.*

Let a few dry cells be inserted into a circuit containing a coil of a large number of turns of wire, the circuit being closed at some point by touching two bare copper wires together. Holding the bare wire in the fingers, break the circuit between the hands and observe the shock due to the current which the E.M.F. of self-induction sends through your body. Without the coil in circuit you will obtain no such shock, though the current stopped when you break the circuit will be many times larger.

The spark coil on an automobile is a good illustration of a device for producing a spark due to self-induction.

376. The induction coil. The induction coil, as usually made (Fig. 326), consists of a soft iron core *C*, composed of a bundle of soft iron wires; a primary coil *p* wrapped around

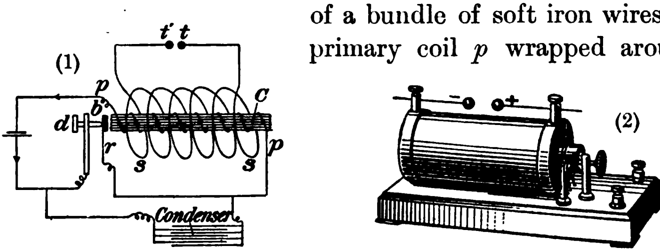


Fig. 326. Induction coil

this core, and consisting of, say, 200 turns of coarse copper wire (for example, No. 16), which is connected into the circuit of a battery through the contact point at the end of the screw *d*; a secondary coil *s* surrounding the primary in the manner

indicated in the diagram, and consisting generally of between 30,000 and 1,000,000 turns of No. 36 copper wire, the terminals of which are the points t and t'; and a hammer b, or other automatic arrangement for making and breaking the circuit of the primary.

Let the hammer b be held away from the opposite contact point by means of the finger, then touched to this point, then pulled quickly away. *A spark will be found to pass between t and t′ at break only — never at make.* This is because, on account of the opposing influence at make of self-induction in the primary, the magnetic field about the primary rises very gradually to its full strength, and hence its lines pass into the secondary coil comparatively slowly. At *break*, however, by separating the contact points very quickly we can make the current in the primary fall to zero in an exceedingly short time, perhaps not more than .00001 second; that is, we can make all of its lines pass out of the coil in this time. Hence the *rate* at which lines thread through or cut the secondary is perhaps 10,000 times as great at break as at make, and therefore the E.M.F. is also something like 10,000 times as great. In the normal use of the coil, the circuit of the primary is automatically made and broken at b by means of the magnet and the spring r, precisely as in the case of the electric bell. Let the student analyze this part of the coil for himself. The condenser, shown in the diagram, with its two sets of plates connected to the conductors on either side of the spark gap between r and d, is not an essential part of a coil, but when it is introduced it is found that the length of the spark which can be sent across between t and t' is considerably increased. The reason is as follows: When the circuit is broken at b the inertia, that is, the self-induction, of the primary current, tends to make a spark jump across from d to b; and if this happens, the current continues to flow through this spark (or arc) until the terminals have become separated through a considerable distance. This makes the current die down gradually instead of suddenly, as it ought to do to produce a high E.M.F. But when a condenser is inserted, as soon as b begins to leave d the current begins to flow into the condenser, and this gives the hammer time to get so far away from d that an arc cannot be formed. This means a sudden break and a high E.M.F. Since a spark passes between t and t' only at break, it must always pass in the same direction. Coils which give 24-inch sparks (perhaps 500,000 volts) are not uncommon. Such coils usually have hundreds of miles of wire upon their secondaries.

377. Laminated cores; Foucault currents. The core of an induction coil should always be made of a bundle of soft iron wires insulated from one another by means of shellac or varnish (see Fig. 327); for whenever a current is started or stopped in the primary p of a coil furnished with a *solid* iron core (see Fig. 328), the change in the magnetic field of the primary induces a current in the conducting core C for the same reason that it induces one in the secondary s. This current flows around the body of the core in the same direction as the induced current in the secondary, that is, in the direction of the arrows. The only effect of these so-called *eddy* or *Foucault* currents is to heat the core. This is obviously a waste of energy. If we can prevent the appearance of these currents, all of the energy which they would waste in heating the core may be made to appear in the current of the secondary. The core is therefore built of varnished iron wires, which run parallel to the axis of the coil, that is, perpendicular to the direction in which the currents would be induced. The induced E.M.F. therefore finds no closed circuits in which to set up a current (Fig. 327). It is for the same reason that the iron cores of dynamo and motor armatures, instead of being solid, consist of iron disks placed side by side, as shown in Fig. 329, and insulated from one another by films of oxide. A core of this kind is called a *laminated* core. It will be seen that in all such cores the spaces or slots between the laminæ must run at right angles to the direction of the induced E.M.F., that is, perpendicular to the conductors upon the core.

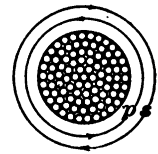


FIG. 327. Core of insulated iron wires

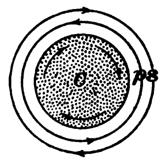


FIG. 328. Diagram showing eddy currents in solid core

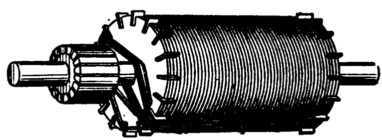


FIG. 329. Laminated drum-armature core with commutator, showing one coil wound on the core

378. The transformer. The commercial transformer is a modified form of the induction coil. The chief difference is that the core R (Fig. 330), instead of being straight, is bent into the form of a ring, or is given some other shape such that the magnetic lines of force have a continuous iron

path, instead of being obliged to push out into the air, as in the induction coil. Furthermore, it is always an alternating instead of an intermittent current which is sent through the primary A. Sending such a current through A is equivalent to magnetizing the core first in one direction, then demagnetizing it, then magnetizing it in the opposite direction, etc. The results of these changes in the magnetism of the core is of course an induced alternating current in the secondary B.

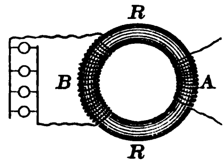


FIG. 330. Diagram of transformer

379. The use of the transformer. The use of the transformer is to convert an alternating current from one voltage to another which, for some reason, is found to be more convenient. For example, in electric lighting where an alternating current is used, the E.M.F. generated by the dynamo is usually either 1100 or 2200 volts, a voltage too high to be introduced safely into private houses. Hence transformers are connected across the main conductors in the manner shown in Fig. 331. The current which passes into the houses to supply the lamps does not come directly from the dynamo. It is an induced current generated in the transformer.

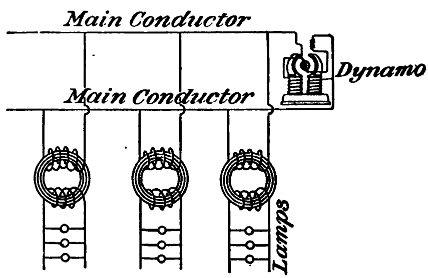


FIG. 331. Alternating current lighting circuit with transformers

380. Pressure in primary and secondary. If there are a few turns in the primary and a large number in the secondary, the transformer is called a *step-up* transformer, because the P.D. produced at the terminals of the secondary is greater than that applied at the terminals of the primary. Thus an induction

coil is a step-up transformer. In electric lighting, however, transformers are mostly of the *step-down* type; that is, a high P.D., say, 2200 volts, is applied at the terminal of the primary, and a lower P.D., say, 110 volts, is obtained at the terminals of the secondary. In such a transformer the primary will have twenty times as many turns as the secondary. In general, *the ratio between the voltages at the terminals of the primary and secondary is the ratio of the number of turns of wire upon the two.*

381. Efficiency of the transformer. In a perfect transformer the efficiency would be unity. This means that the electrical energy put into the primary, that is, the volts applied to its terminals times the amperes flowing through it, would be exactly equal to the energy taken out in the secondary, that is, the volts generated in it times the strength of the induced current; and, in fact, in actual transformers the latter product is often more than 97% of the former; that is, there is less than 3% loss of energy in the transformation. This lost energy appears as heat in the transformer. This transfer, which goes on in a big transformer, of huge quantities of power from one circuit to another entirely independent circuit, without noise or motion of any sort and almost without loss, is one of the most wonderful phenomena of modern industrial life.

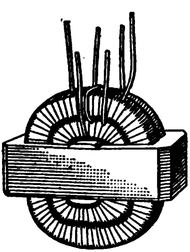


FIG. 332. Commercial transformer

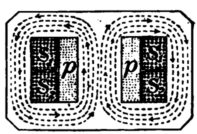


FIG. 333. Cross section of transformer, showing shape of magnetic field

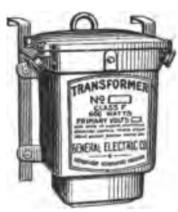


FIG. 334. Transformer case

382. Commercial transformers. Fig. 332 illustrates a common type of transformer used in electric lighting. The core is built up of sheet-iron laminæ about $\frac{1}{2}$ millimeter thick. Fig. 333 shows a section of the same

†

transformer. The closed magnetic circuit of the core is indicated by the arrows. The primary and the two secondaries, which can furnish either 52 or 104 volts, are indicated by the letters p, S_1, and S_2. Fig. 334 is the case in which the transformer is placed. Such cases may be seen attached to poles outside of houses wherever alternating currents are used for electric-lighting (Fig. 335).

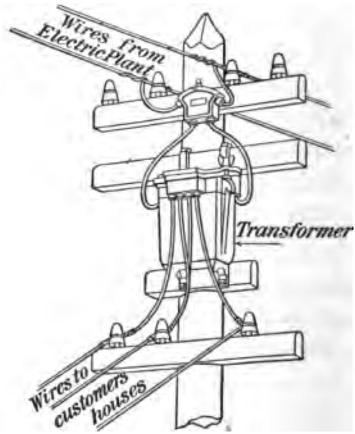


FIG. 335. Transformer on electric-light pole

383. Electrical transmission of power. Since the electrical energy produced by a dynamo is equal to the product of the E.M.F. generated by the current furnished, it is evident that in order to transmit from one point to another a given number of watts, say, 10,000, it is possible to have either an E.M.F. of 100 volts and a current of 100 amperes, or an E.M.F. of 1000 volts and a current of 10 amperes. In the two cases, however, the loss of energy in the wire which carries the current from the place where it is generated to the place where it is used will be widely different. If R represents the resistance of this transmitting wire, the so-called "line," and C the current flowing through it, we have seen in § 351 that the heat developed in it will be proportional to C^2R. Hence the energy wasted in heating the line will be but $\frac{1}{100}$ as much in the case of the high-voltage, 10-ampere current as in the case of the lower-voltage, 100-ampere current. Hence, for long-distance transmission, where line losses are considerable, it is important to use the highest possible voltages.

On account of the difficulty of insulating the commutator segments from one another, voltages higher than 700 or 800 cannot be obtained with direct-current dynamos of the kind which have been described. With alternators, however, the difficulties of insulation are very much less on account of the absence of a commutator. The large 10,000-horse-power alternating-current dynamos on the Canadian side of Niagara Falls generate directly 12,000 volts. This is the highest voltage thus far produced by generators. In all cases where these high pressures are employed they are transformed down at the receiving end of the line to a safe and convenient voltage (from 50 to 500 volts) by means of step-down transformers.

It will be seen from the above facts that only alternating currents are suitable for long-distance transmission. Plants are now in operation which transmit power as far as 150 miles and use pressures as high as 100,000 volts. In all such cases *step-up* transformers, situated at the power house, transfer the electrical energy developed by the generator to the line, and step-down transformers, situated at the receiving end, transfer it to the motors, or lamps, which are to be supplied. The generators used on the American side of Niagara Falls produce a pressure of 2300 volts. For transmission to Buffalo, 20 miles away, this is transformed up to 22,000 volts. At Buffalo it is transformed down to the voltages suitable for operating the street cars, lights, and factories of the city. On the Canadian side the generators produce currents at 12,000 volts, as stated, and these are transformed up, for long-distance transmission, to 22,000, 40,000, and 60,000 volts.

384. The mercury-arc rectifier. The mercury-arc rectifier is a recently developed instrument for changing an alternating to a direct current. It consists of two graphite anodes A and A' (Fig. 336), and a mercury cathode B in an exhausted bulb. It is found that a current will pass through such a bulb when the carbon is made the positive and the mercury the negative electrode, but not in the reverse direction. When, then, an alternating E.M.F. is applied at H and G, the current passes through the circuit first in the direction indicated by the plain arrows, and then, as the E.M.F. reverses, in the direction indicated by the circled arrows. It will be seen that it always passes in the same direction through the storage batteries J which are to be charged. Were it not for the large coils EF the transformer would be short-circuited through PDQ and no current would flow through the path MBD or NBD. But the self-inductions of E and F are so large that most of the current flowing from M to D or N to D is forced over the path MBD or NBD. The extra mercury electrode C and the resistance coil O are merely used for starting the rectifier. This is done by tilting it until the mercury in B and

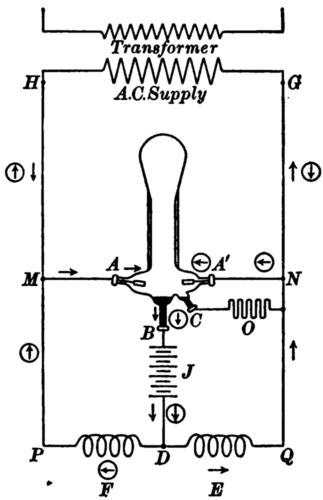


FIG. 336. The mercury-arc rectifier

C makes contact, then righting and thus breaking this contact and forming a temporary arc. This puts the mercury vapor into condition to cause the rectifier to function as described.

385. The simple telephone. The telephone was invented in 1875 by Alexander Graham Bell, of Washington, and Elisha Gray, of Chicago. In its simplest form it consists, at each end, of a permanent bar magnet A (Fig. 337) surrounded by a coil of fine wire B, in series with the line, and an iron disk or diaphragm E mounted close to one end of the magnet. When a sound is made in front of the diaphragm, the vibrations produced by the sounding body are transmitted by the air to the diaphragm, thus causing the latter to vibrate back and forth in front of the magnet. These vibrations of the diaphragm produce slight backward and forward movements of the lines of force which pass into the disk from the magnet in the manner shown in Fig. 338. Some of these lines of force, therefore, cut across the coil B, first in one direction and then in the other, and in so doing induce currents in it. These induced currents are transmitted by the line to the receiving station, where those in one direction pass around B' in such a way as to *increase* the strength of the magnet A', and thus increase the pull which it exerts upon E; while the opposite currents pass around B' in the opposite direction, and therefore *weaken* the magnet A' and diminish its pull upon E'. When, therefore, E moves in one direction, E' also moves in one direction, and when E reverses its motion, the direction of E' is also reversed. In other words, the induced currents, transmitted by the line,

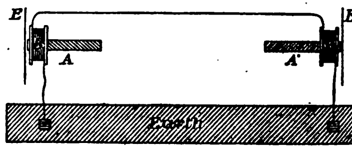


FIG. 337. The simple telephone

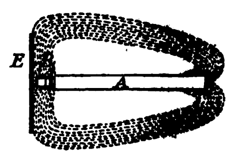


FIG. 338. Magnetic field about a telephone receiver

© Clinedinst

Alexander Graham Bell, Washington, D.C.

Inventor of the telephone, 1875

© Underwood & Underwood

Thomas A. Edison, Orange, New Jersey

Inventor of the phonograph, the incandescent lamp, etc.

Guglielmo Marconi (Italy)

Inventor of commercial wireless telegraphy

Orville Wright, Dayton, Ohio

Inventor, with his brother Wilbur, of the aëroplane

A GROUP OF MODERN INVENTORS

force E' to reproduce the motions of E. E' therefore sends out sound waves exactly like those which fell upon E. In exactly the same way a sound made in front of E' is reproduced at E. Telephones of this simple type will work satisfactorily for a distance of several miles. This simple form of instrument is still used at the receiving end of the modern telephone, the only innovation which has been introduced consisting in the substitution of a U-shaped magnet for the bar magnet. The instrument used at the transmitting end has, however, been changed, as explained in the next paragraph, and the circuit is now completed through a return wire instead of through the earth. A modern telephone receiver is shown in Fig. 339. G is the mouthpiece, E the diaphragm, A the U-shaped magnet, and B the coils, consisting of many turns of fine wire, and having soft iron cores.

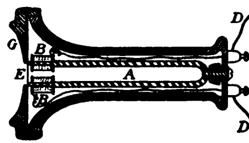


FIG. 339. The modern receiver

386. The modern transmitter. To increase the distance at which telephoning may be done, it is necessary to increase the strength of the induced currents. This is done in the modern transmitter by replacing the magnet and coil by an arrangement which is essentially an induction coil, the current in the primary of which is caused to vary by the

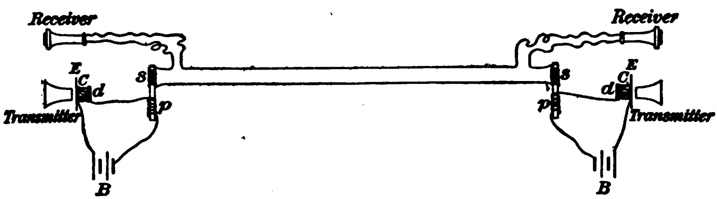


FIG. 340. The telephone circuit (local-battery system)

motion of the diaphragm. This is accomplished as follows: The current from the battery B (Fig. 340) is led first to the back of the diaphragm E, whence it passes through a little chamber C, filled with granular carbon, to the conducting back d of the transmitter, and thence through the primary p of the induction coil, and back to the battery

As the diaphragm vibrates it varies the pressure upon the many contact points of the granular carbon through which the primary current flows. This produces considerable variation in the resistance of the primary circuit, so that as the diaphragm moves forward, that is, toward the carbon, a comparatively large current flows through p, and, as it moves back, a much smaller current. These changes in the current strength in the primary p produce changes in the magnetism of the soft-iron core of the induction coil. Currents are therefore induced in the secondary s of the induction coil, and these currents pass over the line and affect the receiver at the other end in the manner explained in the preceding paragraph. The cross section of a complete long-distance transmitter is shown in Fig. 341.

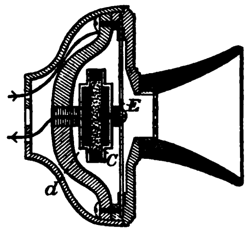


FIG. 341. Cross section of a long-distance telephone transmitter

387. The subscriber's telephone connections. In the most recent practice of the Bell Telephone Company the local battery at the subscriber's end is done away with altogether and the primary current is furnished by a 24-volt battery at the central station. Fig. 342 shows the essential elements of such a system. When the subscriber wishes to call up cen tral, he has only to lift the receiver from the hook. This closes the line circuit at t, and the direct current which at once begins to flow from the

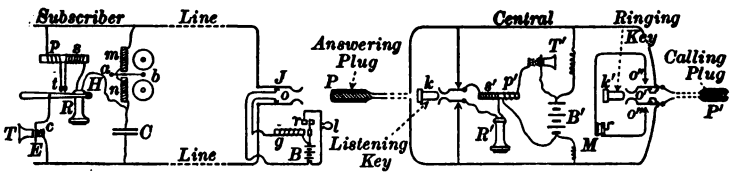


FIG. 342. The modern telephone circuit (central-station system)

battery B through the electromagnet g closes the circuit of B through the glow lamp l and the contact point r. This lights up the lamp l which is upon the switchboard in front of the operator. Upon seeing this signal the latter inserts the answering plug P into the subscribers' "jack" J and connects her own receiver R' into the line by pressing the listening key k. The operation of inserting the plug P extinguishes the lamp l by disconnecting the contact points o. The battery B' is, however, now upon the line (B and B' are, in fact, one and the same battery, shown here separate only for the sake of simplifying the

diagram). As, now, the subscriber talks into the transmitter T', the strength of the direct current from the battery B' through the primary p is varied by the varying pressure of the diaphragm E upon the granular carbon c, and these variations induce in the secondary s the talking currents which pass over the line to the receiver R' of the operator. Although with this arrangement the primary and secondary currents pass simultaneously over the same line, speech is found to be transmitted quite as distinctly as when the two circuits are entirely separate, as is the case with the arrangement of Fig. 340. When the operator finds what number the subscriber wishes, she inserts the calling plug P' into the proper line and presses the ringing key k'. This cuts out the first subscriber, while the ringing is going on, by opening the contact points o' and closing the points o''. When the person called answers, the ringing key k' is released, and the two subscribers are thus connected and the magneto M, which actually runs all the time, is disconnected from both lines. Also the operator releases her key k and thus cuts out her receiver while the conversation is going on. When one of the subscribers "hangs up," another lamp like l is lighted by a mechanism not shown here and the operator then pulls out both plugs P and P'.

The bell b rings when an *alternating* P.D. is thrown upon the line, because, although the circuit is broken at t, an *alternating current* will surge into and out of the condenser C and thus pull the armature first toward m and then toward n. The bell could, of course, not be rung by a direct current.

QUESTIONS AND PROBLEMS

1. Does the spark of an induction coil occur at make or at break? Why?

2. Explain why an induction coil is able to produce such an enormous E.M.F. (Draw a diagram to illustrate the method of operation of the coil.)

3. Why could not an armature core be made of coaxial cylinders of iron running the full length of the armature, instead of flat disks, as shown in Fig. 329?

4. What relation must exist between the number of turns on the primary and secondary of a transformer which feeds 110-volt lamps from a main line whose conductors are at 1000 volts P.D.?

5. The same amount of power is to be transmitted over two lines from a power plant to a distant city. If the heat losses in the two lines are to be the same, what must be the ratio of the cross sections of the two lines if one current is transmitted at 100 volts and the other at 10,000 volts?

CHAPTER XVI *

NATURE AND TRANSMISSION OF SOUND

Speed and Nature of Sound

388. Sources of sound. If a sounding tuning fork provided with a stylus is stroked across a smoked-glass plate, it produces a wavy line, as shown in Fig. 343; if a light, suspended ball is brought into contact with it, the latter is thrown off with considerable violence. If we look about for the source of any sudden noise, we find that some object has fallen, or some collision has occurred, or some explosion has taken place; in a word, that some violent motion of matter has been set up in some way. From these familiar facts we conclude that *sound arises from the motions of matter.*

Fig. 343. Trace made by vibrating fork

389. Media of transmission. Air is ordinarily the medium through which sound comes to our ears, yet the Indians put their ears to the ground to hear a distant noise, and most boys know how loud the clapping of stones sounds under water. If the base of the sounding fork of Fig. 343 is held in a dish of water, the sound will be markedly transmitted by the water. These facts show that a gas like air is certainly no more effective in the transmission of sound than a liquid or a solid. Let us next see whether or not matter is necessary at all for the transmission of sound.

* This chapter should be accompanied by laboratory experiments on the speed of sound in air, the vibration rate of a fork, and the determination of wave lengths. See, for example, Experiments 38, 39, and 40 of the authors' manual.

Let an electric bell be suspended inside the receiver of an air pump by means of two fine springs which pass through a rubber stopper in the manner shown in Fig. 344. Let the air be exhausted from the receiver by means of the pump. The sound of the bell will be found to become less and less pronounced. Let the air be suddenly readmitted. The volume of sound will at once increase.

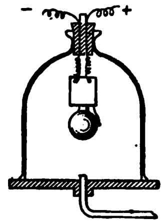


FIG. 344. Sound not transmitted through a vacuum

Since the nearer we approach a vacuum, the less distinct becomes the sound, we infer that sound cannot be transferred through a vacuum and that therefore *the transmission of sound is effected only through the agency of ordinary matter.* In this respect sound differs from heat and light, which evidently pass with perfect readiness through a vacuum, since they reach the earth from the sun and stars.

390. Speed of transmission. The first attempt to measure accurately the speed of sound was made in 1738, when a commission of the French Academy of Sciences stationed two parties about three miles apart and observed the interval between the flash of a cannon and the sound of the report. By taking observations between the two stations, first in one direction and then in the other, the effect of the wind was eliminated. A second commission repeated these experiments in 1832, using a distance of 18.6 kilometers, or a little more than 11.5 miles. The value found was 331.2 meters per second at 0° C. The accepted value is now 331.3 meters. The speed in water is about 1400 meters per second and in iron 5100 meters.

The speed of sound in air is found to increase with an increase in temperature. The amount of this increase is about 60 centimeters per degree centigrade. Hence the speed at 20° C. is about 343.3 meters per second. The above figures are equivalent to 1087 feet per second at 0° C., or 1126 feet per second at 20° C.

391. Mechanism of transmission. When a firecracker or toy cap explodes, the powder is suddenly changed to a gas the volume of which is enormously greater than the volume of the powder. The air is therefore suddenly pushed back in all directions from the center of the explosion. This means that the air particles which lie about this center are given violent outward velocities.* When these outwardly impelled air particles collide with other particles, they give up their outward motion to these second particles, and these in turn pass it on to others, etc. It is clear, therefore, that the motion started by the explosion must travel on from particle to particle to an indefinite distance from the center of the explosion. Furthermore, it is also clear that, although the motion travels on to great distances, the individual particles do not move far from their original positions; for it is easy to show experimentally that whenever an elastic body in motion collides with another similar body at rest, the colliding body simply transfers its motion to the body at rest and comes itself to rest.

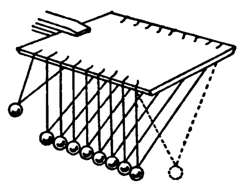


FIG. 345. Illustrating the propagation of sound from particle to particle

Let six or eight equal steel balls be hung from cords in the manner shown in Fig. 345. First, let all of the balls but two adjacent ones be held to one side, and let one of these two be raised and allowed to fall against the other. The first ball will be found to lose its motion in the collision, and the second will be found to rise to practically the same height as that from which the first fell. Next, let all of the balls be placed in line and the end one raised and allowed to fall as before. The motion will be transmitted from ball to ball, each giving up the whole of its motion practically as soon as it receives it, and the last ball will move on alone with the velocity which the first ball originally had.

* These outward velocities are simply superposed upon the velocities of agitation which the molecules already have on account of their temperature. For our present purpose we may ignore entirely the existence of these latter velocities and treat the particles as though they were at rest, save for the velocities imparted by the explosion.

The preceding experiment furnishes a very nice mechanical illustration of the manner in which the air particles which receive motions from an exploding firecracker or a vibrating tuning fork transmit these motions in all directions to neighboring layers of air, these in turn to the next adjoining layers, etc., until the motion has traveled to very great distances, although the individual particles themselves move only very minute distances. When a motion of this sort, transmitted by air particles, reaches the drum of the ear, it produces the sensation which we call *sound.*

392. A train of waves; wave length. In the preceding paragraphs we have confined attention to a single pulse traveling out from a center of explosion. Let us next consider the sort of disturbance which is set up in the air by a continuously vibrating body, like the prong of Fig. 346. Each time that this prong moves to the right it sends out a pulse which travels through the air at the rate of 1100 feet per second, in exactly the manner described in the preceding paragraphs. Hence, if the prong is vibrating uniformly, we shall have a continuous succession of pulses following each other through the air at exactly equal intervals. Suppose, for example, that the prong makes 110 complete vibrations per second. Then at the end of one second the first pulse sent out will have reached a distance of 1100 feet. Between this point and the prong there will be 110 pulses distributed at equal intervals; that is, each two adjacent pulses will be just 10 feet apart. If the prong made 220 vibrations per second, the distance between adjacent pulses would be 5 feet, etc. The *distance between two adjacent pulses in such a train of waves is called a wave length.*

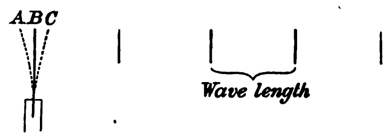


FIG. 346. Vibrating reed sending out a train of equidistant pulses

393. Relation between velocity, wave length, and number of vibrations per second. If n represents the number of vibrations per second of a source of sound, l the wave length, and v the velocity with which the sound travels through the medium, it is evident from the example of the preceding paragraph that the following relation exists between these three quantities:

$$l = v/n, \text{ or } v = nl; \tag{1}$$

that is, *wave length is equal to velocity divided by the number of vibrations per second, or velocity is equal to the number of vibrations per second times the wave length.*

394. Condensations and rarefactions. Thus far, for the sake of simplicity, we have considered a train of waves as a series of thin, detached pulses separated by equal intervals of air at rest. In point of fact, however, the air in front of the prong B (Fig. 346) is being pushed forward not at one particular

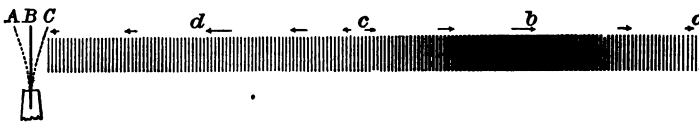


FIG. 347. Illustrating motions of air particles in one complete sound wave consisting of a condensation and a rarefaction

instant only, but during all the time that the prong is moving from A to C, that is, through the time of one half vibration of the fork; and during all this time this forward motion is being transmitted to layers of air which are farther and farther away from the prong, so that when the latter reaches C, all the air between C and some point c (Fig. 347) one-half wave length away is crowding forward, and is therefore in a state of compression or *condensation.* Again, as the prong moves back from C to A, since it tends to leave a vacuum behind it, the adjacent layer of air rushes in to fill up this space, the layer next adjoining follows, etc., so that when the prong reaches A, all the air between A and c (Fig. 347) is moving backward and

is therefore in a state of diminished density or *rarefaction.* During this time the preceding forward motion has advanced one-half wave length to the right, so that it now occupies the region between *c* and *a* (Fig. 347). Hence at the end of one complete vibration of the prong we may divide the air between it and a point one wave length away into two portions, one a region of condensation *ac*, and the other a region of rarefaction *cA*. The arrows in Fig. 347 represent the direction and relative magnitudes of the motions of the air particles in various portions of a complete wave.

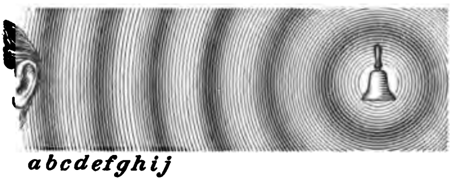


Fig. 348. Illustration of sound waves

At the end of *n* vibrations the first disturbance will have reached a distance *n* wave lengths from the fork, and each wave between this point and the fork will consist of a condensation and a rarefaction, so that sound waves may be said to consist of a series of condensations and rarefactions following one another through the air in the manner shown in Fig. 348.

Wave length may now be more accurately defined as *the distance between two successive points of maximum condensation* (*b* and *f*, Fig. 348) *or of maximum rarefaction* (*d* and *h*).

395. Water-wave analogy. Condensations and rarefactions of sound waves are exactly analogous to the familiar crests and troughs of water waves. Thus the wave length of such a series of waves as that shown in Fig. 349 is defined as the distance *bf* between two crests, or the distance *dh*, or *ae*, or *cg*, or *mn*, between any two points which are in the same condition or *phase* of disturbance. The crests, that is, the shaded portions, which are above the natural level of the water, correspond exactly

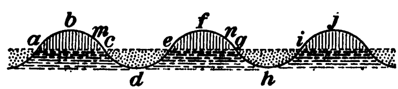


Fig. 349. Illustrating wave length of water waves

to the condensations of sound waves, that is, to the portions of air which are above the natural density. The troughs, that is, the dotted portions, correspond to the rarefactions of sound waves, that is, to the portions of air which are below the natural density. Nevertheless, the analogy breaks down at one point; for in water waves the motion of the particles is transverse to the direction of propagation, while in sound waves, as shown in § 394, the particles move back and forth *in* the line of propagation of the wave. *Water waves are therefore called transverse waves, while sound waves in air are longitudinal waves.*

396. Distinction between musical sounds and noises. Let a current of air from a $\frac{1}{8}$-inch nozzle be directed against a row of forty-eight equidistant $\frac{1}{4}$-inch holes in a metal or cardboard disk, mounted as in Fig. 350 and set into rotation either by hand or by an electric motor. A very distinct musical tone will be produced. Then let the jet of air be directed against a second row of forty-eight holes, which differs from the first only in that the holes are irregularly instead of regularly spaced about the circumference of the disk. The musical character of the tone will altogether disappear.

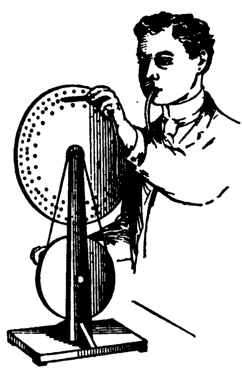


FIG. 350. Regularity of pulses the condition for a musical tone

The experiment furnishes a very striking illustration of the difference between a musical sound and a noise. *Only those sounds possess a musical quality which come from sources capable of sending out pulses, or waves, at absolutely regular intervals.* Therefore it is only sounds possessing a musical quality which may be said to have wave lengths.

397. Pitch. While the apparatus of the preceding experiment is rotating at constant speed, let a current of air be directed first against the outside row of regularly spaced holes and then suddenly turned against the inside row, which is also regularly spaced but which contains a smaller number of holes. The note produced in the second case will

be found to have a markedly lower pitch than the other one. Again, let the jet of air be directed against one particular row, and let the speed of rotation be changed from very slow to very fast. The note produced will gradually rise in pitch.

We conclude, therefore, that *the pitch of a musical note depends simply upon the number of pulses which strike the ear per second.* If the sound comes from a vibrating body, *the pitch of the note depends upon the rate of vibration of the body.*

398. Doppler's principle. When a rapidly moving express train rushes past an observer, he notices a very distinct change in the pitch of the bell as the engine passes him, the pitch being higher as the engine approaches than as it recedes. The explanation is as follows: The bell, of course, sends out pulses at exactly equal intervals of time. As the train is approaching, however, the pulses reach the ear at shorter intervals than the intervals between emissions, since the train comes toward the observer between two successive emissions. But as the train recedes, the interval between the receipt of pulses by the ear is longer than the interval between emissions, since the train is moving away from the ear during the interval between emissions. Hence the pitch of the bell is higher during the approach of the train than during its recession. This phenomenon of the change in pitch of a note proceeding from an approaching or receding body is known as *Doppler's principle.*

399. Loudness. The loudness or intensity of a sound depends upon the rate at which energy is communicated by it to the tympanum of the ear. *Loudness is therefore determined by the distance of the source and the amplitude of its vibration.*

If a given sound pulse is free to spread equally in all directions, at a distance of 100 feet from the source the same energy must be distributed over a sphere of four times as large an area as at a distance of 50 feet. Hence under these ideal conditions *the intensity of a sound varies inversely as the square of the distance from the source.* But when sound is confined within a tube so that the energy is continually communicated from one layer to another of equal area, it will travel to great distances with little loss of intensity. This explains the efficiency of speaking tubes and megaphones.

QUESTIONS AND PROBLEMS

1. A thunderclap was heard $5\frac{1}{2}$ sec. after the accompanying lightning flash was seen. How far away did the flash occur?

2. A bullet fired from a rifle with a speed of 1200 ft. per second is heard to strike the target 6 sec. afterwards. What is the distance to the target, the temperature of the air being 20° C.?

3. A church bell is ringing at a distance of $\frac{1}{2}$ mi. from one man and $\frac{1}{4}$ mi. from another. How much louder would it appear to the second man than to the first, if no reflections of the sound took place?

4. Explain the principle of the ear trumpet.

5. The vibration rate of a fork is 256. Find the wave length of the note given out by it at 20° C.

6. A stone is dropped into a well 200 m. deep. At 20° C. how much time will elapse before the sound of the splash is heard at the top?

7. As a circular saw cuts into a block of wood the pitch of the note given out falls rapidly. Why?

8. Since the music of an orchestra reaches a distant hearer without confusion of the parts, what may be inferred as to the relative velocities of the notes of different pitch?

Reflection, Reënforcement, and Interference

400. Echo. That a sound wave in hitting a wall suffers reflection is shown by the familiar phenomenon of echo. The roll of thunder is due to successive reflections of the original sound from clouds and other surfaces which are at different distances from the observer.

In ordinary rooms the walls are so close that the reflected waves return before the effect of the original sound on the ear has died out. Consequently the echo blends with and strengthens the original sound instead of interfering with it. This is why, in general, a speaker may be heard so much better indoors than in the open air. Since the ear cannot appreciate successive sounds as distinct if they come at intervals shorter than a tenth of a second, it will be seen from the fact that sounds travels about 113 feet in a tenth of a second that a wall which is nearer than about 50 feet cannot possibly

produce a perceptible echo. In rooms which are large enough to give rise to troublesome echoes it is customary to hang draperies of some sort, so as to break up the sound waves and prevent regular reflection.

401. Sound foci. Let a watch be hung at the focus of a large concave mirror. On account of the reflection from the surface of the mirror a fairly well-defined beam of sound will be thrown out in front of the mirror, so that if both watch and mirror are hung on a single support and the whole turned in different directions toward a number of observers, the ticking will be distinctly heard by those directly in front of the mirror, but not by those at one side. If a second mirror is held in the path of this beam, as in Fig. 351, the sound may be again brought to a focus, so that if the ear is placed in the focus of this second mirror, or better still, if a small funnel which is connected with the ear by a rubber tube is held in this focus, the ticking of the watch may sometimes be heard hundreds of feet away. A whispering gallery is a room so arranged as to contain such sound foci. Any two opposite points a few feet from the walls of a dome, like that of St. Peter's at Rome or St. Paul's at London, are sufficiently near to such sound foci to make very low whispers on one side distinctly audible at the other, although at intermediate points no sound can be heard.

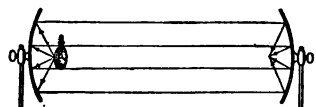


FIG. 351. Sound foci

402. Resonance. Resonance is the *reënforcement* or *intensification* of sound because of the union of *direct* and *reflected* waves.

Thus let one prong of a vibrating tuning fork, which makes, for example, 512 vibrations per second, be held over the mouth of a tube an inch or so in diameter, arranged as in Fig. 352, so that as the vessel A is raised or lowered, the height of the water in the tube may be adjusted at will. It will be found that as the position of the water is slowly lowered from the top of the tube a very marked reënforcement of the sound will occur at a certain point.

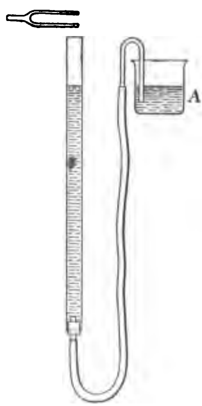


FIG. 352. Illustrating resonance

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Let other forks of different pitch be tried in the same way. It will be found that the lower the pitch of the fork, the lower must be the water in the tube in order to get the best reënforcement. This means that the longer the wave length of the note which the fork produces, the longer must be the air column in order to obtain resonance.

We conclude, therefore, that *a fixed relation exists between the wave length of a note and the length of the air column which will reënforce it.*

403. Best resonant length of a closed pipe is one-fourth wave length. If we calculate the wave length of the note of the fork by dividing the speed of sound by the vibration rate of the fork, we shall find that, in every case, *the length of air column which gives the best response is approximately one-fourth wave length.* The reason for this is evident when we consider that the length must be such as to enable the reflected wave to return to the mouth just in time to unite with the direct wave which is at that instant being sent off by the prong. Thus when the prong is first starting down from the position A (see Fig. 353), it starts the beginning of a condensation down the tube. If this motion is to return to the mouth just in time to unite with the direct wave sent off by the prong, it must get back at the instant that the prong is first starting up from the position C. In other words, the pulse must go down the tube and back again while the prong is making a half vibration. This means that the path down and back must be a half wave length, and hence that the length of the tube must be a fourth of a wave length.

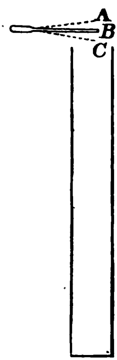


FIG. 353. Resonant length of a closed pipe is ¼ wave length

From the above analysis it will appear that there should also be resonance if the reflected wave does not return to the mouth until the fork is starting back its second time from C, that is, at the end of one and a half vibrations instead of a

half vibration. The distance from the fork to the water and back would then be one and a half wave lengths; that is, the water surface would be a half wave length farther down the tube than at first. The tube length would therefore now be three fourths of a wave length.

Let the experiment be tried. A similar response will indeed be found, as predicted, a half wave length farther down the tube. This response will be somewhat weaker than before, as the wave has lost some of its energy in traveling a long distance through the tube. It may be shown in a similar way that there will be resonance where the tube length is $\frac{5}{4}$, $\frac{7}{4}$, or indeed any odd number of quarter wave lengths.

404. Best resonant length of an open pipe is one-half wave length. Let the same tuning fork which was used in § 403 be held in front of an open pipe (8 or 10 inches long) the length of which is made adjustable by slipping back and forth over it a tightly fitting roll of writing paper (Fig. 354). It will be found that for one particular length this open pipe will respond quite as loudly as did the closed pipe, but *the responding length will be found to be just twice as great as before.* Other resonant lengths can be found when the tube is made 2, 3, etc., times as long.

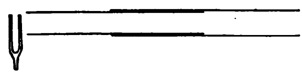


FIG. 354. Resonant length of an open pipe is $\frac{1}{2}$ wave length

We learn, then, that *the shortest resonant length of an open pipe is one-half wave length, and that there is resonance at any multiple of a half wave length.*

The fact that the shortest resonant length of the open pipe is just twice that of the closed one is the experimental proof that a condensation, upon reaching the open end of a pipe, is reflected as a rarefaction. This means that when the lower end of the tube of Fig. 353 is open, a condensation upon reaching it suddenly expands. In consequence of this expansion the new pulse which begins at this instant to travel back through the tube is one in which the particles are moving down instead of up; that is, the particles are moving in a direction opposite to that in which the wave is traveling. This is always the case in a rarefaction (see Fig. 347). In

order then to unite with the motion of the prong this downward motion of the particles must get back to the mouth when the prong is just starting down from A the second time; that is, *after one complete vibration of the prong.* This shows why the pipe length is one-half wave length.

405. Resonators. If the vibrating fork at the mouth of the tubes in the preceding experiments is replaced by a *train of waves* coming from a distant source, precisely the same analysis leads to the conclusion that the waves reflected from the bottom of the tube will reënforce the oncoming waves when the length of the tube is any odd number of quarter wave lengths in the case of a closed pipe, or any number of half wave lengths in the case of an open pipe. It is clear, therefore, that every air chamber will act as a resonator for trains of waves of a *certain* wave length. This is why a conch shell held to the ear is always heard to hum with a particular note. Feeble waves which produce no impression upon the unaided ear gain sufficient strength when reënforced by the shell to become audible. When the air chamber is of irregular form it is not usually possible to calculate to just what wave length it will respond, but it is always easy to determine experimentally what particular wave length it is capable of reënforcing. The resonators on which tuning forks are mounted are air chambers which are of just the right dimensions to respond to the note given out by the fork.

406. Forced vibrations; sounding boards. Let a tuning fork be struck and held in the hand. The sound will be entirely inaudible except to those quite near. Let the base of the sounding fork be pressed firmly against the table. The sound will be found to be enormously intensified. Let another fork be held against the same table. Its sound will also be reënforced. In this case, then, the table intensifies the sound of any fork which is placed against it, while an air column of a certain size could intensify only a single note.

The cause of the response in the two cases is wholly different. In the last case the vibrations of the fork are transmitted

through its base to the table top and force the latter to vibrate in its own period. The vibrating table top, on account of its large surface, sets a comparatively large mass of air into motion and therefore sends a wave of great intensity to the ear; while the fork alone, with its narrow prongs, was not able to impart much energy to the air. Vibrations like those of the table top are called *forced* because they can be produced with any fork, no matter what its period. Sounding boards in pianos and other stringed instruments act precisely as does the table top in this experiment; that is, they are set into forced vibrations by any note of the instrument and reënforce it accordingly.

407. Beats. Since two sound waves are able to unite so as to reënforce each other, it ought also to be possible to make them unite so as to interfere with or destroy each other. In other words, under the proper conditions *the union of two sounds ought to produce silence.*

Let two mounted tuning forks of the same pitch be set side by side, as in Fig. 355. Let the two forks be struck in quick succession with a soft mallet, for example, a rubber stopper on the end of a rod. The two notes will blend and produce a smooth, even tone. Then let a piece of wax or a small coin be stuck to a prong of one of the forks. This diminishes slightly the number of vibrations which this fork makes per second, since it increases its mass. Again, let the two forks be sounded together. The former smooth tone will be replaced by a throbbing or pulsating one. This is due to the alternate destruction and reënforcement of the sounds produced by the two forks. This pulsation is called the phenomenon of *beats.*

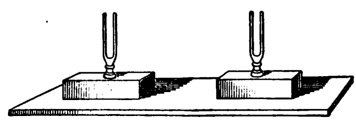


FIG. 355. Arrangement of forks for beats

The mechanism of the alternate destruction and reënforcement may be understood from the following. Suppose that one fork makes 256 vibrations per second (see the dotted line *AC* in Fig. 356), while the other makes 255 (see the heavy line *AC*). If at the beginning of a given second the two forks

are swinging *together* so that they simultaneously send out condensations to the observer, these condensations will of course unite so as to produce a double effect upon the ear (see A', Fig. 356). Since now one fork gains one complete vibration per second over the other, at the end of the second considered the two forks will again be vibrating together, that is, sending out condensations which add their effects as before (see C'). In the middle of this second, however, the two forks are vibrating in opposite directions (see B); that is, one is sending out rarefactions while the other sends out condensations. At the ear of the observer the union of the rarefaction (backward motion of the air particles) produced by one fork with the condensation (forward motion) produced by the other results in no motion at all, provided the two motions have the same energy; that is, *in the middle of the second the two sounds have united to produce silence* (see B'). It will be seen from the above that *the number of beats per second is equal to the difference in the vibration numbers of the two forks.*

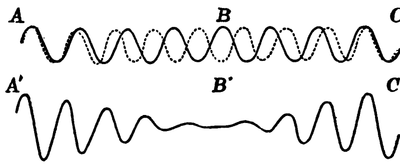


FIG. 356. Graphical illustration of beats

To test this conclusion, let more wax or a heavier coin be added to the weighted prong; the number of beats per second will be increased. Diminishing the weight will reduce the number of beats per second.

408. Interference of sound waves by reflection. Let a thin cork about an inch in diameter be attached to one end of a brass or

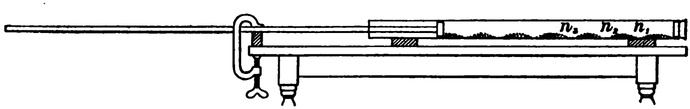


FIG. 357. Interference of advancing and retreating trains of sound waves

glass rod from one to two meters long. Let this rod be clamped firmly in the middle, as in Fig. 357. Let a piece of glass tubing a meter or more long and from an inch to an inch and a half in diameter be slipped

over the cork, as shown. Let the end of the rod be stroked longitudinally with a well-resined cloth. (A wet cloth will answer better if the rod is of glass.) A loud shrill note will be produced.

This note is due to the fact that the slipping of the resined cloth over the surface of the rod sets the latter into longitudinal vibrations, so that its ends impart alternate condensations and rarefactions to the layers of air in contact with them. As soon as this note is started the cork dust inside the tube will be seen to be intensely agitated. If the effect is not marked at first, a slight slipping of the glass tube forward or back will bring it out. Upon examination it will be seen that the agitation of the cork dust is not uniform, but at regular intervals throughout the tube there will be regions of complete rest, n_1, n_2, n_3, etc., separated by regions of intense motion.

The points of rest correspond to the positions in which the reflected train of sound waves returning from the end of the tube neutralizes the effect of the advancing train passing down the tube from the vibrating rod. The points of rest are called *nodes*, the intermediate portions *loops* or *antinodes*. The distance between these nodes is one half wave length, for at

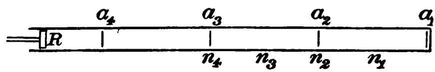


Fig. 358. Distance between nodes is one half wave length

the instant that the first wave front a_1 (Fig. 358) reaches the end of the tube it is reflected and starts back toward R. Since at this instant the second wave front a_2 is just one wave length to the left of a_1, the two wave fronts must meet each other at a point n_1, just one-half wave length from the end of the tube. The exactly equal and opposite motions of the particles in the two wave fronts exactly neutralize each other. Hence the point n_1 is a point of no motion, that is, a node. Again, at the instant that the reflected wave front a_1 met the advancing wave front a_2 at n_1, the third wave front a_3 was just one wave length to the left of n_1. Hence, as the first wave front a_1 continues to travel back toward R it meets a_3 at n_2, just one half wave length from n_1, and produces there a second node. Similarly, a third node is produced at n_3, one half wave length to the

left of n_2, etc. Thus *the distance between two nodes must always be just one half the wave length of the waves in the train.*

In the preceding discussion it has been tacitly assumed that the two oppositely moving waves are able to pass through each other without either of them being modified by the presence of the other. That two opposite motions are, in fact, transferred in just this manner through a medium consisting of elastic particles may be beautifully shown by the following experiment with the row of balls used in § 391.

Let the ball at one end of the row be raised a distance of, say, 2 inches and the ball at the other end raised a distance of 4 inches. Then let both balls be dropped simultaneously against the row. The two opposite motions will pass through each other in the row altogether without modification, the larger motion appearing at the end opposite to that at which it started, and the smaller likewise.

Another and more complete analogy to the condition existing within the tube of Fig. 357 may be had by simply vibrating one end of a two- or three-meter rope, as in Fig. 359. The trains of advancing and reflected waves which continuously travel through each other up and down the rope will unite so as to form a series of nodes and loops. The nodes at *c* and *e* are the points at which the advancing and reflected waves are always urging the cord equally in opposite directions. The distance between them is one half the wave length of the train sent down the rope by the hand.

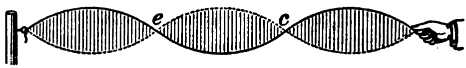


FIG. 359. Nodes and loops in a cord

Black line denotes advancing train; dotted line, reflected train

QUESTIONS AND PROBLEMS

1. Why do the echoes which are prominent in empty halls often disappear when the hall is full of people?

2. A gunner hears an echo $5\frac{1}{2}$ sec. after he fires. How far away was the reflecting surface, the temperature of the air being 20° C.?

3. Find the number of vibrations per second of a fork which, at 20° C., produces resonance in a pipe 1 ft. long closed at one end.

4. A fork making 500 vibrations per second is found to produce resonance in an air column like that shown in Fig. 352, first when the water is a certain distance from the top, and again when it is 34 cm. lower. Find the velocity of sound.

5. Show why an open pipe needs to be twice as long as a closed pipe if it is to respond to the same note.

CHAPTER XVII

PROPERTIES OF MUSICAL SOUNDS

MUSICAL SCALES

409. Physical basis of musical intervals. Let a metal or card board disk 10 or 12 inches in diameter be provided with four concentric rows of equidistant holes, the successive rows containing respectively 24, 30, 36, and 48 holes (Fig. 360). The holes should be about $\frac{1}{4}$ inch in diameter and the rows should be about $\frac{1}{2}$ inch apart. Let the disk be placed in the rotating apparatus and a constant speed imparted. Then let a jet of air be directed, as in § 396, against each row of holes in succession. It will be found that the musical sequence *do, mi, sol, do′* results. If the speed of rotation is increased, each note will rise in pitch, but the sequence will remain unchanged.

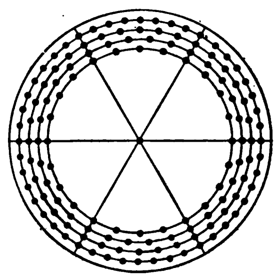


FIG. 360. Disk for producing musical sequence *do, mi, sol, do′*

We learn, therefore, that *the musical sequence do, mi, sol, do′ consists of notes whose vibration numbers have the ratios of* 24, 30, 36, *and* 48, *that is*, 4, 5, 6, 8, *and that this sequence is independent of the absolute vibration numbers of the tones.*

Furthermore, when two notes an octave apart are sounded together, they form the most harmonious combination which it is possible to obtain. These characteristics of notes an octave apart were recognized in the earliest times, long before anything whatever was known about the ratio of their vibration numbers. The preceding experiment showed that *this ratio is the simplest possible*, namely, 24 to 48, or 1 to 2. Again, the next easiest musical interval to produce and the next

most harmonious combination which can be found corresponds to the two notes commonly designated as *do*, *sol*. Our experiment showed that this interval corresponds to the next simplest possible vibration ratio, namely, 24 to 36, or 2 to 3. When *sol* is sounded with *do*′ the vibration ratio is seen to be 36 to 48, or 3 to 4. We see, therefore, that the three simplest possible ratios of vibration numbers, namely, 1 to 2, 2 to 3, and 3 to 4, are used in the production of the three notes *do*, *sol*, *do*′. Again, our experiment shows that another harmonious musical interval, *do*, *mi*, corresponds to the vibration ratio 24 to 30, or 4 to 5. We learn, therefore, that *harmonious musical intervals correspond to very simple vibration ratios.*

410. The major diatonic scale. When the three notes *do*, *mi*, *sol*, which, as seen above, have the vibration ratios 4, 5, 6, are all sounded together, they form a remarkably pleasing combination of tones. This combination was picked out and used very early in the musical development of the race. It is now known as the *major chord. The major diatonic scale is built up of three major chords.* The absolute vibration number taken as the starting point is wholly immaterial, but the explanation of the origin of the eight notes of the octave commonly designated by the letters *C*, *D*, *E*, *F*, *G*, *A*, *B*, *C*′ may be made more simple if we begin, as above, with a note whose vibration number is 24. If this note is designated by the letter *C*, the two other notes of the first major chord, *do-mi-sol*, are designated by *E* and *G*. The second chord is obtained by starting from *C*′, the octave of *C*, and coming down in the ratios 6, 5, 4. The corresponding vibration numbers are 48, 40, and 32, and the corresponding notes, known as *do*, *la*, *fa*, are designated by the letters *C*′, *A*, and *F*. The third chord starts with *G* as the first note and runs up in the ratios 4, 5, 6. The corresponding notes, known as *sol*, *si*, *re*, have the vibration numbers 36, 45, and 54, and are designated by the letters *G*, *B*, and *D*′. It will be seen that the note *D*′ does not fall in the

octave between C and C', for its vibration number is above 48. The note D an octave below it falls between C and C' and has a vibration number 27. This completes the eight notes of the diatonic scale. The chord *do-mi-sol* is called the tonic, *sol-si-re* the dominant, and *fa-la-do* the subdominant. Below is given in tabular form the relations between the notes of an octave.

Syllables	*do*	*re*	*mi*	*fa*	*sol*	*la*	~~*si*~~ ti	*do*
Letters	C	D	E	F	G	A	B	C
Relative vibration numbers	24	27	30	32	36	40	45	48
Vibration ratios in terms of *do*	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Absolute vibration numbers	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512

Standard middle C forks made for physical laboratories all have the vibration number 256. In the so-called *international pitch* middle C has 261 vibrations, and in *concert pitch* 274.

411. The even-tempered scale. If G is taken as *do* and a scale built up as above, it will be found that six of the above notes in each octave can be used in this new key, but that two additional ones are required (see table below). Similarly, to build up scales, as above, in all the keys demanded by modern music would require about fifty notes in each octave. Hence a compromise is made by dividing the octave into twelve equal intervals represented by the eight white and five black keys of a piano. How much this so-called *even-tempered scale* differs from the ideal, or diatonic, scale is shown below.

Note	C	D	E	F	G	A	B	C'	D'	E'	F'	G'
Diatonic	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512	576	640	682.2	768
Diatonic key of G	. . .	. . .	. . .	. . .	384	**432**	480	512	576	640	**720**	768
Tempered	256	287.4	322.7	341.7	383.8	430.7	483.5	512	574.8	645.4	683.4	767.6

VIBRATING STRINGS *

412. Laws of vibrating strings. Let two piano wires be stretched over a box, or a board with pulleys attached so as to form a sonometer (Fig. 361). Let the weights A and B be adjusted until the two wires emit exactly the same note. The phenomenon of beats will make it

* This discussion should be followed by a laboratory experiment on the laws of vibrating strings. See, for example, Experiment 41 of the authors' manual.

possible to do this with great accuracy. Then let the bridge *D* be inserted exactly at the middle of one of the wires, and the two wires plucked in succession. The interval will be recognized at once as *do, do′*. Next let the bridge be inserted so as to make one wire two thirds as long as the other, and let the two be plucked again. The interval will be recognized as *do, sol*.

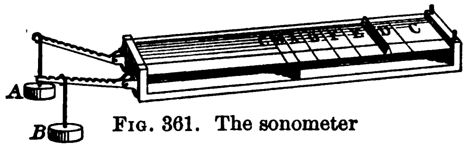


FIG. 361. The sonometer

Now it was shown in § 409 that *do′* has twice as many vibrations per second as *do*, and *sol* has three halves as many. Hence, since the length corresponding to *do′* is one half as great as the first length, and that corresponding to *sol* two thirds as great, we conclude from this experiment that, other things being equal, *the vibration numbers of strings are inversely proportional to their lengths.*

Again, let the two wires be tuned to unison, and then let the weight *A* be increased until the pull which it exerts on the wire is exactly four times as great as that exerted by *B*. The note given out by the *A* wire will again be found to be an octave above that given out by the *B* wire.

We learn, then, that *the vibration numbers of similar strings of equal length are proportional to the square roots of the tensions.*

413. Nodes and loops in vibrating strings. Let a string a meter long be attached to one of the prongs of a large tuning fork which makes in the neighborhood of 100 vibrations per second. Let the other end be attached as in the figure, and the fork set into vibration. If the fork is not electrically driven, which is much to be preferred, it may be bowed with a violin bow or struck with a soft mallet. By making the tension of the thread, for example, proportional to the numbers 9, 4, and 1 it will be found possible to make it vibrate either as a whole, as in Fig. 362, or in two or three parts (Fig. 363).

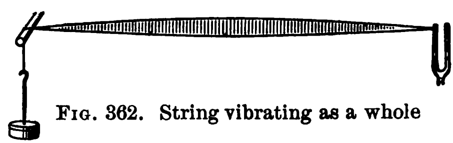


FIG. 362. String vibrating as a whole

This effect is due, as explained in § 408, to the interference of the direct and reflected waves sent down the string from

the vibrating fork. But we shall show in the next paragraph that in considering the effects of the vibrating string on the surrounding air we shall make no mistake if we think of it as clamped at each node, and as actually vibrating in two or three or four separate parts, as the case may be.

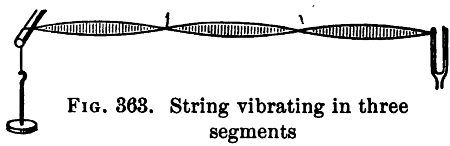


FIG. 363. String vibrating in three segments

FUNDAMENTALS AND OVERTONES

414. Fundamentals and overtones. If the assertion just made be correct, then a string which has a node in the middle communicates twice as many pulses to the air per second as the same string when it vibrates as a whole. This may be conclusively shown as follows:

Let the sonometer wire (Fig. 361) be plucked in the middle and the pitch of the corresponding tone carefully noted. Then let the finger be touched to the middle of the wire, and the latter plucked midway between this point and the end.* The octave of the original note will be distinctly heard. Next let the finger be touched at a point one third of the wire length from one end, and the wire again plucked. The note will be recognized as *sol′*. Since we learned in § 410 that *sol′* has three halves as many vibrations as *do′*, it must have three times as many vibrations as the original note. Hence a wire which is vibrating in three segments sends out three times as many vibrations as when it is vibrating as a whole.

Now when a wire is plucked in the middle it vibrates simply as a whole, and therefore gives forth the lowest note which it is capable of producing. This note is called the *fundamental* of the wire. When the wire is made to vibrate in two parts it gives forth, as has just been shown, a note an octave higher than the fundamental. This is called the *first overtone*. When the wire is made to vibrate in three parts it gives forth a note corresponding to three times the vibration number of the fundamental, namely, *sol′*. This is called the *second overtone*.

* It is well to remove the finger almost simultaneously with the plucking.

When the wire vibrates in four parts it gives forth the third overtone, which is a note two octaves above the fundamental. The overtones of wires are often called *harmonics*. They bear the vibration ratios 2, 3, 4, 5, 6, 7, etc., to the fundamental.*

415. Simultaneous production of fundamentals and overtones. We have thus far produced overtones only by forcing the wire to remain at rest at certain properly chosen points during the bowing.

Now let the wire be plucked at a point one fourth of its length from one end, *without being touched in the middle*. The tone most distinctly heard will be the fundamental, but if the wire is now touched very lightly exactly in the middle, the sound, instead of ceasing altogether, will continue, but the note heard will be an octave higher than the fundamental, showing that in this case there was superposed upon the vibration of the wire as a whole a vibration in two segments also (Fig. 364). By touching the wire in the middle the vibration as a whole was destroyed, but that in two parts remained. Let the experiment be repeated, with this difference, that the wire is now plucked in the middle instead of one fourth its length from one end. If it is now touched in the middle, the sound will entirely cease, showing that when a wire is plucked in the middle there is no first overtone superposed upon the fundamental. Let the wire be plucked again one fourth of its length from one end, and careful attention given to the compound note emitted. It will be found possible to recognize both the fundamental and the first overtone sounding at the same time. Similarly, by plucking at a point one sixth of the length of the wire from one end, and then touching it at a point one third of its length from the end, the second overtone may be made to appear distinctly, and a trained ear will detect it in the note given off by the wire, even before the fundamental is suppressed by touching at the point indicated.

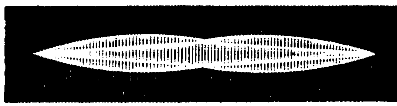


Fig. 364. A wire simultaneously emitting its fundamental and first overtone

*Some instruments, such as bells, can produce higher tones whose vibration numbers are not exact multiples of the fundamental. These notes are still called overtones, but they are not called harmonics, the latter term being reserved for the multiples. Strings produce harmonics only.

The experiments show, therefore, that in general *the note emitted by a string plucked at random is a complex one, consisting of a fundamental and several overtones, and that just what overtones are present in a given case depends on where and how the wire is plucked.*

416. Quality. Let the sonometer wire be plucked first in the middle and then close to one end. The two notes emitted will have exactly the same pitch, and they may have exactly the same loudness, but they will be easily recognized as different in respect to something which we call *quality.* The experiment of the last paragraph shows that the real physical difference in the tones is a difference in the sort of overtones which are mixed with the fundamental in the two cases.

Again, let a mounted C' fork be sounded simultaneously with a mounted C fork. The resultant tone will sound like a rich, full C, which will change into a hollow C when the C' is quenched with the hand.

Every one is familiar with the fact that when notes of the same pitch and loudness are sounded upon a piano, a violin, and a cornet, the three tones can be readily distinguished. The last experiments suggest that the cause of this difference lies in the fact that it is only the *fundamental* which is the same in the three cases, while the *overtones* are different. In other words, the characteristic of a tone which we call its quality is determined simply by *the number and prominence of the overtones which are present.* If there are few and weak overtones present, while the fundamental is strong, the tone is, as a rule, soft and mellow, as when a sonometer wire is plucked in the middle, or a closed organ pipe is blown gently, or a tuning fork is struck with a soft mallet. The presence of comparatively strong overtones up to the fifth adds fullness and richness to the resultant tone. This is illustrated by the ordinary tone from a piano, in which several if not all of the first five overtones have a prominent place. When overtones higher than the sixth are present a sharp metallic quality begins to appear. This is illustrated when a tuning fork is

struck, or a wire plucked, with a hard body. It is in order to avoid this quality that the hammers which strike against piano wires are covered with felt.

417. Analysis of tones by the manometric flame. A very simple and beautiful way of showing the complex character of most tones is furnished by the so-called *manometric flames.* This device consists of the following parts: a chamber in the block *B* (Fig. 365), through which gas is led by way of the

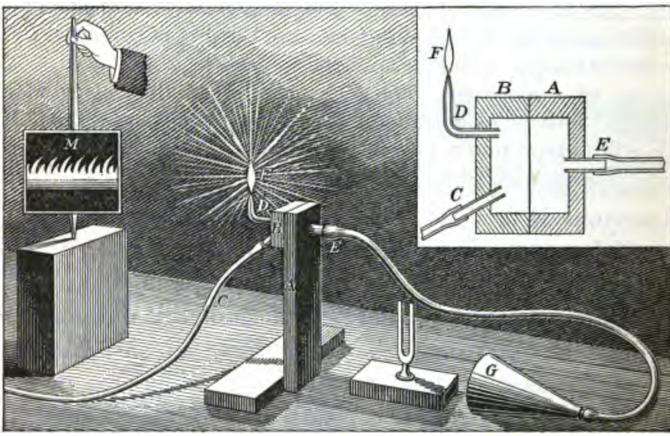


FIG. 365. Analysis of sounds with manometric flames

tubes *C* and *D* to the flame *F*; a second chamber in the block *A*, separated from the first chamber by an elastic diaphragm made of very thin sheet rubber or paper, and communicating with the source of sound through the tube *E* and trumpet *G*; and a rotating mirror *M* by which the flame is observed. When a note is produced before the mouthpiece *G* the vibrations of the diaphragm produce variations in the pressure of the gas coming to the flame through the chamber in *B*, so that when condensations strike the diaphragm the height of the flame is increased, and when rarefactions strike it the height of the

flame is diminished. If these up-and-down motions of the flame are viewed in a rotating mirror, the longer and shorter images of the flame, which correspond to successive intervals of time, appear side by side, as in Fig. 366. If a rotating mirror is not to be had, a piece of ordinary mirror glass held in the hand and oscillated back and forth about a vertical axis will be found to give perfectly satisfactory results.

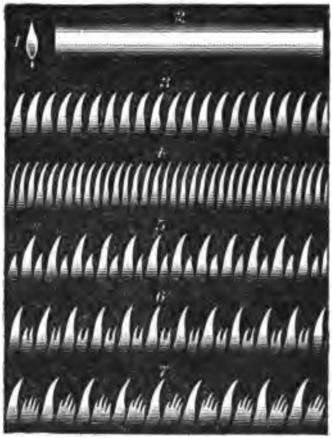


FIG. 366. Vibration forms shown by manometric flames

First let the mirror be rotated when no note is sounded before the mouthpiece. There will be no fluctuations in the flame, and its image, as seen in the moving mirror, will be a straight band, as shown in *2*, Fig. 366. Next let a mounted *C* fork be sounded, or some other simple tone produced in front of *G*. The image in the mirror will be that shown in *3*. Then let another fork *C′* be sounded in place of the *C*. The image will be that shown in *4*. The images of the flame are now twice as close together as before, since the blows strike the diaphragm twice as often. Next let the open ends of the resonance boxes of the two tuning forks *C* and *C′* be held together in front of *G*. The image of the flame will be as shown in *5*. If the vowel *o* be sung in the pitch *B♭* before the mouthpiece, a figure exactly similar to *5* will be produced, thus showing that this last note is a complex, consisting of a fundamental and its first overtone.

The proof that most other tones are likewise complex lies in the fact that when analyzed by the manometric flame they show figures not like *3* and *4*, which correspond to simple tones, but like *5*, *6*, and *7*, which may be produced by sounding combinations of simple tones. In the figure, *6* is produced by singing the vowel *e* on C''; *7* is obtained when *o* is sung on C''.

†

418. Helmholtz's experiment. If the loud pedal on a piano is held down and the vowel sounds $\overline{oo}$, *ī*, *ā*, *ah*, *ē*, sung loudly into the strings, these vowels will be caught up and returned by the instrument with sufficient fidelity to make the effect almost uncanny.

It was by a method which may be considered as merely a refinement of this experiment that Helmholtz proved conclusively that quality is determined simply by the number and prominence of the overtones which are blended with the fundamental. He first constructed a large number of resonators, like that shown in Fig. 367, each of which would respond to a note of some particular pitch. By holding these resonators in succession to his ear while a musical note was sounding, he picked out the constituents of the note; that is, he found out just what overtones were present and what were their relative intensities. Then he put these constituents together and reproduced the original tone. This was done by sounding simultaneously, with appropriate loudness, two or more of a whole series of tuning forks which had the vibration ratios 1, 2, 3, 4, 5, 6, 7. In this way he succeeded not only in imitating the qualities of different musical instruments, but even in reproducing the various vowel sounds.

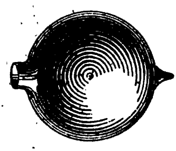


FIG. 367. Helmholtz's resonator

419. Sympathetic vibrations. Let two mounted tuning forks of the same pitch be placed with the open ends of their resonators facing each other. Let one be set into vigorous vibration with a soft mallet and then quickly quenched by grasping the prongs with the hand. The other fork will be found to be sounding loudly enough to be heard over a large room. Next let a penny be waxed to one prong of the second fork and the experiment repeated. When the sound of the first fork is quenched, no sound whatever will be found to be coming from the second fork.

The experiment illustrates the phenomenon of *sympathetic vibrations* and shows what conditions are essential to its appearance. If two bodies capable of emitting musical notes have

HERMANN LUDWIG FERDINAND VON HELMHOLTZ (1821–1894)

Noted German physicist and physiologist; professor of physiology and anatomy at Bonn and at Heidelberg from 1855 to 1871; professor of physics at Berlin from 1871 to 1894; published in 1847 a famous paper on the conservation of energy, which was most influential in establishing that doctrine; invented the ophthalmoscope; discovered the physical significance of tone quality, and made other contributions to acoustics and optics; was preëminent also as a mathematical physicist

exactly the same natural period of vibration, the pulses communicated to the air when one alone is sounding, beat upon the second at intervals which correspond exactly to its own natural period. Each pulse therefore adds its effect to that of the preceding pulses, and though the effect due to a single pulse is very slight, a great number of such pulses produce a large resultant effect. In the same way a large number of very feeble pulls may set a heavy pendulum into vibrations of considerable amplitude if the pulls come at intervals exactly equal to the natural period of the pendulum. On the other hand, if the two sounding bodies have even a slight difference of period, the effect of the first pulses is neutralized by the effect of succeeding pulses as soon as the two bodies, on account of their difference in period, get to swinging in opposite directions.

Let notes of different pitches be sung into a piano when the dampers are lifted. The wire which has the pitch of the note sounded will in every case respond. Sing a little off the key and the response will cease.

420. Sympathetic vibrations produced by overtones. It is not essential, in order that a body may be set into sympathetic vibrations, that it have the same pitch as the sounding body, provided its pitch corresponds exactly with the pitch of one of the *overtones* of that body.

Thus, if the damper is lifted from the C string of a piano and the octave below, C_1, is sounded loudly, C will be heard to sound after C_1 has been quenched by the damper. In this case it is the first overtone of C_1 which is in exact tune with C, and which therefore sets it into sympathetic vibration. Again, if the damper is lifted from the G string while C_1 is sounded, this note will be found to be set into vibration by the second overtone of C_1. A still more interesting case is obtained by removing the damper from E while C_1 is sounded. When C_1 is quenched, the note which is heard is not E, but an octave above E; that is, E'. This is because there is no overtone of C_1 which corresponds to the vibration of E; but the fourth overtone of C_1, which has five times the vibration number of C_1, corresponds exactly to the vibration number of E', the first overtone of E. Hence E is set into vibration not as a whole but in halves.

421. Physical significance of harmony and of discord. Let two pieces of glass tubing about an inch in diameter and a foot and a half long be supported vertically, as shown in Fig. 368. Let two gas jets, made by drawing down pieces of one-fourth inch glass tubing until, with full gas pressure, the flame is about an inch long, be thrust inside these tubes to a height of about three or four inches from the bottom. Let the gas be turned down until the tubes begin to sing. Without attempting to discuss the part which the flame plays in the production of the sound, we wish simply to call attention to the fact that the two tones are either quite in unison, or so near it that but a few beats are produced per second. Now let the length of one of the tubes be slightly increased by slipping the paper cylinder S up over its end. The number of beats will be rapidly increased until they will become indistinguishable as separate beats and will merge into a jarring, grating discord.

FIG. 368. Illustrating the production of discords

The experiment teaches that *discord is simply a phenomenon of beats.* If the vibration numbers do not differ by more than five or six, that is, if there are not more than five or six beats per second, the effect is not particularly unpleasant. From this point on, however, as the difference in the vibration numbers, and therefore in the number of beats per second, increases, the unpleasantness increases, and becomes worst at a difference of about thirty. Thus the notes B and C', which differ by about thirty-two beats per second, produce about the worst possible discord. When the vibration numbers differ by as much as seventy, which is about the difference between C and E, the effect is again pleasing or harmonious. Moreover, in order that two notes may harmonize well, it is necessary not only that the notes themselves shall not produce an unpleasant number of beats, but also that such beats shall not arise from their overtones. Thus C and B are very

discordant, although they differ by a large number of vibrations per second. The discord in this case arises between B and C', the first overtone of C.

Again, there are certain classes of instruments, of which bells are a striking example, which produce insufferable discords when even such notes as *do*, *sol*, *do'*, are sounded simultaneously upon them. This is because these instruments, unlike strings and pipes, have overtones which are not harmonics, that is, which are not multiples of the fundamental; and these overtones produce beats either among themselves or with one of the fundamentals. It is for this reason that in playing chimes the bells are struck in succession, not simultaneously.

QUESTIONS AND PROBLEMS

1. At what point must the G_1 string be pressed by the finger of the violinist in order to produce the note C?

2. If one wire has twice the length of another and is stretched by four times the stretching force, how will their vibration numbers compare?

3. A wire gives out the note G. What is its fourth overtone?

4. What is the wave length of middle C when the speed of sound is 1152 ft. per second?

5. What is the pitch of a note whose wave length is 5.4 in., the speed being 1152 ft. per second?

6. If middle C had 300 vibrations per second, how many vibrations would F and A have?

7. What is the fourth overtone of C? the fifth overtone?

8. A wire gives out the note C when the tension on it is 4 kg. What tension will be required to give out the note G?

9. A wire 50 cm. long gives out 400 vibrations per second. How many vibrations will it give when the length is reduced to 10 cm.? What syllable will represent this note if *do* represents the first note?

10. There are seven octaves and two notes on an ordinary piano, the lowest note being A_4 and the highest one C''''. If the vibration number of the lowest note is 27, find the vibration number of the highest.

11. Find the wave length of the lowest note on the piano; the wave length of the highest note. (Take the speed of sound in air as 1130 ft. per second.)

12. A violin string is commonly bowed about one seventh of its length from one end. Why is this better than bowing in the middle?

WIND INSTRUMENTS

422. Fundamentals of closed pipes. Let a tightly fitting rubber stopper be inserted in a glass tube *a* (Fig. 369), eight or ten inches long and about three fourths of an inch in diameter. Let the stopper be pushed along the tube until, when a vibrating C' fork is held before the mouth, resonance is obtained as in § 402. (The length will be six or seven inches.) Then let the fork be removed and a stream of air blown across the mouth of the tube through a piece of tubing *b*, flattened at one end as in the figure.* The pipe will be found to emit strongly the note of the fork.

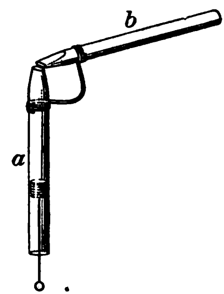


FIG. 369. Musical notes from pipes

In every case it is found that a note which a pipe may be made to emit is always a note to which it is able to respond when used as a resonator. Since, in § 403, the best resonance was found when the wave length given out by the fork was four times the length of the pipe, we learn that *when a current of air is suitably directed across the mouth of a closed pipe, it will emit a note which has a wave length four times the length of the pipe.* This note is called the *fundamental* of the pipe. It is the lowest note which the pipe can be made to produce.

423. Fundamentals of open pipes. Since we found in § 404 that the lowest note to which a pipe open at the lower end can respond is one the wave length of which is twice the pipe length, we infer that an open pipe when suitably blown ought to *emit* a note the wave length of which is twice the pipe length. This means that if the same pipe is blown first when closed at the lower end and then when open, the first note ought to be an octave lower than the second.

* If the arrangement of Fig. 369 is not at hand, simply blow with the lips across the edge of a piece of ordinary glass tubing within which a rubber stopper may be pushed back and forth.

Let the pipe *a* (Fig. 369) be closed at the bottom with the hand and blown; then let the hand be removed and the operation repeated. The second note will indeed be found to be an octave higher than the first.

We learn, therefore, that *the fundamental of an open pipe has a wave length equal to twice the pipe length.*

424. Overtones in pipes. It was found in § 403 that there are a whole series of pipe lengths which respond to a given fork, and that these lengths bear to the wave length of the fork the ratios $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc. This is equivalent to saying that a closed pipe of *fixed* length can respond to a whole series of notes whose vibration numbers have the ratios 1, 3, 5, 7, etc. Similarly, in § 404, we found that in the case of an open pipe the series of pipe lengths which will respond to a given fork bear to the wave length of the fork the ratios $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, etc. This again is equivalent to saying that an open pipe can respond to a series of notes whose vibration numbers have the ratios 1, 2, 3, 4, 5, etc. Hence we infer that it ought to be possible to cause both open and closed pipes to emit notes of higher pitch than their fundamentals, that is, overtones, and that the first overtone of an *open* pipe should have twice the rate of vibration of the fundamental, that is, that it should be *do′*, the fundamental being considered as *do*; that the second overtone should vibrate three times as fast as the fundamental, that is, it should be *sol′*; that the third overtone should vibrate four times as fast, that is, it should be *do″*; that the fourth overtone should vibrate five times as fast, that is, it should be *mi″*, etc. In the case of the *closed* pipe, however, the first overtone should have a vibration rate three times that of the fundamental, that is, it should be *sol′*; the second overtone should vibrate five times as fast, that is, it should be *mi″*, etc. In other words, while an open pipe ought to give forth *all* the harmonics, both odd and even, a closed pipe ought to produce the *odd* harmonics, but be entirely incapable of producing the *even* ones.

Let the pipe of Fig. 369 be blown so as to produce the fundamental when the lower end is open. Then let the strength of the air blast be increased. The note will be found to spring to *do′*. By blowing still harder it will spring to *sol′*, and a still further increase will probably bring out *do″*. When the lower end is closed, however, the first overtone will be found to be *sol′* and the next one *mi″*, just as our theory demands.

425. Mechanism of emission of notes by pipes. A musical note is produced by blowing across the mouth of a pipe because the jet of air vibrates back and forth across the lip in a period which is determined wholly by the natural resonance period of the pipe. Thus suppose that the jet *a* (Fig. 370) first strikes just inside the edge or *lip* of the pipe. A condensational pulse starts down the pipe. When it returns to the mouth after reflection at the closed end it pushes the jet outside the lip. This starts a rarefaction down the pipe, which, after return from the lower end, pulls the jet in again. There are thus sent out into the room regularly timed puffs the period of which is controlled by the reflected pulses coming back from the lower end; that is, by the natural resonance period of the pipe.

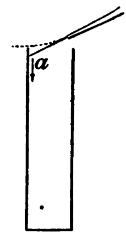


FIG. 370. Vibrating air jet

By blowing more violently it is possible to create, by virtue of the friction of the walls, so great and so sudden a compression in the mouth of the pipe that the jet is forced out over the edge before the return of the first reflected pulse. In this case no note will be produced unless the blowing is of just the right intensity to cause the jet to swing out in the period corresponding to an overtone. In this case the reflected pulses will return from the end at just the right intervals to keep the jet swinging in this period. This shows why a current of a particular intensity is required to start any particular overtone.

Another way of looking at the matter is to think of the pipe as being filled up with air until the pressure within it is

great enough to force the jet outside the lip, upon which a period of discharge follows, to be succeeded in turn by another period of charge. These periods are controlled by the length of the pipe and the violence of the blowing precisely as described above.

With open pipes the situation is in no way different save that the reflection of a condensation as a rarefaction at the lower end makes the natural period twice as high, since the pipe length is now one-half wave length instead of one-fourth wave length (see § 404).

426. Vibrating air-jet instruments. The mechanism of the production of musical tones by the ordinary organ pipe, the flute, the fife, the piccolo, and all whistles is essentially the same as in the case of the pipe of Fig. 370. In all these instruments an air jet is made to play across the edge of an opening in an air chamber, and the reflected pulses returning from the other end of the chamber cause it to vibrate back and forth, first into the chamber and then out again. In this way a series of regularly timed puffs of air is made to pass from the instrument to the ear of the observer precisely as in the case of the rotating disk of § 396. The air chamber may be either open or closed at the remote end. In the flute it is open, in whistles it is usually closed, and in organ pipes it may be either open or closed. Fig. 371 shows a cross section of two types of organ pipes. The jet of air from S vibrates across the lip L in obedience to the pressure exerted on it by waves reflected from O. Pipe organs are provided with a different pipe for each note, but the flute, piccolo, or fife is made to produce a whole series of notes, either by blowing overtones or by opening holes in the tube, an operation which is equivalent to cutting the tube off at the hole.

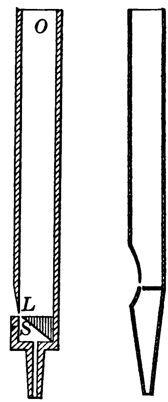


Fig. 371. Organ pipes

427. Vibrating reed instruments. In reed instruments the vibrating air jet is replaced by a vibrating reed or tongue which opens and closes, at absolutely regular intervals, an opening against which the performer is directing a current of air. In the clarinet, the oboe, the bassoon, etc., the reed is placed at the upper end of the tube (see l, Fig. 372), and the

theory of its opening and closing the orifice so as to admit successive puffs of air to the pipe is identical with the theory of the fluctuation of the air jet into and out of the organ pipe. For in these instruments the reed has practically no rigidity and consequently no natural period. Hence its vibrations are controlled entirely by the reflected pulses.

In other reed instruments, like the mouth organ, the common reed organ, or the accordion, it is the elasticity of the reed alone (see z, Fig. 373) which

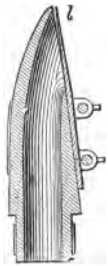


FIG. 372. Mouthpiece of a clarinet, showing the tongue l, which opens and closes the upper end of the pipe

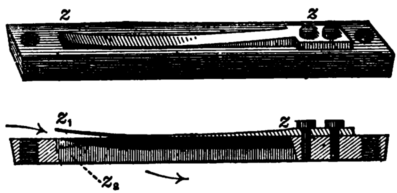


FIG. 373. The vibrating tongue of the mouth organ, accordion, etc.

controls the emission of pulses. In such instruments there is no necessity for air chambers. The arrows of Fig. 373 indicate the direction of the air current which is interrupted as the reed vibrates between the positions z_1 and z_2.

In still other reed instruments, like the reed pipes used in large organs (Fig. 374), the period of the pulses is controlled partly by the elasticity of the reed and partly by the return of the reflected waves; in other words, the natural period of the reed is more or less coerced by the period of the reflected pulses. Within certain limits, therefore, such instruments may be tuned by changing the length of the vibrating reed l without changing the length of the pipe. This is done by pushing the wire r up or down.

FIG. 374. The reed-organ pipe

428. Vibrating lip instruments. In instruments of the bugle and cornet type the vibrating reed is replaced by the vibrating lips of the musician, the period of their vibration being controlled, precisely as in the organ pipe or the clarinet, by the period of the returning pulses. In the bugle the pipe length is fixed, and hence the only notes of which such an instrument is

capable are the fundamental and about five overtones. In the cornet (Fig. 375) and in most forms of horns, valves *a, b, c,* worked by the fingers vary the length of the pipe, and hence such instruments can produce as many series of fundamentals and overtones as there are possible tube lengths. In the trombone the variation of pitch is accomplished by blowing overtones and by changing the tube length by a sliding portion.

FIG. 375. The cornet

429. The phonograph. In the original form of the phonograph the sound waves, collected by the cone, are carried to a thin metallic disk *C* (Fig. 376), exactly like a telephone diaphragm, which takes up very nearly the vibration form of the wave which strikes it. This vibration form is permanently impressed on the wax-coated cylinder *M* by means of a stylus *D* which is attached to the back of the disk. When the stylus is run a second time over the groove which it first made in the wax, it receives again and imparts to the disk the vibration form which first fell upon it.

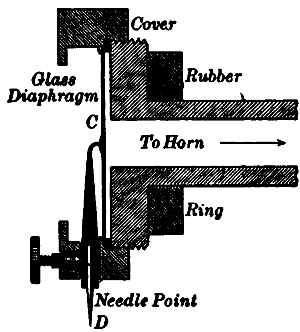


FIG. 377. Mechanism for forming gramophone records

In the most familiar of the modern forms of the phonograph (gramophone, etc.) the needle point *C*, instead of digging a groove in wax, vibrates back and forth (see Fig. 377) over a greased zinc disk. This wavy trace in the disk is etched out with chromic acid. Then a copper mold is made by the electrotyping process, and as many as a thousand facsimiles of the original wavy line are impressed on hard rubber disks by heat and pressure. When the needle is again run over the disk, it follows along the wavy groove and transmits to the diaphragm *C* vibrations exactly like those which originally fell upon it. Spoken words, vocal and orchestral music, are reproduced in pitch, loudness, and quality with wonderful exactness. This instrument is one of the many inventions of Thomas Edison.

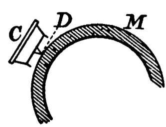


FIG. 376

QUESTIONS AND PROBLEMS

1. What will be the relative lengths of a series of organ pipes which produce the eight notes of a diatonic scale?

2. What must be the length of a closed organ pipe which produces the note *E*? (Take the speed of sound as 340 m. per second.)

3. Will the pitch of a pipe organ be the same in summer as on a cold day in winter? What could cause a difference?

4. What is the first overtone which can be produced in an open *G* organ pipe?

5. What is the first overtone which can be produced by a closed *C* organ pipe?

6. Explain how an instrument like the bugle, which has an air column of unchanging length, may be made to produce several notes of different pitch.

7. When water is poured into a deep bottle, why does the pitch of the sound rise as the bottle fills?

8. Why is the quality of an open organ pipe different from that of a closed organ pipe?

9. The velocity of sound in hydrogen is about four times as great as it is in air. If a *C* pipe is blown with hydrogen, what will be the pitch of the note emitted?

10. What effect will be produced on a phonograph record made with the instrument of Fig. 377 if the loudness of a note is increased? if the pitch is lowered an octave?

CHAPTER XVIII

NATURE AND PROPAGATION OF LIGHT

TRANSMISSION OF LIGHT

430. Speed of light. Before the year 1675 light was thought to pass instantaneously from the source to the observer. In that year, however, Olaf Roemer, a young Danish astronomer, made the following observations. He had observed accurately the instant at which one of Jupiter's satellites *M* (Fig. 378) passed into Jupiter's shadow when the earth was at *E*, and predicted, from the known mean time between such eclipses, the exact instant at which a given eclipse should occur six months later when the earth was at *E′*. It actually took place 16 minutes 36 seconds (996 seconds) later. He concluded that the 996 seconds' delay represented the time required for light to travel across the earth's orbit, a distance known to be about 180,000,000 miles. The most precise of modern determinations of the speed of light are made by laboratory methods. The generally accepted value, that of Michelson, of The University of Chicago, is 299,860 kilometers per second. It is sufficiently correct to remember it as 300,000 kilometers, or 186,000 miles.

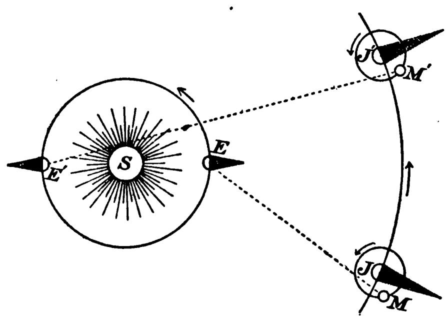


FIG. 378. Illustrating Roemer's determination of the velocity of light

Though this speed would carry light around the earth $7\frac{1}{2}$ times in a second, yet it is so small in comparison with interstellar distances that the light which is now reaching the earth from the nearest fixed star, Alpha Centauri, started 4.4 years ago. If an observer on the pole star had a telescope powerful enough to enable him to see events on the earth, he would not see the battle of Gettysburg (which occurred in July, 1863) until January, 1918.

Both Foucault in France and Michelson in America have measured directly the velocity of light in water and have found it to be only three fourths as great as in air. It will be shown later that in all transparent liquids and solids it is less than it is in air.

431. Reflection of light.* Let a beam of sunlight be admitted to a darkened room through a narrow slit. The straight path of the beam will be rendered visible by the brightly illumined dust particles suspended in the air. Let the beam fall on the surface of a mirror. Its direction will be seen to be sharply changed, as shown in Fig. 379. Let the mirror be held so that it is perpendicular to the beam. The beam will be seen to be reflected directly back on itself. Let the mirror be turned through an angle of 45°. The reflected beam will move through 90°.

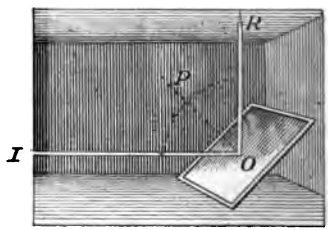


Fig. 379. Illustrating law of reflection of light

The experiment shows roughly, therefore, that the angle IOP, between the incident beam and the normal to the mirror, is equal to the angle POR between the reflected beam and the normal to the mirror. The first angle, IOP, is called the angle of incidence, and the second, POR, the angle of reflection. Hence the law of the reflection of light may be stated thus: *The angle of reflection is equal to the angle of incidence.*

* An exact laboratory experiment on the law of reflection should either precede or follow this discussion. See, for example, Experiment 42 of the authors' manual.

A. A. MICHELSON, CHICAGO

Distinguished for extraordinarily accurate experimental researches in light. First American scientist to receive the Nobel prize

LORD RAYLEIGH (ENGLAND)

Distinguished for the discovery of argon, for very accurate determinations in electricity and sound and for profound theoretical studies

HENRY A. ROWLAND, JOHNS HOPKINS

Distinguished for the invention of the concave grating and for epoch-making studies in heat and electricity

SIR WILLIAM CROOKES, LONDON

Distinguished for his pioneer work (1875) in the study and interpretation of cathode rays (pp. 418 and 423)

A GROUP OF MODERN PHYSICISTS

432. Diffusion of light. In the last experiment the light was reflected by a very smooth plane surface. Let the beam be now allowed to fall upon a rough surface like that of a sheet of unglazed white paper. No reflected beam will be seen; but, instead, the whole room will be brightened appreciably, so that the outline of objects before invisible may be plainly distinguished.

The beam has evidently been scattered in all directions by the innumerable little reflecting surfaces of which the surface of the paper is composed. The effect will be much more noticeable if the beam is allowed to fall alternately on a piece of dead black cloth and on the white paper.

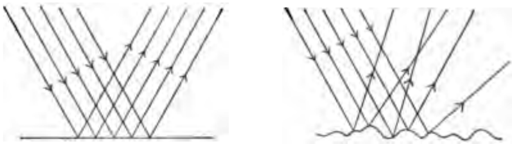


FIG. 380. Regular and irregular reflection

The light is largely absorbed by the cloth, while it is scattered or *diffusely reflected* by the paper. The difference between a smooth reflector and a rough one is illustrated in greatly magnified form in Fig. 380.

433. Visibility of nonluminous bodies. Every one is familiar with the fact that certain classes of bodies, such as the sun, a gas flame, etc., are self-luminous, that is, visible on their own account; while other bodies, like books, chairs, tables, etc., can be seen only when they are in the presence of luminous bodies. The above experiment shows how such nonluminous, diffusing bodies become visible in the presence of luminous bodies. For, since a diffusing surface scatters in all directions the light which falls upon it, each small element of such a surface is sending out light in a great many directions, in much the same way in which each point on a luminous surface is sending out light in all directions. Hence we always see the *outline* of a diffusing surface as we do that of an emitting surface, no matter where the eye is placed. On the other hand, when light comes to the eye from a polished reflecting surface, since the form of

the beam is wholly undisturbed by the reflection, we see the outline not of the mirror but rather of the source from which the light came to the mirror, whether this source is itself self-luminous or is only acting, because of its light-scattering power, like a self-luminous source. Points on the mirror which are not in line with this source can send no light whatever to the eye. Hence the mirror itself must be invisible. The fact that one often runs into a large mirror or plate-glass window is sufficient confirmation of the truth of the statement that neither a perfect reflector nor a perfectly transparent body is itself visible. All bodies other than self-luminous ones are visible only by the light which they diffuse. Black bodies send no light to the eye, but their outlines can be distinguished by the light which comes from the background. Any object *which can be seen*, therefore, may be regarded as itself sending rays to the eye; that is, it may be treated as a luminous body.

434. Refraction. Let a narrow beam of sunlight be allowed to fall on a thick glass plate with a polished front and whitened back* (Fig. 381). It will be seen to split into a reflected and a transmitted portion. The transmitted portion will be seen to be bent toward the perpendicular OP drawn into the glass. Upon emergence into the air it will be bent again, but this time away from the perpendicular $O'P'$ drawn into the air. Let the incident beam strike the surface at different angles. It will be seen that *the greater the angle of incidence the greater the bending.* At normal incidence there will be no bending at all.. If the upper and lower faces of the glass are parallel, the bending at the two faces will always be the same, so that the emergent beam is parallel to the incident beam.

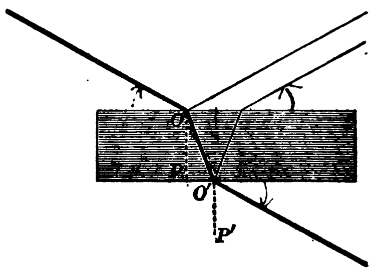


FIG. 381. Path of a ray through a medium bounded by parallel faces

* All of these experiments on reflection and refraction may be done effectively and conveniently by using disks of glass, like those used with the Hartl Optical Disk, through which the beam can be traced.

Similar experiments made with other substances have brought out the general law that *whenever light travels obliquely from one medium into another in which the speed is less, it is bent toward the perpendicular, and when it passes from one medium to another in which the speed is greater, it is bent away from the perpendicular drawn into the second medium.*

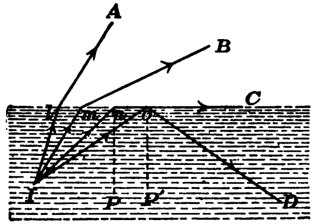


FIG. 382. Rays coming from a source I under water to the boundary between air and water at different angles of incidence

435. Total reflection; critical angle. Since rays emerging from a medium like water into one of less density like air are always bent *from* the perpendicular (see IlA, ImB, etc., Fig. 382), it is clear that if the angle of incidence on the under surface of the water is made larger and larger, a point must be reached at which the refracted ray is *parallel to the surface* (see InC, Fig. 382). It is interesting to inquire what will happen to a ray Io which strikes the surface at a still greater angle of incidence IoP'. It will not be unnatural to suppose that since the ray nC just grazed the surface, the ray Io will not be able to emerge at all. The following experiment will show that this is indeed the case.

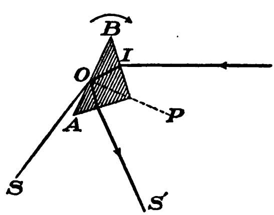


FIG. 383. Transmission and reflection of light at surface AB of a right-angled prism

Let a prism with three polished edges, a polished front, and a whitened back be held in the path of a narrow beam of sunlight, as shown in Fig. 383. If the angle of incidence IoP is small, the beam will divide at O into a reflected and a transmitted portion, the former going to S', the latter to S (neglect the color for the present). Let the prism be rotated slowly in the direction of the arrow. A point will be reached at which the transmitted beam disappears completely, while at the same time the spot at S' shows an appreciable increase in brightness. Since

the transmitted ray OS has totally disappeared, the whole of the light incident at O must be in the reflected beam. The angle of incidence IOP at which this occurs is called the *critical angle.* This angle for crown glass is 42.5°, for water 48.5°, for diamond 23.7°.

We learn then that *when a ray of light traveling in any medium meets another in which the speed is greater, it is totally reflected if the angle of incidence is greater than a certain angle called the critical angle.*

QUESTIONS AND PROBLEMS

1. Sirius, the brightest star, is about 52,000,000,000,000 miles away. If it were suddenly annihilated, how long would it shine on for us?

2. In Fig. 384 the portion *acdb* of the shadow is called the umbra, the portions *aec* and *bdf* the penumbra. What kind of a source has no penumbra?

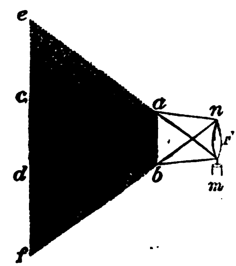


FIG. 384. Shadow from a broad source

3. If the opaque body in Fig. 384 is moved nearer to the screen *ef*, how does the penumbra change?

4. The sun is much larger than the earth. Draw a diagram showing the shape of the earth's umbra and penumbra.

5. The diameter of the moon is 2000 miles, that of the sun 860,000 miles, and the sun is 93,000,000 miles away. What is the length of the moon's umbra?

6. Will it ever be possible for the moon to totally eclipse the sun from the whole of the earth's surface at once?

7. If the distance from the center of the earth to the center of the moon were exactly equal to the length of the moon's umbra, over how wide a strip on the earth's surface would the sun be totally eclipsed at any one time?

8. Why is a room with white walls much lighter than a similar room with black walls?

9. If the word "white" be painted with white paint (or whiting moistened with alcohol) across the face of a mirror and held in the path of a beam of sunlight in a darkened room, in the middle of the spot on the wall which receives the reflected beam the word "white" will appear in black letters. Explain.

10. Look at the reflected image of an electric-light filament in a piece of red glass. Why are there two images, one red and one white?

11. The earth reflects sixteen times as much light to the moon as the moon does to the earth. Trace from the sun to the eye of the observer the light by which he is able to see the dark part of the new moon. Why can we not see the dark part of a third-quarter moon?

12. If a penny is placed in the bottom of a vessel in such a position that the edge just hides it from view (Fig. 385), it will become visible as soon as water is poured into the vessel. Explain.

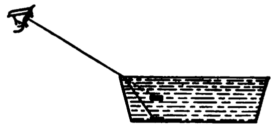


FIG. 385

13. A stick held in water appears bent, as shown in Fig. 386. Explain.

14. Should a man who wishes to spear a fish aim high or low?

15. A glass prism placed in the position shown in Fig. 387 is the most perfect reflector known. Why is it better than an ordinary mirror?

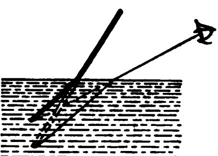


FIG. 386

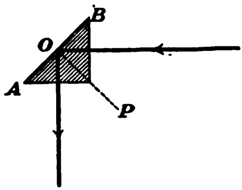


FIG. 387

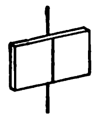


FIG. 388

16. What is the principal reflecting medium in an ordinary mirror?

17. Explain why a straight wire seen obliquely through a piece of glass appears broken, as in Fig. 388.

18. In what direction must a fish look in order to see the setting sun? (See Fig. 389.)

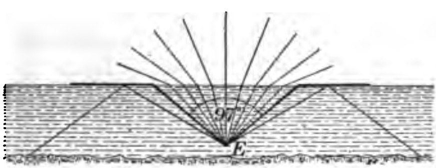


FIG. 389. To an eye under water all external objects appear to lie within a cone whose angle is 97°

FIG. 390. Luxfer prism glass

19. Fig. 390 represents a section of a plate of Luxfer prism glass. Explain why glass of this sort is so much more efficient than ordinary window glass in illuminating the rears of dark stores on the ground floor in narrow streets.

The Nature of Light

436. The corpuscular theory of light. All of the properties of light which have so far been discussed are perhaps most easily accounted for on the hypothesis that light consists of streams of very minute particles, or *corpuscles*, projected with the enormous velocity of 300,000 kilometers per second from all luminous bodies. The facts of straight-line propagation and reflection are exactly as we should expect them to be if this were the nature of light. The facts of refraction can also be accounted for, although somewhat less simply, on this hypothesis. As a matter of fact, this theory of the nature of light, known as the *corpuscular theory*, was the one most generally accepted up to about 1800.

437. The wave theory of light. A rival hypothesis, which was first completely formulated by the great Dutch physicist Huygens (1629–1695), regarded light, like sound, as a *form of wave motion*. This hypothesis met at the start with two very serious difficulties. In the first place, light, unlike sound, not only travels with perfect readiness through the best vacuum which can be obtained with an air pump, but it travels without any apparent difficulty through the great interstellar spaces which are probably infinitely better vacua than can be obtained by artificial means. If, therefore, light is a wave motion, it must be a wave motion of some medium which fills all space and yet which does not hinder the motion of the stars and planets. Huygens assumed such a medium to exist, and called it the *ether*.

The second difficulty in the way of the wave theory of light was that it seemed to fail to account for the fact of straight-line propagation. Sound waves, water waves, and all other forms of waves with which we are most familiar bend readily around corners, while light apparently does not. It was this difficulty chiefly which led many of the most famous of the

CHRISTIAN HUYGENS (1629–1695)

Great Dutch physicist, mathematician, and astronomer; discovered the rings of Saturn; made important improvements in the telescope; invented the pendulum clock (1656); developed with marvelous insight the wave theory of light; discovered in 1690 the "polarization" of light. (The fact of double refraction was discovered by Erasmus Bartholinus in 1669, but Huygens first noticed the polarization of the doubly refracted beams, and offered an explanation of double refraction from the standpoint of the wave theory)

early philosophers, including the great Sir Isaac Newton, to reject the wave theory and to support the projected-particle theory. Within the last hundred years, however, this difficulty has been completely removed, and in addition other properties of light have been discovered for which the wave theory offers the only satisfactory explanation. The most important of these properties will be treated in the next paragraph.

438. Interference of light. Let two pieces of plate glass about half an inch wide and four or five inches long be separated at one end by a thin sheet of paper in the manner shown in Fig. 391, while the other end is clamped or held firmly together, so that a very thin wedge of air exists between the plates. Let a piece of asbestos or blotting paper be soaked in a solution of common salt (sodium chloride) and placed over the tube of a Bunsen burner so as to touch the flame in the manner shown. The flame will be colored a bright yellow by the sodium in the salt. When the eye looks at the reflection of the flame from the glass surfaces, a series of fine black and yellow lines will be seen to cross the plate.

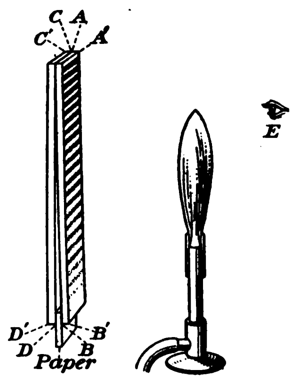


FIG. 391. Interference of light waves

The wave theory offers the following explanation of these effects. Each point of the flame sends out light waves which travel to the glass plate and are in part reflected and in part transmitted at all the surfaces of the glass, that is, at $A'B'$, at AB, at CD, and at $C'D'$ (Fig. 391). We will consider, however, only those reflections which take place at the two faces of the air wedge, namely, at AB and CD. Let Fig. 392 represent a greatly magnified section of these two surfaces. Let the wavy line *as* represent a light wave reflected from the surface AB at the point a, and returning thence to the eye. Let the dotted wavy line *ir* represent a light wave reflected from the surface CD

at the point i, and returning thence to the eye. Similarly, let all the continuous wavy lines of the figure represent light waves reflected from different points on AB to the eye, and let all the dotted wavy lines represent waves reflected from corresponding points on CD to the eye. Now, in precisely the same way in which two trains of sound waves from two tuning forks were found, in the experiment illustrating beats (see § 407), to interfere with each other so as to produce silence whenever the two waves corresponded to motions of the air particles in opposite directions, so in this experiment the two sets of light waves from AB and CD interfere with each other so as to produce darkness wherever these two waves correspond to motions of the light-transmitting medium in opposite directions.

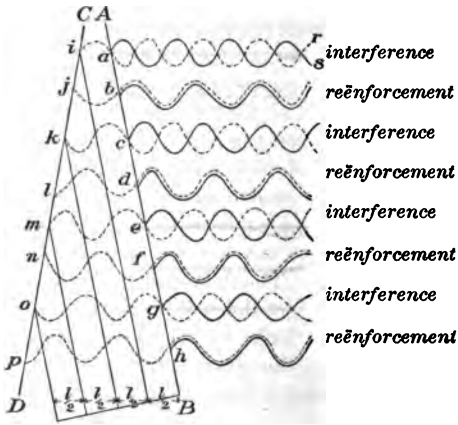


FIG. 392. Explanation of formation of dark and light bands by interference of light waves

The dark bands, then, of our experiment are simply the places at which the two beams reflected from the two surfaces of the air film neutralize or destroy each other, while the light bands correspond to the places at which the two beams reënforce each other and thus produce illumination of double intensity.

Now the condition for destructive interference is obviously that the wave which passes through the film and is reflected from any point on CD, such, for example, as i, shall return to AB, after its double passage through the film, in the condition

or *phase* of vibration which is exactly opposite to that of the wave which is being reflected at that instant from the corresponding point on *AB*, namely from *a*. If this condition occurs at *a*, it must occur again at a point *c* enough farther down the wedge to make the double path through the film just one wave length more, and again at *e* where the double path is two wave lengths more, etc. In other words, the dark bands ought to follow one another at equal intervals down the wedge, precisely as we observed them to do. Between each two successive points of interference there must, of course, be a point like *b*, *d*, *f*, or *h*, at which the waves reflected from the two surfaces unite in *like* phases and therefore reënforce each other. This phenomenon of the interference of light is met with in many different forms, and in every case the wave theory furnishes at once a wholly satisfactory explanation of the observed effects; while the corpuscular theory, on the other hand, is unable to account for any of these interference effects without the most fantastic and violent assumptions. Hence *the corpuscular theory is now practically abandoned, and light is universally regarded by physicists as a form of wave motion.*

439. The ether. We have already indicated that if the wave theory is to be accepted, we must conceive, with Huygens, that all space is filled with a medium, called the *ether*, in which the waves can travel. This medium cannot be like any of the ordinary forms of matter; for if any of these forms existed in interplanetary space, the planets and the other heavenly bodies would certainly be retarded in their motions. As a matter of fact, in all the hundreds of years during which astronomers have been making accurate observations of the motions of heavenly bodies no such retardation has ever been observed. The medium which transmits light waves must therefore have a density which is infinitely small even in comparison with that of our lightest gases. The existence of such a medium is now universally assumed by physicists.

Further, in order to account for the transmission of light through transparent bodies, it is necessary to assume that the ether penetrates not only all interstellar spaces but all intermolecular spaces as well.

440. Wave length of yellow light. Although light, like sound, is a form of wave motion, light waves differ from sound waves in several important respects. In the first place, an analysis of the preceding experiment, which seems to establish so conclusively the correctness of the wave theory, shows that the wave length of the light waves used in that experiment is extremely minute in comparison with that of ordinary sound waves. Thus, suppose the air wedge was 10 centimeters long, and that the upper edges of the glass strips were in contact, while the lower edges were held apart by a sheet of paper .03 millimeter thick. Suppose, further, that the black bands were found to be 1 millimeter apart. Now, since the air wedge is 100 millimeters long, the difference in its thickness at two points such as *a* and *c* (see Fig. 392), 1 millimeter apart, must be .01 of its thickness at the base, that is, $\frac{1}{100}$ of .03 millimeter, or .0003 millimeter. Since it is the *double* path through the air wedge at *c* which must be exactly one wave length longer than the double path at *a* (see § 438), we see that the difference in the thicknesses of the wedge at *c* and at *a* must be $\frac{1}{2}$ wave length. Hence the wave length of yellow light must be $2 \times .0003 = .0006$ millimeter. Careful measurements by better methods give .000589 millimeter as the correct value.

The *number of vibrations per second* made by the little particles which send out the light waves may be found, as in the case of sound, by dividing the velocity by the wave length. Since the velocity of light is 30,000,000,000 centimeters per second and the wave length is .00006 centimeter, the number of vibrations per second of the particles which emit yellow light has the enormous value 500,000,000,000,000.

441. Wave theory explanation of refraction. Let one look vertically down upon a glass or tall jar full of water and place his finger on the side of the glass at the point at which the bottom appears to be, as seen through the water (Fig. 393). In every case it will be found that the point touched by the finger will be about one fourth of the depth of the water above the bottom.

According to the wave theory this effect is due to the fact that the speed of light is less in water than in air. Thus consider a wave which originates at any point P (Fig. 394)

beneath a surface of water and spreads from that point with equal speed in all directions. At the instant at which the front of this wave first touches the surface *mn* it will, of course, be of spherical form, having P as its center. Let *aob* be a section of this sphere. An instant later, if the speed had not changed in passing into air, the wave would have still had P as its center, and its form would have coincided with the dotted line co_1d, so drawn that ac, oo_1, and bd are all equal. But if the velocity in air is *greater* than in water, then at the instant considered the disturbance will have reached some point o_2 instead of o_1, and hence the emerging wave will actually have the form of the heavy line co_2d instead of the dotted line co_1d. Now this wave co_2d is more curved than the old wave *aob*, and hence it has its center at some point P' above P. In other words, the wave has bulged upward in passing from water into air. Therefore, when a section of this wave enters the eye at E, the wave appears to originate not at P but at P', for the light actually comes to the eye from P' as a center rather than from P. We conclude, therefore, *that if light travels slower in water than in air, all objects beneath the surface of water ought to appear nearer to the eye than they actually are.* This is precisely what we found to be the case in our experiment.

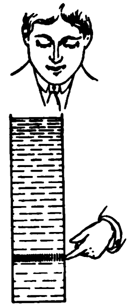


FIG. 393. Apparent elevation of the bottom of a body of water

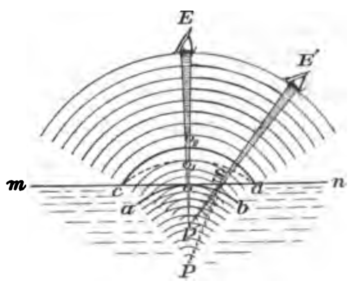


FIG. 394. Representing a wave emerging from water into air

Furthermore, since when the eye is in any position other than E, for example E', the light travels to it over the broken path PSE', the construction shows that light is always bent

away from the perpendicular when it passes obliquely into a medium in which the speed is greater. If it had passed into a medium of less speed, the point P would evidently have appeared depressed below its natural position, and hence the oblique rays would have appeared to be bent *toward* the perpendicular, as we found in § 434 to be the case.

442. Ratio of the speeds of light in air and water. The last experiment not only indicates qualitatively that the speed of light is greater in air than in water, but it furnishes a simple means of determining the precise ratio of the two speeds. Thus in Fig. 394 the line oo_2 represents just how far the wave travels in air while it is traveling the distance ac $(= oo_1)$ in water. Hence $\frac{oo_2}{oo_1}$ is the ratio of the speeds of light in air and in water. Now it may be shown that when the arc cod is small, a condition which is in general realized in experimental work, $\frac{oo_2}{oo_1}$ is equal to $\frac{oP}{oP'}$.* But in our experiment we found that the bottom was raised one fourth of the depth; that is, that $\frac{oP}{oP'} = \frac{4}{3}$. We conclude, therefore, that light travels three fourths as fast in water as in air.

The fact that the value of this ratio, as determined by this indirect method, is exactly the same as that found by Foucault and Michelson, by direct measurement (§ 430), furnishes one of the strongest proofs of the correctness of the wave theory.

*For as the wave goes through the surface at o its curvature changes from that of an arc which has its center at P to that of an arc which has its center at P'. Now the curvatures of these two arcs are by definition the reciprocals of their radii; that is, they are $\frac{1}{oP}$ and $\frac{1}{oP'}$, respectively. The naturalness of this definition may be seen from the fact that if at a point like o one arc is curving twice as fast as another, it is evident that this means merely that its center is but half as far away; that is, the curvatures of the two arcs are inversely proportional to their radii. But so long as the arcs co_1d and co_2d are very short, their curvatures are also directly proportional to oo_1 and oo_2; that is, they are proportional to the amounts by which these two curved lines depart from the straight line cod.

Hence
$$oo_1/oo_2 = oP'/oP.$$

443. Index of refraction. The *ratio of the speed of light in air to its speed in any medium is called the index of refraction of that medium.* It is evident that the method employed in the last paragraph for determining the index of refraction of water can be easily applied to any transparent medium whether liquid or solid. The refractive indices of some of the commoner substances are as follows:

Water	1.33	Crown glass	1.53
Alcohol	1.36	Flint glass	1.67
Turpentine	1.47	Diamond	2.47

444. Light waves are transverse. Thus far we have discovered but two differences between light waves and sound waves; namely, the former are disturbances in the ether and are of very short wave length, while the latter are disturbances in ordinary matter and are of relatively great wave length. There exists, however, a further radical difference which follows from a capital discovery made by Huygens in the year 1690. It is this: While sound waves consist, as we have already seen, of *longitudinal* vibrations of the particles of the transmitting medium, that is, vibrations back and forth in the line of propagation of the wave, light waves are like the water waves of Fig. 349, p. 319, in that they consist of *transverse* vibrations, that is, vibrations of the medium at right angles to the direction of the line of propagation.

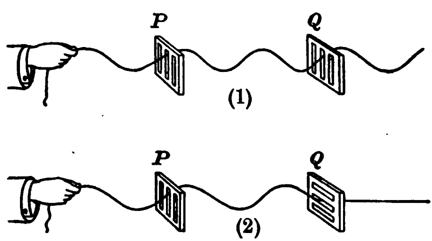


FIG. 395. Tranverse waves passing through slits

In order to appreciate the difference between the behavior of waves of these two types under certain conditions, conceive of transverse waves in a rope to be made to pass through two gratings in succession, as in Fig. 395. So long as the slits in

both gratings are parallel to the plane of vibration of the hand, as in Fig. 395, (1), the waves can pass through them with perfect ease; but if the slits in the first grating P are parallel to the direction of vibration, while those of the second grating Q are turned at right angles to this direction, as in Fig. 395, (2), it is evident that the waves will pass readily through P, but will be stopped completely by Q, as shown in the figure. In other words, these gratings P and Q will let through only such vibrations as are parallel to the direction of their slits.

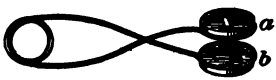


FIG. 396. Tourmaline tongs

If, on the other hand, a longitudinal instead of a transverse wave — such, for example, as a sound wave — had approached such a grating, it would have been as much transmitted in one position of the grating as in another, since a *to-and-fro* motion of the particles can evidently pass through the slits with exactly the same ease, no matter how they are turned.

Now two crystals of tourmaline are found to behave with respect to light waves precisely as the two gratings behave with respect to the waves on the rope.

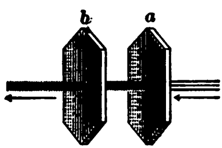


FIG. 397. Light passing through tourmaline crystals

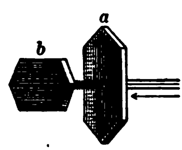


FIG. 398. Light cut off by crossed tourmaline crystals

Let one such crystal a (Fig. 396) be held in front of a small hole in a screen through which a beam of sunlight is passing to a neighboring wall; or, if the sun is not shining, simply let the crystal be held between the eye and a source of light. The light will be readily transmitted, although somewhat diminished in intensity. Then let a second crystal b be held in line with the first. The light will still be transmitted, *provided the axes of the crystals are parallel*, as shown in Fig. 397. When, however, one of the crystals is rotated in its ring through 90° (Fig. 398), the light *is cut off*. This shows that a crystal of tourmaline is capable of transmitting only light which is vibrating in one particular plane.

From this experiment, therefore, we are forced to conclude that *light waves are transverse rather than longitudinal vibrations.*

The above experiment illustrates what is technically known as the *polarization of light*, and the beam which, after passage through *a*, is unable to pass through *b* if the axes of *a* and *b* are crossed, is known as a *polarized beam.* It is, then, the phenomenon of the *polarization of light* upon which we base the conclusion that light waves are transverse.

445. Intensity of light. Let four candles be set as close together as possible in such a position *B* as to cast upon a white screen *C*, placed in a well-darkened room, a shadow of an opaque object *O* (Fig. 399). Let one single candle be placed in a position *A* such that it will cast another shadow of *O* upon the screen. Since light from *A* falls on the shadow cast by *B*, and light from *B* falls on the shadow cast by *A*, it is clear that the two shadows will appear equally dark only when light of equal intensity falls on each; that is, when *A* and *B* produce equal illumination upon the screen. Let the positions of *A* and *B* be shifted until this condition is fulfilled. Then let the distances from *B* to *C* and from *A* to *C* be measured. If all five candles are burning with flames of the same size, the first distance will be found to be just twice as great as the second. Hence the illumination produced upon the screen by each one of the candles at *B* is but one fourth as great as that produced on the screen by one candle at *A*, one half as far away.

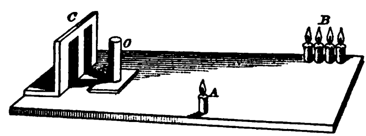


FIG. 399. Rumford's photometer

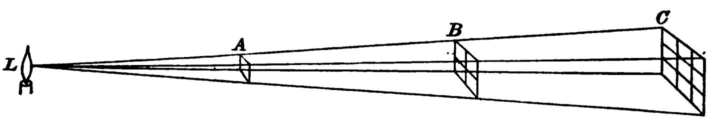


FIG. 400. Proof of law of inverse squares

The above is the direct experimental proof that *the intensity of light varies inversely as the square of the distance from the source.*

The theoretical proof of the law is furnished at once by Fig. 400, for since all the light which falls from the candle *L*

on A is spread over four times as large an area when it reaches B, twice as far away, and over nine times as large an area when it reaches C, three times as far away, obviously the *intensities* at B and at C can be but one fourth and one ninth as great as at A.

The above method of comparing experimentally the intensities of two lights was first used by Count Rumford. The arrangement is therefore called the *Rumford photometer* (light measurer).

446. Candle power. The last experiment furnishes a method of comparing the *light-emitting powers* of various sources of light. For example, suppose that the four candles at B are replaced by a gas flame, and that for the condition of equal illumination upon the screen the two distances BC and AC are the same as above, namely 2 to 1. We should then know that the gas flame, which is able to produce the same illumination at a distance of two feet as a candle at a distance of one foot, has a light-emitting power equal to four candles. In general, then, the candle power of any two sources which produce equal illumination on a given screen are *directly proportional to the squares of the distances of the sources from the screen.*

It is customary to express the intensities of all sources of light in terms of candle power, one candle power being defined as the amount of light emitted by a sperm candle $\frac{7}{8}$ inch in diameter and burning 120 grains (7.776 grams) per hour. The candle power of an ordinary gas flame burning 5 cubic feet per hour is from 16 to 25, depending on the quality of the gas. A Welsbach lamp burning 3 cubic feet per hour has a candle power of from 50 to 100. Most incandescent electric lamps which are used for domestic purposes are of 16 candle power. The average arc light has a candle power of about 500, although when measured in the direction of greatest intensity the illuminating power may be as great as that of 1000 or 1200 candles.

447. Bunsen's photometer. Let a drop of oil or melted paraffin be placed in the middle of a sheet of unglazed white paper to render it translucent. Let the paper be held near a window and the side *away from* the window observed. The oiled spot will appear *lighter* than the remainder of the paper. Then let the paper be held so that the side nearest the window may be seen. The oiled spot will appear *darker* than the rest of the paper. We learn, therefore, that *when the paper is viewed from the side of greater illumination, the oiled spot appears dark;* but *when it is viewed from the side of lesser illumination, the spot appears light.* If, then, the two sides of the paper are equally illuminated, the spot ought to be of the same brightness when viewed from either side. Let the room be darkened and the oiled paper placed between two gas flames, two electric lights, or any two equal sources of light. It will be observed that when the paper is held closer to one than the other, the spot will appear dark when viewed from the side next the closer light; but if it is then moved until it is nearer the other source, the spot will change from dark to light when viewed always from the same side. It is always possible to find some position for the oiled paper at which the spot either disappears altogether or at least appears the same when viewed from either side. This is the position at which the illuminations from the two sources are equal. Hence, to find the candle power of any unknown source it is only necessary to set up a candle on one side and the unknown source on the other, as in Fig. 401, and to move the spot A to the position of equal illumination. The candle power of the unknown source at C will then be the square of the distance from C to A, divided by the square of the distance from B to A.

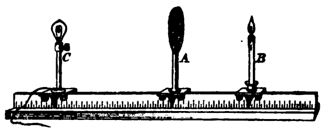


FIG. 401. Bunsen's photometer

This arrangement is known as the *Bunsen photometer.*

QUESTIONS AND PROBLEMS

1. What is the speed of light in water? (Index of refraction is 1.33.)

2. Will a beam of light going from water into flint glass be bent toward or away from the perpendicular drawn into the glass?

3. If the wedge-shaped film of air in Fig. 391 were replaced by water, would the distance between successive fringes be greater or less than in air? Why?

†

4. When light passes obliquely from air into carbon bisulphide it is bent more than when it passes from air into water at the same angle. Is the speed of light in carbon bisulphide greater or less than in water?

5. Does a man above the surface of water appear to a fish below it farther from or nearer to the surface than he actually is?

6. How far from a screen must a 4-candle-power light be placed to give the same illumination as a 16-candle-power electric light 3 m. away?

7. A Bunsen photometer placed between an arc light and an incandescent light of 32 candle power is equally illuminated on both sides when it is 10 ft. from the incandescent light and 36 ft. from the arc light. What is the candle power of the arc?

8. A 5-candle-power and a 30-candle-power source of light are 2 m. apart. Where must the oiled disk of a Bunsen photometer be placed in order to be equally illuminated on the two sides by them?

9. If the sun were at the distance of the moon from the earth, instead of at its present distance, how much stronger would sunlight be than at present? The moon is 240,000 mi. and the sun 93,000,000 mi. from the earth.

10. If a gas flame is 300 cm. from the screen of a Rumford photometer, and a standard candle 50 cm. away gives a shadow of equal intensity, what is the candle power of the gas flame?

CHAPTER XIX

IMAGE FORMATION

Images formed by Lenses

448. Focal length of a convex lens. Let a convex lens be held in the path of a beam of sunlight which enters a darkened room, where it is made plainly visible by means of chalk dust or smoke. The beam will be found to converge to a focus F, as shown in Fig. 402.

The explanation is as follows: The waves from the sun or any distant object are without any appreciable curvature when they strike the lens, that is, they are so called *plane waves* (see Fig. 402). Since the speed of light is less in glass than in air, the central portion of these waves is retarded more than the outer portions in passing through the lens. Hence on emerging from the lens the waves are concave instead of plane, and close in to a center or *focus* at F.

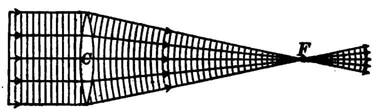


Fig. 402. Principal focus F and focal length CF of a convex lens

A second way of looking at the phenomenon is to think of the "rays" which strike the lens as being bent by it, in accordance with the laws given in § 434, so that they all pass through the point F.

The distance CF from the center of the lens to the point at which incident plane waves (parallel rays) are brought to a focus is called the *focal length* (f) of the lens.

The line through the middle C (the *optical center*) of the lens, perpendicular to its faces, is called the *principal axis*.

The point F at which rays parallel to the principal axis are brought to a focus is called the *principal focus*.

The plane $F'FF''$ (Fig. 403) in which plane waves (parallel rays) coming to the lens from slightly different directions, as from the top and bottom of a distant house, all have their foci F', F'', etc. is called the *focal plane* of the lens.

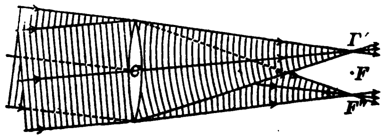


FIG. 403. Focal plane of a convex lens

Since the curvature of any arc is defined as the reciprocal of its radius (see footnote, p. 364), *the curvature which a lens impresses on an incident plane wave is equal to* $\frac{1}{f}$. Moreover, no matter what the curvature of an incident wave may be, *the lens will always change the curvature by the same amount,* $\frac{1}{f}$.

Let the focal length of a convex lens be accurately determined by measuring the distance from the middle of the lens to the image of a distant house.

449. Conjugate foci. If a point source of light is placed at F (Fig. 402), it is obvious that the light which goes through the lens must exactly retrace its former path; that is, its waves

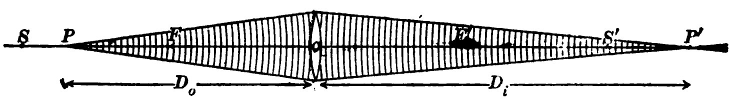


FIG. 404. Conjugate foci

will be rendered plane or its rays parallel by the lens. But if the point source is at a distance D_o greater than f (Fig. 404), then the waves upon striking the lens have a curvature $\frac{1}{D_o}$ (since the curvature of an arc is defined as the reciprocal of its radius), which is less than their former curvature, $\frac{1}{f}$. Since the lens was able to subtract all the curvature from waves coming

from F and render them plane, by subtracting the same curvature from the flatter waves from P it must render them concave; that is, the rays after passing through the lens are converging and intersect at P'. If the source is placed at P', obviously the rays will meet at P. The points P and P' are called *conjugate foci.*

450. Formula for conjugate foci; secondary foci. The relation between the distances of P and P' from the lens is obtained at once from the consideration that the curvature which the wave has after it gets through the lens, namely $\frac{1}{D_i}$, is the difference between the curvature impressed by the lens, namely $\frac{1}{f}$, and that which the wave had when it reached the lens, namely $\frac{1}{D_o}$; that is,

$$\frac{1}{D_i}=\frac{1}{f}-\frac{1}{D_o}, \quad \text{or} \quad \frac{1}{D_o}+\frac{1}{D_i}=\frac{1}{f}. \qquad (1)$$

If $D_o = D_i$, then the last equation shows that both D_o and D_i are equal to $2f$.

The two conjugate foci S and S' which are at equal distances from the lens are called the *secondary foci*, and their distance from the lens is twice the focal distance.

451. Images of objects. Let a candle or electric-light bulb be placed between the principal focus F and the secondary focus S at PQ (Fig. 405), and let a screen be placed at $P'Q'$. An enlarged inverted image will be seen upon the screen.

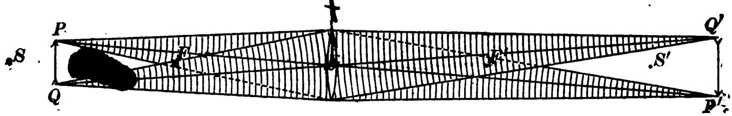


FIG. 405. Formation of a real image by a lens

This image is formed as follows: All the light which strikes the lens from the point P is brought together at a point P'. The location of this image P' must be on a straight

line drawn from P through C; for any ray passing through C will remain parallel to its original direction, since the portions of the lens through which it enters and leaves may be regarded as small parallel planes (see § 434). The image $P'Q'$ is therefore always formed between the lines drawn from P and Q through C. If the focal length f and the distance of the object D_o are known, the distance of the image D_i may be obtained easily from formula (1).

The position of the image may also be found graphically as follows: Of the cone of rays passing from P to the lens, that

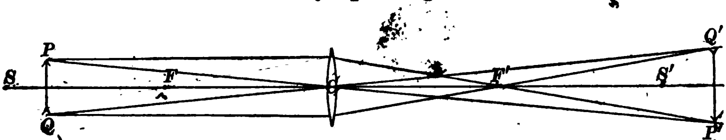


FIG. 406. Ray method of constructing image

ray which is parallel to the principal axis must, by § 448, pass through the principal focus F'. The intersection of this line with the straight line through C locates the image P' (see Fig. 406). Q', the image of Q, is located similarly.

452. Size of image. Since the image and object are always between the intersecting straight lines PP' and QQ', the similar triangles PCQ and $P'CQ'$ show that

$$\frac{PQ}{P'Q'} = \frac{D_o}{D_i}; \qquad (2)$$

that is, $\dfrac{\text{Length of object}}{\text{Length of image}} = \dfrac{\text{Distance of object from lens}}{\text{Distance of image from lens}}.$

It may be seen from Fig. 406, as well as from formulas (1) and (2), that

1. When the object is at S the image is at S', and image and object are of the same size.

2. As the object moves out from S to a great distance the image moves from S' up to F', becoming smaller and smaller.

3. As the object moves from S up to F the image moves out to a very great distance to the right, becoming larger and larger.

4. When the object is at F the emerging waves are plane (the emerging rays are parallel), and no real image is formed.

453. Virtual image. We have seen that when the object is at F the waves after passing through the lens are plane. If, then, the object is nearer to the lens than F, the emerging waves, although reduced in curvature by an amount $\frac{1}{f}$, will still be convex, and if received by an eye at E, will appear to come from a point P' (Fig. 407). Since, however, there is actually no source of light at P', this sort of image is called a *virtual image.* Such an image cannot be projected upon a screen as a real image can, but must be observed by an eye.

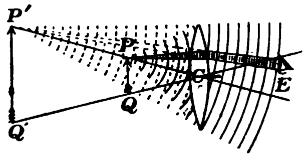


FIG. 407. Virtual image formed by a convex lens

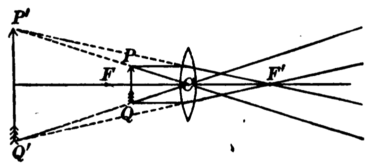


FIG. 408. Ray method of locating virtual image in convex lens

The graphical location of a virtual image may be accomplished precisely as in the case of a real image (§ 451). It will be seen that in this case (Figs. 407 and 408) *the image is enlarged and erect.*

454. Image in concave lens. When a plane wave strikes a concave lens it must emerge as a divergent wave, since the middle of the wave is retarded by the glass less than the edges (Fig. 409). The point F from which plane waves appear to come after passing through such a lens is the principal focus of the lens. For the same reason as in the case of the convex lens the centers of the transmitted waves from P and Q

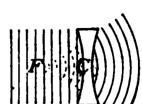


FIG. 409. Virtual focus of a concave lens

(Fig. 410), that is, the images P' and Q', must lie upon the lines PC and QC, and since the curvature is increased by the lens, they must lie closer to the lens than P and Q. Fig. 410

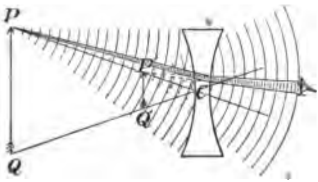


FIG. 410. Image in a concave lens

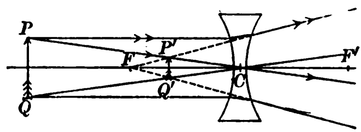


FIG. 411. Ray method of locating image in concave lens

shows the way in which such a lens forms an image. This image is always *virtual, erect, and diminished.* The graphical method of locating the image is shown in Fig. 411.

IMAGES IN MIRRORS

455. Image of a point in a plane mirror. We are all familiar with the fact that to an eye at E (Fig. 412), looking into a plane mirror mn, a pencil point at P appears to be at some point P' behind the mirror. We are able in the laboratory to find experimentally the exact location of this image P' with respect to P and the mirror, but we may also obtain this location from theory as follows: Consider a light wave which originates in the point P (Fig. 412) and spreads in all directions. Let aob be a section of the wave at the instant at which it reaches the reflecting surface mn. An instant later, if there were no reflecting surface, the wave would have reached the position of the dotted line co_1d.

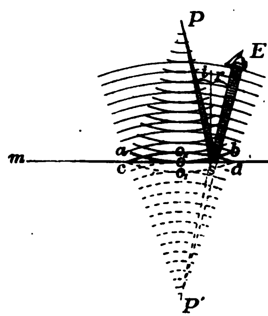


FIG. 412. Wave reflected from a plane surface

Since, however, reflection took place at mn, and since the reflected wave is propagated backward with exactly the same velocity with which the original wave would have been propagated forward, at the proper instant, the reflected wave must have reached the position of the line co_2d, so drawn that oo_1 is equal to oo_2. Now the wave co_2d has its center at some point P', and it will be seen that P' must lie just as far below mn as P lies above it, for co_1d and co_2d are arcs of equal circles having the common chord cd. For the same reason, also, P' must lie on the perpendicular drawn from P through mn. When, then, a section of this reflected wave co_2d enters the eye at E, the wave appears to have originated at P' and not at P, for the light actually comes to the eye from P' as a center rather than from P. Hence P' is the image of P. We learn, therefore, that *the image of a point in a plane mirror lies on the perpendicular drawn from the point to the mirror, and is as far back of the mirror as the point is in front of it.*

456. Construction of image of object in a plane mirror. The image of an object in a plane mirror (Fig. 413) may be located by applying the law proved above for each of its points, that is, *by drawing from each point a perpendicular to the reflecting surface, and extending it an equal distance on the other side.*

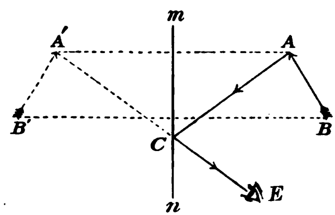


FIG. 413. Construction of image of object in plane mirror.

To find the path of the rays which come to an eye placed at E from any point such as A of the object, we have only to draw a line from the image A' of this point to the eye and connect the point of intersection of this line with the mirror, namely C, with the original point A. ACE is then the path of the ray.

Let a candle (Fig. 414) be placed exactly as far in front of a pane of window glass as a bottle full of water is behind it, both objects being on the same perpendicular drawn through the glass. The candle will appear to be burning inside the water. This explains a large class of familiar optical illusions, such as "the figure suspended in mid-air," the "bust of a person without a trunk," the "stage ghost," etc. In the last case the illusion is produced by causing the audience to look at the actors obliquely through a sheet of very clear plate glass, the edges of which are concealed by draperies. Images of strongly illuminated figures at one side then appear to the audience to be in the midst of the actors.

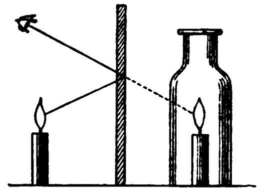


FIG. 414. Position of image in a plane mirror

457. Focal length of a curved mirror half its radius of curvature. The effect of a convex mirror on plane waves incident upon it is shown in Fig. 415. The wave which would at a given instant have been at co_1d is at co_2d where $oo_1 = oo_2$. The center F from which the waves appear to come to the eye E is the *focus* of the mirror.

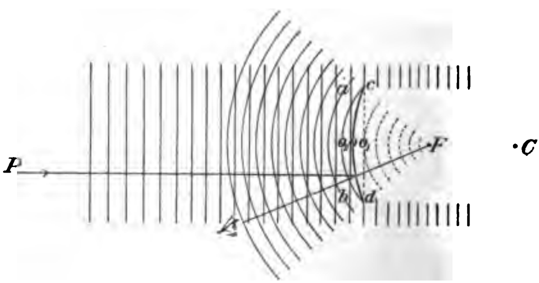


FIG. 415. Reflection of a plane wave from a convex mirror

Now so long as the arc cod is small its curvature may, without appreciable error, be measured by o_1o (see footnote, p. 364); that is, by the departure of the curved line cod from the straight line co_1d. Since o_1o was made equal to oo_2, we have $o_1o_2 = 2\,o_1o$; that is, the curvature $\frac{1}{f}$ of the reflected wave is equal to twice the curvature of the mirror, or $f = \frac{R}{2}$. In other words, *the focal length of a mirror is equal to one half its radius.*

458. Image in a convex mirror. We are all familiar with the fact that a convex mirror always forms behind the mirror a virtual, erect, and diminished image. The reason for this is shown clearly in Fig. 416. The image of the point P lies, as in plane mirrors, always on the perpendicular to the mirror, but now necessarily nearer to the mirror than the focus F', since the mirror has increased the curvature of the reflected waves by an amount equal to $\frac{1}{f}$. The image $P'Q'$ of an object PQ is always diminished because it lies between the converging lines PC and QC. It can be located by the ray method (Fig. 417) exactly as in the case of concave lenses. In fact, a convex mirror and a concave lens have exactly the same optical properties. This is because *each always increases the curvature of the incident waves by an amount* $\frac{1}{f}$.

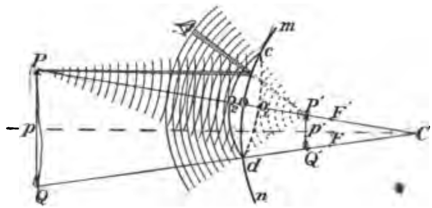


FIG. 416. Construction of image in a convex mirror

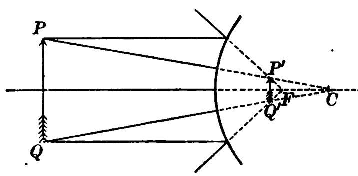


FIG. 417

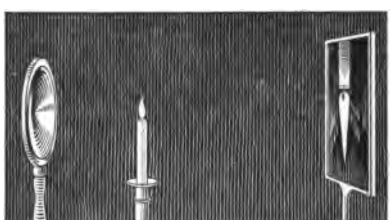


FIG. 418. Real image of candle formed by a concave mirror

459. Images in concave mirrors. Let the images obtainable with a concave mirror be studied precisely as were those obtainable from a convex lens. It will be found that exactly the same series of images is obtained; that is, when the object is between the mirror and the principal focus the image is virtual, enlarged,

and erect. When it is at the focus the reflected waves are plane; that is, the rays from each point are a parallel bundle. When it is between the principal focus and the center of curvature, the image is inverted, enlarged, and real (Figs. 418 and 419). When it is at a distance R ($= oC$) from the mirror, the image is also at a distance R and of the same size

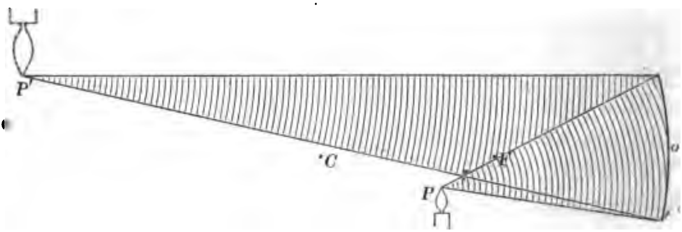


FIG. 419. Method of formation of a real image by a concave mirror

as the object, though inverted (see Fig. 423). As the object is moved from R out to a great distance the image moves from C up to F, and is always real, inverted, and diminished. The most convenient way of finding the focal length is to find where the image of a distant object is formed.

We learn, then, that a concave mirror has exactly the optical properties of a convex lens. This is because, like the convex

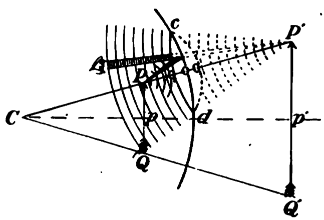


FIG. 420. Virtual image in a concave mirror

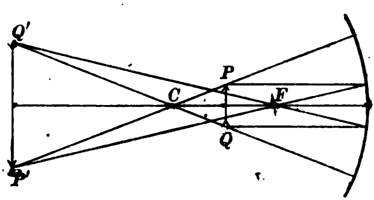
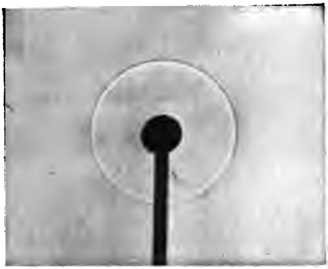


FIG. 421. Ray method of locating real image in concave mirror

lens, it always *diminishes* the curvature of the waves. The same formulas hold throughout, and the same constructions are applicable (see Figs. 420 and 421).

460. Summary for lenses and mirrors. REAL IMAGES, *always inverted.* Formed by convex lenses and concave mirrors if $D_o > f$. Enlarged if



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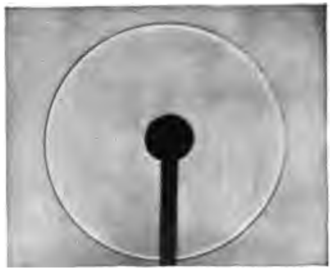

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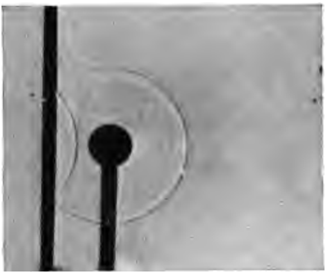

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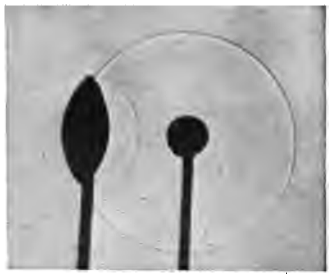

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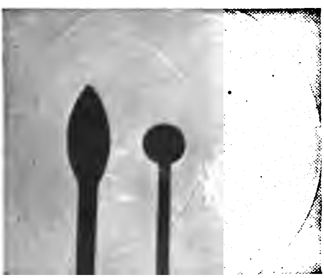


5

6

PHOTOGRAPHS OF SOUND WAVES HAVING THEIR ORIGIN IN AN ELECTRIC SPARK BEHIND THE MIDDLE OF THE BLACK DISK

1. A spherical sound wave. 2. The same wave .00007 second later. 3. A wave reflected from a plane surface, curvature unchanged. 4. A wave reflected from a convex surface, curvature increased. 5. The source at the focus of a SO_2 lens. The photograph shows first, the original wave on the right; second, the reflected wave, with its increased curvature; and third, the transmitted plane wave. 6. Source at focus of a concave mirror; the reflected wave is plane. (Taken by Professor A. L. Foley and Wilmer H. Souder, of the University of Indiana)

$2f > D_o > f$; diminished if $D_o > 2f$. The resulting curvature is equal to the curvature impressed by the lens, diminished by the initial curvature.

$$\frac{1}{D_i} = \frac{1}{f} - \frac{1}{D_o} \quad \text{or} \quad \frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}.$$

VIRTUAL IMAGES, *always erect.* (1) Formed by convex lenses and concave mirrors if $D_o < f$; always enlarged. The resulting curvature is equal to the initial curvature diminished by the curvature impressed by the lens.

$$\frac{1}{D_i} = \frac{1}{D_o} - \frac{1}{f} \quad \text{or} \quad \frac{1}{D_o} - \frac{1}{D_i} = \frac{1}{f}.$$

(2) Formed by concave lenses and convex mirrors; always diminished. The resulting curvature is equal to the initial curvature increased by the curvature impressed by the lens.

$$\frac{1}{D_i} = \frac{1}{D_o} + \frac{1}{f} \quad \text{or} \quad \frac{1}{D_i} - \frac{1}{D_o} = \frac{1}{f}.$$

The size of all images is given by

$$\frac{L_o}{L_i} = \frac{D_o}{D_i}$$

where L_o and L_i denote the size of object and image respectively, and D_o and D_i their distances from the lens or mirror.*

QUESTIONS AND PROBLEMS

1. A man runs toward a plane mirror at the rate of 12 ft. per second. How fast does he approach his image?

2. A man is standing squarely in front of a plane mirror which is very much taller than he is. The mirror is tipped toward him until it makes an angle of 45° with the horizontal. He still sees his full length. What position does his image occupy?

3. Show from a construction of the image that a man cannot see his entire length in a vertical mirror unless the mirror is half as tall as he is. Decide from a study of the figure whether or not the distance of the man from the mirror affects the case.

4. How tall is a tree 200 ft. away, if the image of it formed by a lens of focal length 4 in. is 1 in. long? (Consider the image to be formed in the focal plane.)

* Laboratory experiments on the formation of images by concave mirrors and by lenses should follow this discussion. See, for example, Experiments 45 and 46 of the authors' manual.

5. How long an image of the same tree will be formed in the focal plane of a lens having a focal length of 9 in.?

6. Why does the nose appear relatively large in comparison with the ears when the face is viewed in a convex mirror?

7. Can a convex mirror ever form an inverted image? Give reason for your answer.

8. When does a convex lens form a real, and when a virtual, image? When an enlarged, and when a diminished, image? When an erect, and when an inverted, one?

9. Describe the image formed by a concave lens. Why can it never be larger than the object?

10. What is the difference between a real and a virtual image?

11. A candle placed 20 cm. in front of a concave mirror has its image formed 50 cm. in front of the mirror. Find the radius of the mirror.

12. Find the relative sizes of image and object in Problem 11.

13. An object is 15 cm. in front of a convex lens of 12 cm. focal length. What will be the nature of the image, its size, and its distance from the lens?

14. What is the focal length of a lens if the image of an object 10 ft. away is 3 ft. from the lens?

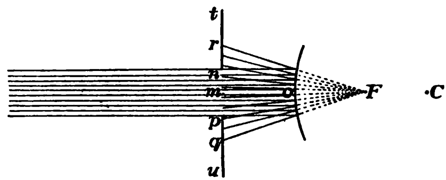


FIG. 422. Determination of focal length of a convex mirror

15. If the object in Problem 14 is 6 in. long, how long will the image be?

16. A beam of sunlight falls on a convex mirror through a circular hole in a sheet of cardboard, as in Fig. 422. Prove that when the diameter of the reflected beam *rq* is twice the diameter of the hole *np*, the distance *mo* from the mirror to the screen is equal to the focal length *oF* of the mirror.

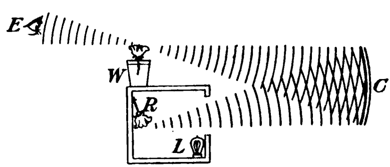


FIG. 423. Image of object at center of curvature

17. If a rose *R* is pinned upside down in a brightly illuminated box, a real image may be formed in a glass of water *W* by a concave mirror *C* (Fig. 423). Where must the eye be placed to see the image?

18. How far is the rose from the mirror in the arrangement of Fig. 423?

Optical Instruments

461. The photographic camera. A fairly distinct, though dim, image of a candle can be obtained with nothing more elaborate than a pinhole in a piece of cardboard (Fig. 424). If the receiving screen is replaced by a photographic plate, the arrangement becomes a *pinhole camera*, with which good pictures may be taken if the exposure is sufficiently long. If we try to increase the brightness of the image by enlarging the hole, the image becomes blurred because the narrow pencils $a_1a'_1$, $a_2a'_2$, etc. become cones whose bases a'_1, a'_2, overlap and thus destroy the distinctness of the outline.

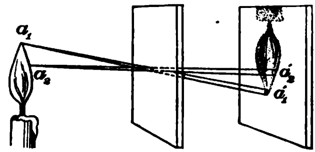


FIG. 424. Image formed by a small opening

It is possible to gain the increased brightness due to the larger hole, without sacrificing distinctness of outline, by placing a lens in the hole (Fig. 425). If the receiving screen is now a sensitive plate, the arrangement becomes a *photographic camera* (Fig. 426). But while, with the pinhole camera, the screen may be at any distance from the hole, with a lens the plate and the object must be at conjugate foci of the lens.

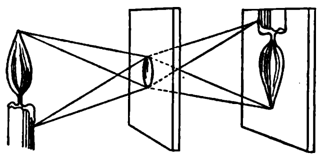


FIG. 425. Principle of the photographic camera

Let a lens of, say, 4 feet focal length be placed in front of a hole in the shutter of a darkened room and a semitransparent screen (for example, architect's tracing paper) placed at the focal plane. A perfect reproduction of the opposite landscape will appear.

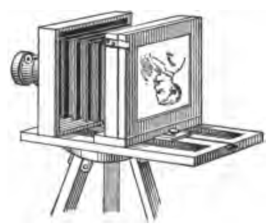


FIG. 426. The photographic camera

462. The projecting lantern. The projecting lantern is essentially a camera in which the position of object and image have been interchanged; for in the use of the camera the object is at a considerable distance and a small inverted image is formed on a plate placed somewhat farther from the lens than the focal distance. In the use of the projecting lantern the object *P* (Fig. 427) is placed a trifle farther from the lens *L'* than

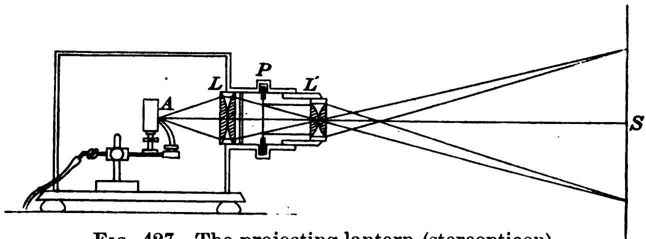


FIG. 427. The projecting lantern (stereopticon)

its focal length, and an enlarged inverted image is formed on a distant screen *S*. In both instruments the optical part is simply a convex lens, or a combination of lenses which is equivalent to a convex lens.

The object *P*, whose image is formed on the screen, is usually a transparent slide which is illuminated by a powerful light *A*. The image is as many times larger than the object as the distance from *L'* to *S* is greater than the distance from *L'* to *P*. The light *A* is usually either a calcium light or an electric arc.

The above are the only essential parts of a projecting lantern. In order, however, that the slide may be illuminated as brilliantly as possible, a so-called condensing lens *L* is always used. This concentrates light upon the transparency and directs it toward the screen.

In order to illustrate the principle of the instrument, let a beam of sunlight be reflected into the room and fall upon a lantern slide. When a lens is placed a trifle more than its focal distance in front of the slide, a brilliant picture will be formed on the opposite wall.

463. The eye. The eye is essentially a camera in which the cornea C (Fig. 428), the aqueous humor l, and the crystalline lens o act as one single lens which forms an inverted image $P'Q'$ on the retina, an expansion of the optic nerve covering the inside of the back of the eyeball.

In the case of the camera the images of objects at different distances are obtained by placing the plate nearer to or farther from the lens. In the eye, however, the distance from the retina to the lens remains constant, and the adjustment for different distances is effected by changing the focal length of the lens itself in such a way as always to keep the image upon the retina. Thus, when the normal eye is perfectly relaxed, the lens has just the proper curvature to focus plane waves upon the retina; that is, to make distant objects distinctly visible. But by directing attention upon near objects we cause the muscles which hold the lens in place to contract in such a way as to make the lens more convex, and thus bring into distinct focus objects which may be as close as eight or ten inches. This power of adjustment, however, varies greatly in different individuals.

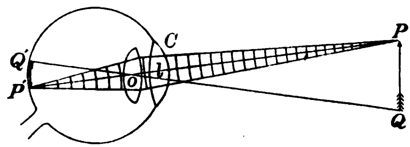


FIG. 428. The human eye

464. The apparent size of a body. The apparent size of a body depends simply upon the size of the image formed upon the retina by the lens of the eye, and hence upon the size of *the visual angle pCq* (Fig. 429). The size of this angle evidently increases as the object is brought nearer to the eye (see PCQ). Thus the image formed on the retina when a man is 100 feet from the eye is in reality only one tenth as large as the image

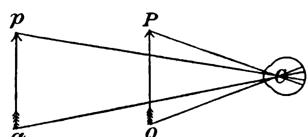


FIG. 429. The visual angle

formed of the same man when he is but 10 feet away. We do not actually interpret the larger image as representing a larger man simply because we have been taught by lifelong experience to take account of the known distance of an object in forming our estimate of its actual size. To an infant who has not yet formed ideas of distance the man 10 feet away doubtless appears ten times as large as the man 100 feet away.

465. Distance of most distinct vision. When we wish to examine an object minutely we bring it as close to the eye as possible in order to increase the size of the image on the retina. But there is a limit to the size of the image which can be produced in this way; for when the object is brought nearer to the normal eye than about 10 inches, the curvature of the incident wave becomes so great that the eye lens is no longer able, without too much strain, to thicken sufficiently to bring the image into sharp focus upon the retina. Hence a person with normal eyes holds an object which he wishes to see as distinctly as possible at a distance of about 10 inches.

Although this so-called *distance of most distinct vision* varies somewhat with different people, for the sake of having a standard of comparison in the determination of the magnifying powers of optical instruments, some exact distance had to be chosen. The distance so chosen is 10 inches, or 25 centimeters.

466. Magnifying power of a convex lens. If a convex lens is placed immediately before the eye, the object may be brought much closer than 25 centimeters without loss of distinctness, for the curvature of the wave is partly, or even wholly, overcome by the lens before the light enters the eye.

If we wish to use a lens as a magnifying glass to the best advantage, we place the eye as close to it as we can, so as to gather as large a cone of rays as possible, and then place the object at a distance from the lens equal to its focal length, so that the waves after passing through it are plane. They are then focused by the eye with the least possible effort. The

visual angle in such a case is PcQ [Fig. 430, (1)], for, since the emergent waves are plane, the rays which pass through the center of the eye from P and Q are parallel to the lines through Pc and Qc. But if the lens were not present, and if the object were 25 centimeters from the eye, the visual angle would be pcq [Fig. 430, (2)]. The ratio of these two angles is approximately $25/f$, where f is the focal length of the lens expressed in centimeters. Now *the magnifying power of a lens or microscope is defined as the ratio of the angle actually subtended by the image when viewed through the instrument, to the angle subtended by the object when viewed with the unaided eye at a distance of* 25 *centimeters.* Therefore the magnifying power of a simple lens is $25/f$. Thus, if a lens has a focal length of 2.5 centimeters, it produces a magnification of 10 diameters when the object is placed at its principal focus. If the lens has a focal length of 1 centimeter, its magnifying power is 25, etc.

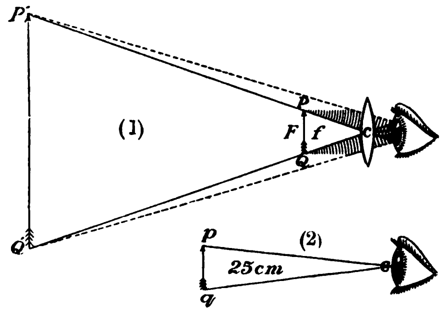


FIG. 430. Magnifying power of a lens

467. Magnifying power of an astronomical telescope. In the astronomical telescope the *objective*, or forward lens, forms at its focus an image of a distant object. Suppose that this image were viewed directly by

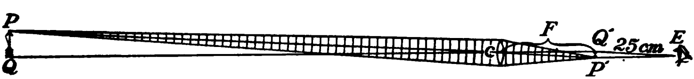


FIG. 431. Magnifying power of a telescope objective is $F/25$

an eye 25 centimeters from the image, as in Fig. 431. The angle subtended by the image at the eye would then be $P'EQ'$; but the angle subtended by the object is PEQ, which is practically the same as $P'cQ'$;

for $P'cQ' = PcQ$, and since the object is very distant, $PcQ = PEQ$ approximately. But $P'EQ'$ divided by $P'cQ'$ is equal to $F/25$, F being the focal length of the objective measured in centimeters. Hence the forward lens alone enables us to increase the visual angle of the object $F/25$ times.

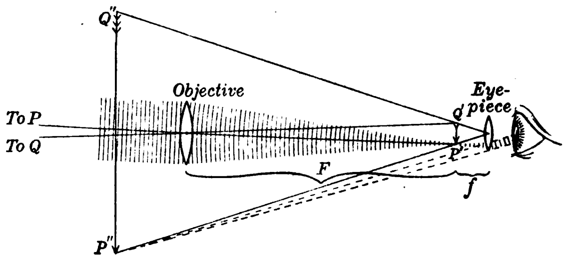


FIG. 432. Magnifying power of a telescope is F/f

In practice, however, the image is not viewed with the unaided eye, but with a simple magnifying glass called an eyepiece (Fig. 432), placed so that the image is at its focus. Since we have seen in § 466 that the simple magnifying glass increases the visual angle $25/f$ times, f being the focal length of the eyepiece, it is clear that the total magnification produced by both lenses, used as above, is $F/25 \times 25/f = F/f$. *The magnifying power of an astronomical telescope is therefore the focal length of the objective divided by the focal length of the eyepiece.* It will be seen, therefore, that to get a high magnifying power it is necessary to use an objective of as great focal length as possible and an eyepiece of as short focal length as possible. The focal length of the great lens at the Yerkes Observatory is about 62 feet and its diameter 40 inches. The great diameter enables it to collect a very large amount of light.

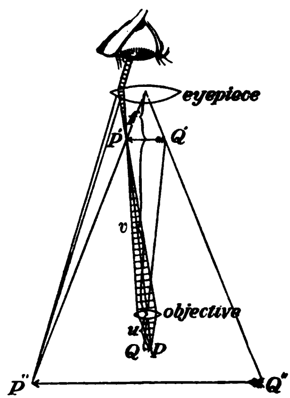


FIG. 433. The compound microscope

Eyepieces often have focal lengths as small as $\frac{1}{4}$ inch. Thus the Yerkes telescope when used with a $\frac{1}{4}$-inch eyepiece has a magnifying power of 2976.

468. The magnifying power of the compound microscope. The compound microscope is like the telescope in that the front lens, or *objective*, forms a real image of the object at the focus of the eyepiece. The size of the image $P'Q'$ (Fig. 433) formed by the objective is as many times the size of the object PQ as v, the distance

from the objective to the image, is times u, the distance from the objective to the object (see § 452). Since the eyepiece magnifies this image $25/f$ times, the total magnifying power of a compound microscope is $\frac{v}{u}\frac{25}{f}\cdot$ Ordinarily v is practically the length L of the microscope tube, and u is the focal length F of the objective. Wherever this is the case, then, the magnifying power of the compound microscope is $\frac{25\,L}{Ff}\cdot$

The relation shows that in order to get a high magnifying power with a compound microscope the focal length of both eyepiece and objective should be as short as possible, while the tube length should be as long as possible. Thus, if a microscope has both an eyepiece and an objective of 6 millimeters focal length and a tube 15 centimeters long, its magnifying power will be $\frac{25 \times 15}{.6 \times .6} = 1042$. Magnifications as high as 2500 or 3000 are sometimes used, but it is impossible to go much farther for the reason that the image becomes too faint to be seen when it is spread over so large an area.

469. The terrestrial telescope. In both the microscope and the telescope, since the image formed by the objective is a real image, it is inverted. Since the eyepiece forms a virtual image of this real image, the object as seen by the eye will appear upside down. This is a serious

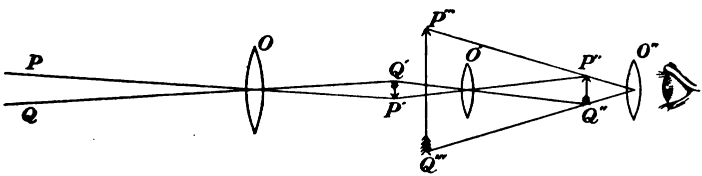


FIG. 434. The terrestrial telescope

objection when it is desired to use the telescope as a field glass. Hence the *terrestrial telescope* is constructed with an objective exactly like that of the astronomical telescope, but with an eyepiece which is essentially a compound microscope. Since, then, the image is twice inverted, once by the objective O (Fig. 434) and once by O', it appears erect.

470. The opera glass. On account of the large number of lenses which must be used in the terrestrial telescope, it is too bulky and awkward for many purposes, and hence it is often replaced by the opera glass (Fig. 435). This instrument consists of an objective like that of the telescope, and an eyepiece which is a concave lens of the same focal

length as the eye of the observer. The effect of the eyepiece is therefore to just neutralize the lens of the eye. Hence the objective, in effect, forms its image directly upon the retina. It will be seen that the size of the image formed upon the retina by the objective of the opera glass

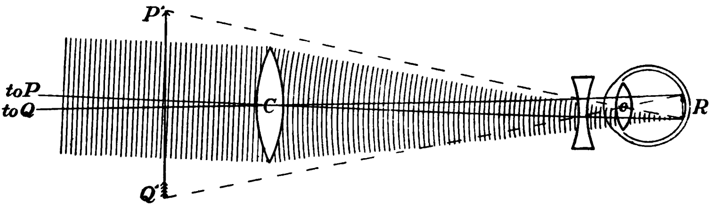


FIG. 435. The opera glass

is as much larger than the size of the image formed by the naked eye as the focal length CR of the objective is greater than the focal length cR of the eye. Since the focal length of the eye is the same as that of the eyepiece, *the magnifying power of the opera glass, like that of the astronomical telescope, is the ratio of the focal lengths of the objective and eyepiece.* Objects seen with an opera glass appear erect, since the image formed on the retina is inverted, as is the case with images formed by the lens of the eye unaided.

471. The stereoscope. Binocular vision. When an object is seen with both eyes the images formed on the two retinas differ slightly, because of the fact that the two eyes, on account of their lateral separation, are viewing the object from slightly different angles. It is this difference in the two images which gives to an object or landscape viewed with two eyes an appearance of depth, or solidity, which is wholly wanting when one eye is closed. The stereoscope is an instrument which reproduces in photographs this effect of binocular vision. Two photographs of the same object are taken from slightly different points of view. These photographs are mounted at A and B (Fig. 436), where they are simultaneously viewed by the two eyes through the two prismatic lenses m and n. These two lenses superpose the two images at C because of their action as prisms, and at the same time magnify them because of their action as simple magnifying lenses. The result is that the observer is conscious of viewing but one photograph; but this differs

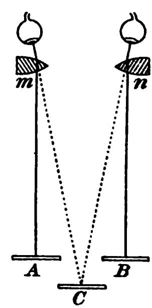


FIG. 436. Principle of the stereoscope

from ordinary photographs in that, instead of being flat, it has all of the characteristics of an object actually seen with both eyes.

The opera glass has the advantage over the terrestrial telescope of affording the benefit of binocular vision; for while telescopes are usually constructed with one tube, opera glasses always have two, one for each eye.

472. The Zeiss binocular. The greatest disadvantage of the opera glass is that the field of view is very small. The terrestrial telescope has a larger field, but is of inconvenient length. An instrument called the Zeiss binocular (Fig. 437) has recently come into use, which combines the compactness of the opera glass with the wide field of view of the terrestrial telescope. The compactness is gained by causing the light to pass back and forth through total reflecting prisms, as in the figure. These reflections also perform the function of reinverting the image, so that the real image which is formed at the focus of the eyepiece is erect. It will be seen, therefore, that the instrument is essentially an astronomical telescope in which the image is reinverted by reflection, and in which the tube is shortened by letting the light pass back and forth between the prisms.

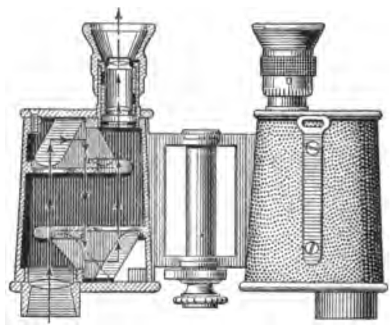


FIG. 437. The Zeiss binocular

A further advantage which is gained by the Zeiss binocular is due to the fact that the two objectives are separated by a distance which is greater than the distance between the eyes, so that the stereoscopic effect is more prominent than with the unaided eye or with the ordinary opera glass.*

QUESTIONS AND PROBLEMS

1. If a photographer wishes to obtain the full figure on a plate of cabinet size, does he place the subject nearer to or farther from the camera than if he wishes to take the head only? Why?

2. The image, on the retina, of a book held a foot from the eye is larger than that of a house on the opposite side of the street. Why do we not judge that the book is actually larger than the house?

* Laboratory experiments on the magnifying powers of lenses and on the construction of microscopes and telescopes should follow this chapter. See, for example, Experiments 47, 48, and 49 of the authors' manual.

3. What is the magnifying power of a $\frac{1}{4}$-in. lens used as a simple magnifier?

4. A telescope has an objective of 30 ft. focal length and an eyepiece of 1 in. focal length. What is its magnifying power?

5. A stereopticon is provided with two lenses, one of 6-in. and the other of 12-in. focal length. Which lens should be used if it is desired to get as large an image as possible on a screen at a fixed distance?

6. A compound microscope has a tube length of 8 in., an objective of focal length $\frac{1}{2}$ in., and an eyepiece of focal length 1 in. What is its magnifying power?

7. Explain why a terrestrial telescope shows objects erect rather than inverted.

8. If the focal length of the eye is 1 in., what is the magnifying power of an opera glass whose objective has a focal length of 4 in.?

9. If the length of a microscope tube is increased after an object has been brought into focus, must the object be moved nearer to or farther from the lens in order that the image may be again in focus?

10. The magnifying power of a microscope is 1000, the tube length is 8 in., and the focal length of the eyepiece is $\frac{1}{2}$ in. What is the focal length of the objective?

11. What sort of lenses are necessary to correct shortsightedness? longsightedness?

CHAPTER XX

COLOR PHENOMENA

Color and Wave Length

473. Wave lengths of different colors. Let a soap film be formed across the top of an ordinary drinking glass, care being taken that both the solution and the glass are as clean as possible. Let a beam of sunlight or the light from a projecting lantern pass through a piece of red glass at A, fall upon the soap film F, and be reflected from it to a white screen S (see Fig. 438). Let a convex lens L of from six to twelve inches focal length be placed in the path of the reflected beam in such a position as to produce an image of the film upon the screen S, that is, in such a position that film and screen are at conjugate foci of the lens. The system of red and black bands upon the screen is formed precisely as in § 438, by the interference of the two beams of light coming from the front and back surfaces of the wedge-shaped film. Let now the red glass be held in one half of the beam and a piece of green glass in the other half, the two pieces being placed edge to edge, as shown at A. Two sets of fringes will be seen side by side on the screen. The fringes will be red and black on one side of the image and green and black on the other; but it will be noticed at once that the dark bands on the green side are closer together than the dark bands on the other side;

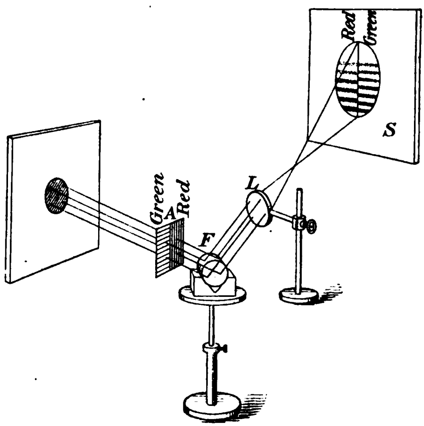


Fig. 438. Projection of soap-film fringes

in fact, seven fringes on the side of the film which is covered by the green glass will be seen to cover about the same distance as six fringes on the red side.*

Since it was shown in § 440 that the distance between two dark bands corresponds to an increase of one-half wave length in the thickness of the film, we conclude, from the fact that the dark bands on the red side are farther apart than those on the green side, that red light must have a longer wave length than green light. The wave length of the central portion of each colored region of the spectrum is roughly as follows:

Red	.000068 cm.	Yellow	.000058 cm.
Green	.000052 cm.	Blue	.000046 cm.
Violet	.000042 cm.		

Let the red and green glasses be removed from the path of the beam. The red and green fringes will be seen to be replaced by a series of bands brilliantly colored in different hues. These are due to the fact that the lights of different wave length form interference bands at different places on the screen. Notice that the upper edges of the bands (lower edges in the inverted image) are reddish, while the lower edges are bluish. We shall find the explanation of this fact in § 482.

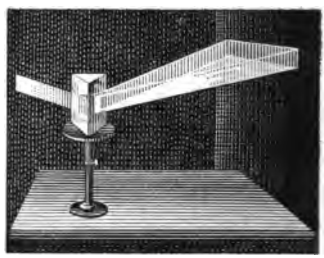


FIG. 439. White light decomposed by a prism

474. Composite nature of white light. Let a beam of sunlight pass through a narrow slit and fall on a prism, as in Fig. 439. The beam which enters the prism as white light is broken up into red, yellow, green, blue, and violet lights, although each color merges, by insensible gradations, into the next. This band of color is called a *spectrum*.

We conclude from this experiment that *white light is a mixture of all the colors of the spectrum, from red to violet inclusive.*

* The experiment may be performed at home by simply looking through red and green glasses at a soap film so placed as to reflect white light to the eye.

475. Color of bodies in white light. Let a piece of red glass be held in the path of the colored beam of light in the experiment of the preceding section. All the spectrum except the red will disappear, thus showing that all the wave lengths except red have been absorbed by the glass. Let a green glass be inserted in the same way. The green portion of the spectrum will remain strong, while the other portions will be greatly enfeebled. Hence green glass must have a much less absorbing effect upon wave lengths which correspond to green than upon those which correspond to red and blue. Let the green and red glasses be held one behind the other in the path of the beam. The spectrum will almost completely vanish, for the red glass has absorbed all except the red rays, and the green glass has absorbed these.

We conclude, therefore, that the color which a body has in ordinary daylight is determined by the wave lengths which the body has *not* the power of absorbing. Thus, if a body appears white in daylight, it is because it diffuses or reflects all wave lengths equally to the eye, and does not absorb one set more than another. For this reason the light which comes from it to the eye is of the same composition as daylight or sunlight. If, however, a body appears red in daylight, it is because it absorbs the red rays of the white light which falls upon it less than it absorbs the others, so that the light which is diffusely reflected contains a larger proportion of red wave lengths than is contained in ordinary light. Similarly, a body appears yellow, green, or blue when it absorbs less of one of these colors than of the rest of the colors contained in white light, and therefore sends a preponderance of some particular wave length to the eye.

476. Color of bodies placed in colored lights. Let a body which appears white in sunlight be placed in the red end of the spectrum. It will appear to be red. In the blue end of the spectrum it will appear to be blue, etc. This confirms the conclusion of the last paragraph, that a white body has the power of diffusely reflecting all the colors of the spectrum equally.

Next let a skein of red yarn be held in the blue end of the spectrum. It will appear nearly black. In the red end of the spectrum

it will appear strongly red. Similarly, a skein of blue yarn will appear nearly black in all the colors of the spectrum except blue, where it will have its proper color.

These effects are evidently due to the fact that the red yarn, for example, has the power of diffusely reflecting red wave lengths copiously, but of absorbing, to a large extent, the others. Hence, when held in the blue end of the spectrum, it sends but little color to the eye, since no red light is falling upon it.

477. Compound colors. It must not be inferred from the preceding paragraphs that every color except white has one definite wave length, for the same effect may be produced on the eye by a mixture of several different wave lengths as is produced by a single wave length. This statement may be proved by the use of an apparatus known as Newton's color disk (Fig. 440). The arrangement makes it possible to rotate differently colored sectors so rapidly before the eye that the effect is precisely the same as though the colors came to the eye simultaneously. If one half of the disk is red and the other half green, the rotating disk will appear yellow, the color being very similar to the yellow of the spectrum. If green and violet are mixed in the same way, the result will be light blue. Although the colors produced in this way are not distinguishable by the eye from spectral colors, it is obvious that their physical constitution is wholly different; for while a spectral color consists of waves of a single wave length, the colors produced by mixture are compounds of several wave lengths. For this reason the spectral colors are called pure and the others compound. In order to tell whether the color of an object is pure or

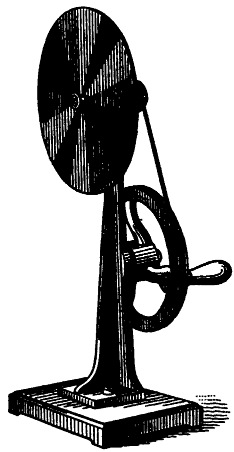


FIG. 440. Newton's color disk

compound, it is only necessary to observe it through a prism. If it is compound, the colors will be separated, giving an image of the object for each color. If it is pure, the object will appear through the prism exactly as it does without the prism.

By compounding colors in the way described above we can produce many which are not found in the spectrum. Thus mixtures of red and blue give purple or crimson; mixtures of black with red, orange, or yellow give rise to the various shades of brown. Lavender may be formed by adding three parts of white to one of blue; lilac, by adding to fifteen parts of white four parts of red and one of blue; olive, by adding one part of black to two parts of green and one of red.

478. Complementary colors. Since white light is a combination of all the colors from red to violet inclusive, it might be expected that if one or several of these colors were subtracted from white light, the residue would be colored light.

To test this experimentally let a beam of sunlight be passed through a slit s, a prism P, and a lens L, to a screen S, arranged as in Fig. 441. A spectrum will be formed at RV, the position conjugate to the slit s, and a pure white spot will appear on the screen when it is at the position which is conjugate to the prism face ab. Let a card be slipped into the path of the beam at R, so as to cut off the red portion of the light. The spot on S will appear a brilliant shade of greenish blue. This is the compound color left after red is taken from the white light. This shade of blue is therefore called the *complementary color* of the red which has been subtracted. Two *complementary colors* are therefore defined as any two colors which produce white when added to each other.

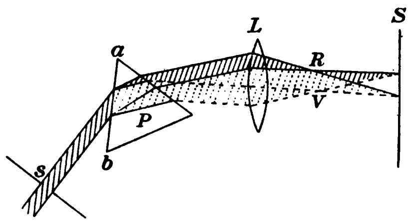


FIG. 441. Recombination of spectral colors into white light

Let the card be slipped in from the side of the blue rays at V. The spot will first take on a yellowish tint when the violet alone is cut out;

and as the card is slipped farther in, the image will become a deep shade of red when violet, blue, and part of the green are cut out.

Next let a lead pencil be held vertically between *R* and *V* so as to cut off the middle part of the spectrum; that is, the yellow and green rays. The remaining red, blue, and violet will unite to form a brilliant purple. In each case the color on the screen is the complement of that which is cut out.

479. Retinal fatigue. Let the gaze be fixed intently for not less than twenty or thirty seconds on a point at the center of a block of any brilliant color — for example, red. Then look off at a dot on a white wall or a piece of white paper, and hold the gaze fixed there for a few seconds. The brilliantly colored block will appear on the white wall, but its color will be the *complement* of that first looked at.

The explanation of this phenomenon, due to so-called "retinal fatigue," is found in the fact that although the white surface is sending waves of all colors to the eye, the nerves which responded to the color first looked at have become fatigued, and hence fail to respond to this color when it comes from the white surface. Therefore the sensation produced is that due to white light minus this color; that is, to the complement of the original color. A study of the spectral colors by this method shows that the following colors are complementary.

Red	Orange	Yellow	Violet	Green
Bluish green	Greenish blue	Blue	Greenish yellow	Crimson

480. Color of pigments. When yellow light is added to the proper shade of blue, white light is produced, since these colors are complementary. But if a yellow pigment is added to a blue one, the color of the mixture will be green. This is because the yellow pigment removes the blue and violet by absorption, and the blue pigment removes the red and yellow, so that only green is left.

When pigments are mixed, therefore, each one *subtracts* certain colors from white light, and the color of the mixture is that color which escapes absorption by the different ingredients. Adding *pigments* and adding *colors*, as in § 477, are therefore wholly dissimilar processes and produce widely different results.

481. Three-color printing. It is found that all colors can be produced by suitably mixing with the color disk (Fig. 440) three spectral colors, namely red, green, and blue-violet. These are therefore called the *three primary colors.* The so-called primary pigments are simply the complements of these three primary colors. They are, in order, peacock blue, crimson, and light yellow. The three primary colors when mixed yield white. The three primary pigments when mixed yield black, because together they subtract all the ingredients from white light. The process of three-color printing consists in mixing on a white background, that is, on white paper, the three primary *pigments* in the following way: Three different photographs of a given-colored object are taken, each through a *filter* of gelatin stained the color of one of the primary colors. From these photographs half-tone "blocks" are made in the usual way. The colored picture is then made by carefully superposing prints from these blocks, using with each an ink whose color is the complement of that of the "filter" through which the original negative was taken. The plate opposite page 400 illustrates fully the process. It will be interesting to examine differently colored portions with a lens of moderate magnifying power.

482. Colors of thin films. The study of complementary colors has furnished us with the key to the explanation of the fact, observed in § 473, that the upper edge of each colored band produced by the water wedge is reddish, while the lower edge is bluish. The red on the upper edge is due to the fact that there the shorter blue waves are destroyed by interference and a complementary red color is left; while on the lower edge of each fringe, where the film is thicker, the longer red waves interfere, leaving a complementary blue. In fact, each wave length of the incident light produces a set of fringes, and it is the superposition of these different sets which gives the resultant colored fringes. Where the film is too thick the overlapping

is so complete that the eye is unable to detect any trace of color in the reflected light.

In films which are of uniform thickness, instead of wedge-shaped, the color is also uniform, so long as the observer does not change the angle at which the film is viewed. With any change in this angle the thickness of film through which the light must pass in coming to the observer changes also, and hence the color changes. This explains the beautiful play of iridescent colors seen in soap bubbles, thin oil films, mother of pearl, etc.

483. Chromatic aberration. It has heretofore been assumed that all the waves which fall on a lens from a given source are brought to one and the same focus. But since blue rays are bent more than red ones in passing through a prism, it is clear that in passing through a lens the blue light must be brought to a focus at some point v (Fig. 442) nearer to the lens than r, where the red light is focused, and that the foci for intermediate colors must fall in intermediate positions. It is for this reason that an image formed by a simple lens is always fringed with color.

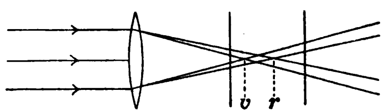


FIG. 442. Chromatic aberration in a lens

Let a card be held at the focus of a lens placed in a beam of sunlight (Fig. 442). If the card is slightly nearer the lens than the focus, the spot of light will be surrounded by a red fringe, for the red rays, being least refracted, are on the outside. If the card is moved out beyond the focus, the red fringe will be found to be replaced by a blue one; for, after crossing at the focus, it will be the more refrangible rays which will then be found outside.

This dispersion of light produced by a lens is called *chromatic aberration.*

484. Achromatic lenses. The color effect caused by the chromatic aberration of a simple lens greatly impairs its usefulness. Fortunately, however, it has been found possible to

Three-Color Printing

1. Yellow impression; negative made through a blue-violet filter. 2. Crimson impression; negative made through a green filter. 3. Crimson on yellow. 4. Blue impression; negative made through a red filter. 5. Yellow, crimson, and blue combined; the final product.

The squares at the left show the colors of ink used in making each impression. Notice the different colors in 5, which are made by combining the yellow, crimson, and blue.

eliminate this effect almost completely by combining into one lens a convex lens of crown glass and a concave lens of flint glass (Fig. 443). The first lens then produces both bending and dispersion, while the second almost completely overcomes the dispersion without entirely overcoming the bending. Such lenses are called *achromatic lenses*. The first one was made by John Dollond in London in 1758. They are used in the construction of all good telescopes and microscopes.

FIG. 443. An achromatic lens

QUESTIONS AND PROBLEMS

1. If a soap film is illuminated with red, green, and yellow strips of light, side by side, how will the distance between the yellow fringes compare with that between the red fringes? with that between the green fringes? (See table on page 394.)

2. What will be the apparent color of a red body when it is in a room to which only green light is admitted?

3. Why do white bodies look blue when seen through a blue glass?

4. What color would a yellow object appear to have if looked at through a blue glass? (Assume that the yellow is a pure color.)

5. A gas flame is distinctly yellow as compared with sunlight. What wave lengths, then, must be comparatively weak in the spectrum of a gas flame?

6. If the green and the yellow are subtracted from white light, what will be the color of the residue?

7. Will a reddish spot on an oil film be thinner or thicker than an adjacent bluish portion?

SPECTRA

485. The rainbow. A very beautiful spectrum with which every one is familiar is the *rainbow*.

Let a spherical bulb F (Fig. 444) $1\frac{1}{2}$ or 2 inches in diameter be filled with water and held in the path of a beam of sunlight which enters the room through a hole in a piece of cardboard AB. A miniature rainbow will be formed on the screen around the opening, the violet edge of the bow being toward the center of the circle and the red outside. A beam of light which enters the flask at C is there both refracted and dispersed;

†

at *D* it is totally reflected; and at *E* it is again refracted and dispersed on passing out into the air. Since in both of the refractions the violet is bent more than the red, it is obvious that it must return nearer to the direction of the incident beam than the red rays. If the flask were a perfect sphere, the angle included between the incident ray *OC* and the emergent red ray *ER* would be 42°; and the angle between the incident ray and the emergent violet ray *EV* would be 40°.

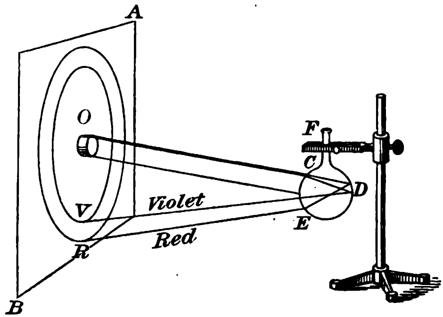


FIG. 444. Artificial rainbow

The actual rainbow seen in the heavens is due to the refraction and reflection of light in the drops of water in the air, which act exactly as did the flask in the preceding experiment. If the observer is standing at *E* with his back to the sun, the light which comes from the drops so as to make an angle of 42° (Fig. 445) with the line drawn from the observer to the sun must be red light; while the light which comes from drops which are at an angle of 40° from the eye must be violet light. It is clear that those drops whose direction from the eye makes any particular angle with the line drawn from the eye to the sun must lie on a circle whose center is on that line. Hence we see a circular arc of light which is violet on the inner edge and red on the outer edge.

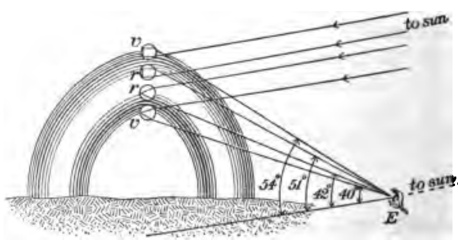


FIG. 445. Primary and secondary rainbows

486. The secondary bow. A second bow having the red on the inside and the violet on the outside is often seen outside

of the one just described, and concentric with it. This bow arises from rays which have suffered two internal reflections and two refractions, in the manner shown in Fig. 445. Since in this bow the emergent ray crosses the incident ray, it will be seen that the color which suffers the largest refraction must make the largest angle with the incident ray. Hence the violet which comes to the eye must come from drops which are farther from the center of the circle than those which send the red. The red rays come from an angle of 51° and the violet rays from an angle of 54°.

487. Continuous spectra. Let a Bunsen burner arranged to produce a white flame be placed behind a slit *s* (Fig. 446). Let the slit be viewed through a prism *P*. The spectrum will be a continuous band of color. If, then, the air is admitted at the base of the burner, and if a clean platinum wire is held in the flame directly in front of the slit, the white-hot platinum will also give a continuous spectrum.*

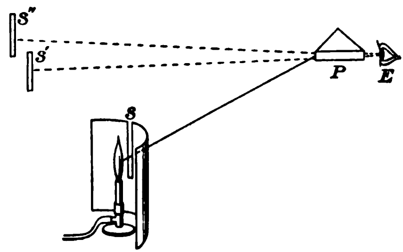


FIG. 446. Arrangement for viewing spectra

All incandescent solids and liquids are found to give spectra of this type which contain all the wave lengths from the extreme red to the extreme violet. The continuous spectrum of a luminous gas flame is due to the incandescence of solid particles of carbon suspended in the flame. The presence of these solid particles is proved by the fact that soot is deposited on bodies held in a white flame.

488. Bright-line spectra. Let a bit of asbestos or a platinum wire be dipped into a solution of common salt (sodium chloride) and held in the flame, care being taken that the wire itself is held so low that the

* By far the most satisfactory way of performing these experiments with spectra is to provide the class with cheap plate-glass prisms like those used in Experiment 50 of the authors' manual, rather than to attempt to project line spectra.

spectrum due to it cannot be seen. The continuous spectrum of the preceding paragraph will be replaced by a clearly defined yellow image of the slit which occupies the position of the yellow portion of the spectrum. This shows that the light from the sodium flame is not a compound of a number of wave lengths, but is rather of just the wave length which corresponds to this particular shade of yellow. The light is now coming from the incandescent sodium vapor and not from an incandescent solid, as in the preceding experiments.

Let another platinum wire be dipped in a solution of lithium chloride and held in the flame. Two distinct images of the slit, s' and s'' (Fig. 446), will be seen, one in yellow and one in red. Let calcium chloride be introduced into the flame. One distinct image of the slit will be seen in the green and another in the red. Strontium chloride will give a blue and a red image, etc. (The yellow sodium image will probably be present in each case, because sodium is present as an impurity in nearly all salts.)

These narrow images of the slit in the different colors are called the characteristic *spectral lines* of the substances. The experiments show that incandescent vapors and gases give rise to *bright-line spectra*, and not continuous spectra like those produced by incandescent solids and liquids. The method of analyzing compound substances through a study of the lines in the spectra of their vapors is called *spectrum analysis*. It was first used by the German chemist Bunsen in 1859.

489. The solar spectrum. Let a beam of sunlight pass first through a narrow slit S (Fig. 447), not more than $\frac{1}{2}$ millimeter in width, then through a prism P, and finally let it fall on a screen S', as shown in Fig. 447. Let the position of the prism be changed until a beam of white light is reflected from one of its faces to that portion of the screen which was previously occupied by the central portion of the spectrum. Then let a lens L be placed between the prism and the slit, and moved back and forth until a perfectly sharp white image of the slit is formed on the screen. This adjustment is made in order to get the slit S and the screen S' in the positions of conjugate foci of the lens. Now let the prism be turned to its original position. The spectrum on the screen will then consist of a series of colored images of the slit arranged side by side. This is called a pure spectrum, to distinguish it from the spectrum shown in Fig. 439, in which no lens was used to bring the rays of

each particular color to a particular point, and in which there was therefore much overlapping of the different colors. If the slit and screen are exactly at conjugate foci of the lens, and if the slit is sufficiently narrow, the spectrum will be seen to be crossed vertically by certain dark lines.

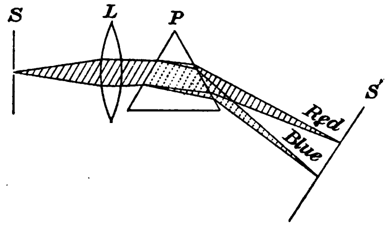


FIG. 447. Arrangement for obtaining a pure spectrum

These lines were first observed by the Englishman Wollaston in 1802, and were first studied carefully by the German Fraunhofer in 1814, who counted and mapped out as many as seven hundred of them. They are called after him, the *Fraunhofer lines*. Their existence in the solar spectrum shows that certain wave lengths are absent from sunlight, or, if not entirely absent, are at least much weaker than their neighbors. When the experiment is performed as described above it will usually not be possible to count more than five or six distinct lines.

490. Explanation of the Fraunhofer lines. Let the solar spectrum be projected as in § 489. Let a few small bits of metallic sodium be laid upon a loose wad of asbestos which has been saturated with alcohol. Let the asbestos so prepared be held to the left of the slit, or between the slit and the lens, and there ignited. A black band will at once appear in the yellow portion of the spectrum, at the place where the color is exactly that of the sodium flame itself; or if the focus was sufficiently sharp so that a dark line could be seen in the yellow before the sodium was introduced, this line will grow very much blacker when the sodium is burned. Evidently then this dark line in the yellow part of the solar spectrum is due in some way to sodium vapor through which the sunlight has somewhere passed.

The experiment at once suggests the explanation of the Fraunhofer lines. The white light which is emitted by the hot nucleus of the sun, and which contained all wave lengths, has had certain wave lengths weakened by absorption as it

passed through the vapors and gases surrounding the sun and the earth. For it is found that *every gas or vapor will absorb exactly those wave lengths which it itself is capable of emitting when incandescent.* This is for precisely the same reason that a tuning fork will respond to, that is, absorb, only vibrations which have the same period as those which it is itself able to emit. Since, then, the dark line in the yellow portion of the sun's spectrum is in exactly the same place as the bright yellow line produced by incandescent sodium vapor, or the dark line which is produced whenever white light shines through sodium vapor, we infer that sodium vapor must be contained in the sun's atmosphere. By comparing in this way the positions of the lines in the spectra of different elements with the positions of various dark lines in the sun's spectrum, many of the elements which exist on the earth have been proved to exist also in the sun. For example, the German physicist Kirchhoff showed that the four hundred sixty bright lines of iron which were known to him were all exactly matched by dark lines in the solar spectrum. Fig. 448 shows a copy of a photograph of a portion of the solar spectrum in the middle, and the corresponding bright-line spectrum of iron each side of it. It will be seen that the coincidence of bright and dark lines is perfect.

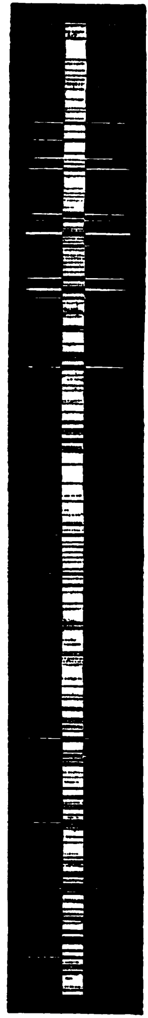


FIG. 448. Comparison of solar and iron spectra

491. Doppler's principle applied to light waves. We have seen (see Doppler's principle, § 398, p. 321) that the effect of the motion of a sounding body toward an observer is to shorten slightly the wave length of the

note emitted, and the effect of motion away from an observer is to increase the wave length. Similarly, when a star is moving toward the earth each particular wave length emitted will be slightly less than the wave length of the corresponding light from a source on the earth's surface. Hence in this star's spectrum all the lines will be displaced slightly toward the violet end of the spectrum. If a star is moving away from the earth, all its lines will be displaced toward the red end. From the direction and amount of displacement, therefore, we can calculate the velocity with which a star is moving toward or receding from the solar system. Observations of this sort have shown that some stars are moving through space toward the solar system with a velocity of 150 miles per second, while others are moving away with almost equal velocities. The whole solar system appears to be sweeping through space with a velocity of about 12 miles per second; but even at this rate it would be at least 1,000,000 years before the earth would come into the neighborhood of the nearest star, even if it were moving directly toward it.

QUESTIONS AND PROBLEMS

1. In what part of the sky will a rainbow appear if it is formed in the early morning?

2. Is a bow seen at 4 o'clock in the afternoon higher or lower than a bow seen at 5 o'clock on the same day?

3. Why is a rainbow never seen during the middle part of the day?

4. If you look at a broad sheet of white paper through a prism, it will appear red at one edge and blue at the other, but white in the middle. Explain why the middle appears uncolored.

5. What evidence have we for believing that there is sodium in the sun?

6. What sort of a spectrum should moonlight give? (The moon has no atmosphere.)

7. If you were given a mixture of a number of salts, how would you proceed with a Bunsen burner, a prism, and a slit, to determine whether or not there was any calcium in the mixture?

8. Can you see any reason why the vibrating molecules of an incandescent gas might be expected to give out a few definite wave lengths, while the particles of an incandescent solid give out all possible wave lengths?

9. Can you see any reason why it is necessary to have the slit narrow and the slit and screen at conjugate foci of the lens in order to show the Fraunhofer lines in the experiment of § 489?

CHAPTER XXI

INVISIBLE RADIATIONS

Radiation from a Hot Body

492. Invisible portions of the spectrum. When a spectrum is photographed the effect on the photographic plate is found to extend far beyond the limits of the shortest visible violet rays. These so-called *ultra-violet rays* have been photographed and measured by Lyman of Harvard down to a wave length of .00001 centimeter, which is only one fourth the wave length of the shortest violet waves.

The longest rays visible in the extreme red have a wave length of about .00008 centimeter, but delicate thermoscopes reveal a so-called *infra-red* portion of the spectrum, the investigation of which was carried in 1912, by Rubens and von Bæyer of Berlin, to wave lengths as long as .03 centimeter, 400 times as long as the longest visible rays.

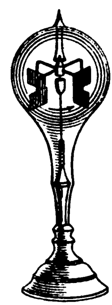


Fig. 449. The Crookes radiometer

The presence of these long heat rays may be detected by means of the radiometer (Fig. 449), an instrument perfected by E. F. Nichols of Dartmouth. In its common form it consists of a partially exhausted bulb, within which is a little aluminium wheel carrying four vanes blackened on one face and polished on the other. When the instrument is held in sunlight or before a lamp, the vanes rotate in such a way that the blackened faces always move away from the source of radiation because they absorb ether waves better than do the polished faces, and thus become hotter. The heated air in contact with these faces then exerts a greater pressure against them than does the air in contact with the polished faces. The more intense the radiation, the faster is the rotation.

A still simpler way of studying these long heat waves was devised in 1912 by Trowbridge of Princeton. A rubber band AC (Fig. 450) a millimeter wide is stretched to double its length over a glass plate $FGHI$, and the thinnest possible glass staff ED, carrying a light mirror E about 2 millimeters square, is placed under the rubber band at its middle point B. When the spectrum is thrown upon the portion AB of the band, the change in its length produced by the heating causes ED to roll, and a spot of light reflected from E to the wall to shift its position by an amount proportional to the heating.

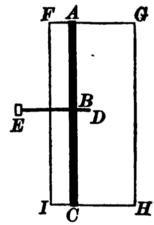


FIG. 450. A simple thermoscope

Let either the radiometer or the thermoscope described above be placed just beyond the red end of the spectrum. It will indicate the presence of heat rays here of even greater energy than those in the visible spectrum. Again, let a red-hot iron ball and one of the detectors be placed at conjugate foci of a large mirror (Fig. 451). The invisible heat rays will be found to be reflected and focused just as are light rays. Next let a flat bottle filled with water be inserted between the detector and any source of heat. It will be found that water, although transparent to light rays, absorbs nearly all of the infra-red rays. But if the water is replaced by carbon bisulphide, the infra-red rays will be freely transmitted, even though the liquid is rendered opaque to light waves by dissolving iodine in it.

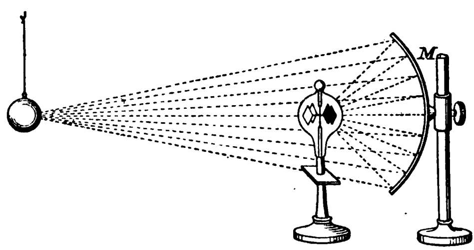


FIG. 451. Reflection of infra-red rays

493. Radiation and temperature. All bodies, even such as are at ordinary temperatures, are continually radiating energy in the form of ether waves. This is proved by the fact that even if a body is placed in the best vacuum obtainable, it continually falls in temperature when surrounded by a colder body, such, for example, as liquid air. The ether waves emitted at ordinary temperatures are doubtless very long as compared with light waves. As the temperature is

raised, more and more of these long waves are emitted, but shorter and shorter waves are continually added. At about 525° C. the first visible waves, that is, those of a dull red color, begin to appear. From this temperature on, owing to the addition of shorter and shorter waves, the color changes continuously — first to orange, then to yellow, and finally, between 800° C. and 1200° C., to white. In other words, all bodies get "red-hot" at about 525° C. and "white-hot" at from 800° C. to 1200° C.

Some idea of how rapidly the total radiation of ether waves increases with increase of temperature may be obtained from the fact that a hot platinum wire gives out thirty-six times as much light at 1400° C. as it does at 1000° C., although at the latter temperature it is already white-hot. The radiations from a hot body are sometimes classified as heat rays, light rays, and chemical or actinic rays. The classification is, however, misleading, since all ether waves are heat waves, in the sense that when absorbed by matter they produce heating effects; that is, molecular motions. *Radiant heat is, then, the radiated energy of ether waves of any and all wave lengths.*

494. Radiation and absorption. Although all substances begin to emit waves of a given wave length at approximately the same temperature, the total rate of emission of energy at a given temperature varies greatly with the nature of the radiating surface. In general, experiment shows that *surfaces which are good absorbers of ether radiations are also good radiators.* From this it follows that *surfaces which are good reflectors*, like the polished metals, *must be poor radiators.*

Thus, let two sheets of tin, 5 or 10 centimeters square, one brightly polished and the other covered on one side with lampblack, be placed in vertical planes about 10 centimeters apart, the lampblacked side of one facing the polished side of the other. Let a small ball be stuck with a bit of wax to the outer face of each. Then let a hot metal

plate or ball (Fig. 452) be held midway between the two. The wax on the tin with the blackened face will melt and its ball will fall first, showing that the lampblack absorbs the heat rays faster than does the polished tin. Now let two blackened glass bulbs be connected, as in Fig. 453, through a U-tube containing colored water, and let a well-polished tin can, one side of which has been blackened, be filled with boiling water and placed between them. The motion of the water in the U-tube will show that the blackened side of the can is radiating heat much more rapidly than the other, although the two are at the same temperature.

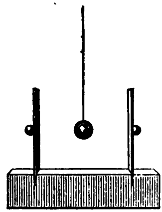


FIG. 452. Good reflectors are poor absorbers

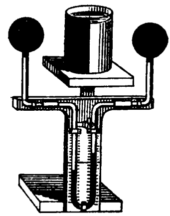


FIG. 453. Good absorbers are good radiators

QUESTIONS AND PROBLEMS

1. When one is sitting in front of an open grate fire does he receive most heat by conduction, convection, or radiation?

2. The atmosphere is transparent to most of the sun's rays. Why are the upper regions of the atmosphere so much colder than the lower regions?

3. Sunlight in coming to the eye travels a much longer air path at sunrise and sunset than it does at noon. Since the sun appears red or yellow at these times, what rays are absorbed most by the atmosphere?

4. Glass transmits all the visible waves, but does not transmit the long infra-red rays. Hence explain the principle of the hotbed.

5. Which will be cooler on a hot day, a white hat or a black one?

6. Will tea cool more quickly in a polished or in a tarnished metal vessel?

7. Which emits the more red rays, a white-hot iron or the same iron when it is red-hot?

8. Liquid air flasks and thermos bottles are doubled-walled glass vessels with a vacuum between the walls. Liquid air will keep many times longer if the glass walls are silvered than if they are not. Why? Why is the space between the walls evacuated?

Electrical Radiations

495. Proof that the discharge of a Leyden jar is oscillatory. We found in § 419, p. 340, that the sound waves sent out by a sounding tuning fork will set into vibration an adjacent fork, provided the latter has the same natural period as the former. The following is the complete electrical analogy of this experiment.

Let the inner and outer coats of a Leyden jar *A* (see Fig. 454) be connected by a loop of wire *cdef*, the sliding crosspiece *de* being arranged so that the length of the loop may be altered at will. Also let a strip of tin foil be brought over the edge of this jar from the inner coat to within about 1 millimeter of the outer coat at *C*. Let the two coats of an exactly similar jar *B* be connected with the knobs *n* and *n′* by a second similar wire loop of fixed length. Let the two jars be placed side by side with their loops parallel, and let the jar *B* be successively charged and discharged by connecting its coats with a static machine or an induction coil. At each discharge of jar *B* through the knobs *n* and *n′* a spark will appear in the other jar at *C*, provided the crosspiece *de* is so placed that the areas of the two loops are equal. When *de* is slid along so as to make one loop considerably larger or smaller than the other, the spark at *C* will disappear.

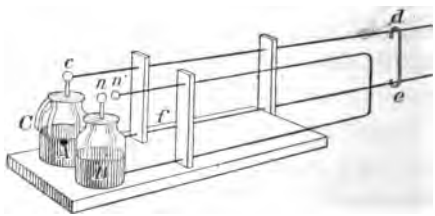


Fig. 454. Sympathetic electrical vibrations

The experiment therefore demonstrates that two electrical circuits, like two tuning forks, can be *tuned* so as to respond to each other sympathetically, and that just as the tuning forks will cease to respond as soon as the period of one is slightly altered, so this *electric resonance* disappears when the exact symmetry of the two circuits is destroyed. Since, obviously, this phenomenon of resonance can occur only between systems which have *natural periods* of vibration, the experiment proves that the discharge of a Leyden jar is a vibratory, that is, an

oscillatory, phenomenon. As a matter of fact, when such a spark is viewed in a rapidly revolving mirror it is actually found to consist of from ten to thirty flashes following each other at equal intervals. Fig. 455 is a photograph of such a spark.

In spite of these oscillations the whole discharge may be made to take place in the incredibly short time of $\frac{1}{10,000,000}$ of a second. This fact coupled with the extreme brightness of the spark has made possible the surprising results of so-called *instantaneous electric-spark photography*. The plate opposite page 414 shows the passage of a bullet through a soap bubble. The film was rotated continuously instead of intermittently, as in ordinary moving-picture photography. The illuminating flashes, 5000 per second, were so nearly instantaneous that the outlines are not blurred.

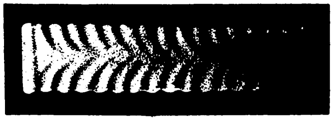


FIG. 455. Oscillations of the electric spark

496. Electric waves. The experiment of § 495 demonstrates not only that the discharge of a Leyden jar is oscillatory, but also that these electrical oscillations set up in the surrounding medium disturbances, or waves of some sort, which travel to a neighboring circuit and act upon it precisely as the air waves acted on the second tuning fork in the sound experiment. Whether these are waves in the air, like sound waves, or disturbances in the ether, like light waves, can be determined by measuring their velocity of propagation. The first determination of this velocity was made by Heinrich Hertz in Germany in 1888. He found it to be precisely the same as that of light; that is, 300,000 kilometers per second. *This result shows, therefore, that electrical oscillations set up waves in the ether.* These waves are now known as Hertz waves.

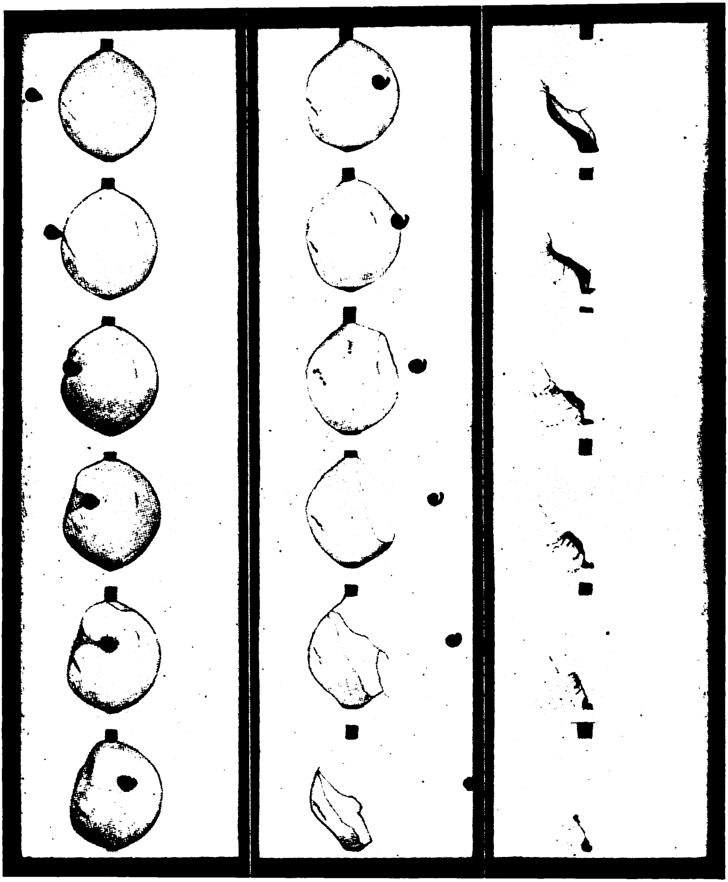
The length of the waves emitted by the oscillatory spark of instantaneous photography is evidently very great, namely, about $\frac{300,000,000}{10,000,000} = 30$ meters, since the velocity of light is

300,000,000 meters per second, and since there are 10,000,000 oscillations per second; for we have seen in § 393, p. 318, that wave length is equal to velocity divided by the number of oscillations per second. By diminishing the size of the jar and the length of the circuit the length of the waves may be greatly reduced. By causing the electrical discharges to take place between two balls only a fraction of a millimeter in diameter, instead of between the coats of a condenser, electrical waves have been obtained as short as .3 centimeter, only ten times as long as the longest measured heat waves.

497. The coherer. In the experiment of § 495 we detected the presence of the electrical waves by means of a small spark gap C in a circuit almost identical with that in which the oscillations were set up. This same means may be employed for the detection of waves many feet away from the source, but the instrument with which electromagnetic waves were first detected hundreds of miles away from the source was *the coherer.* Its principle is illustrated in the following experiment:

Let a glass tube several centimeters long and 6 or 8 millimeters in diameter be filled with fine brass or nickel filings, and let copper wires be thrust into these filings within a distance of about a centimeter of each other. Let these wires be connected in series with a Daniell cell and a simple D'Arsonval galvanometer. The resistance of the loose contacts of the filings will be so great that very little current will flow through the circuit. Now let a static machine be started many feet away. The galvanometer will show a strong deflection as soon as a spark passes between the knobs of the electrical machine. This is because the electric waves, as soon as they fall upon the filings, cause them to cohere or cling together, so that the electrical resistance of the tube of filings is reduced to a small fraction of what it was before. If the tube is tapped with a pencil, the old resistance will be restored, because the filings have been broken apart by the jar. The experiment may then be repeated.

498. Wireless telegraphy. The last experiment illustrates completely the method of transmitting wireless messages during the first decade after Marconi, in 1896, had realized commercial wireless telegraphy. At present the essential elements of the Marconi system of wireless



CINEMATOGRAPH FILM OF A BULLET FIRED THROUGH A SOAP BUBBLE

The flight of the missile may be followed easily. It will be seen that the bubble breaks, not when the bullet enters, but when it emerges. (From "Moving Pictures," by F. A. Talbot. Courtesy of J. B. Lippincott Company)

telegraphy are as follows: The transmitter consists of an ordinary induction coil or transformer T_1, through the primary of which [Fig. 456, (1)] a current is sent from the alternator A. The secondary S of this transformer charges the condenser C_1 until its potential rises high enough to cause a spark discharge to take place across the gap s. This discharge of C_1 is oscillatory (§ 495), the frequency being of the order of 1,000,000 per second, but subject to the control of the operator through the sliding contacts c, precisely as in the case illustrated in Fig. 454. The

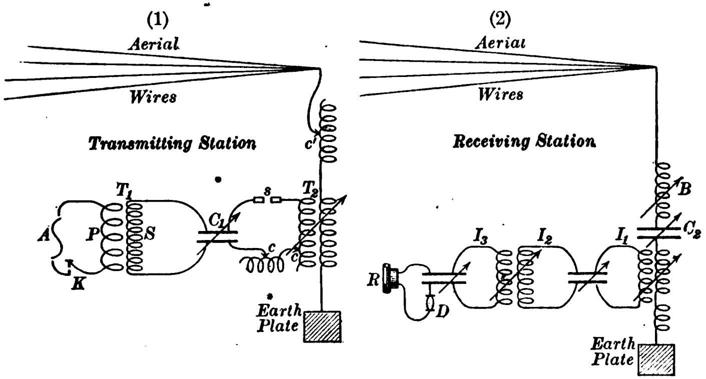


FIG. 456. Transmitting and receiving stations for wireless telegraphy

(1) Transmitting station; (2) receiving station

oscillations in this condenser circuit induce oscillations in the aërial-wire system, which is tuned to resonance with it through the sliding contact c'.*

The waves sent out by this aërial system induce like oscillations in the aërial system of the receiving station [Fig. 456, (2)], it may be thousands of miles away, which is tuned to resonance with it through the variable capacity C_2 and "inductance" B. These oscillations induce exactly similar ones in the condenser circuits I_1, I_2, and I_3, all of which are tuned to resonance with the receiving aërial system. The detector of the oscillations in I_3 is simply a crystal of carborundum D in series with a telephone receiver R. This crystal, like the mercury arc of § 384,

* In the diagram an arrow drawn diagonally across a condenser indicates that for the sake of tuning the condenser is made adjustable. Similarly, an arrow across two circuits coupled inductively, like the primary and secondary of the "oscillation transformer," T_2, indicates that the amount of interaction of the two circuits can be varied, as, for example, by sliding one coil a larger or smaller distance inside the other.

has the property of transmitting a current in one direction only. Were it not for this property the telephone could not be used as a detector because the frequency is so high — of the order of a million. In view of this property, however, while the oscillations of a given spark last, an intermittent current passes in one direction and then ceases until the oscillations of the next spark arrive. Since from 300 to 1000 sparks pass at s per second when the key K is closed, the operator hears a musical note of this frequency as long as K is depressed. Long and short notes then correspond to the dots and dashes of ordinary telegraphy.

The stretching of the aërial wires horizontally instead of vertically, as was formerly done, permits to some extent of directive sending and receiving, for as in the experiment of § 495 the sending and receiving wires work best when they are parallel.

The three tuned circuits, I_1, I_2, I_3, are used because such a series of tuned circuits does not pick up waves of other periods. For "nonselective" receiving these circuits are omitted and the detector and telephone are placed directly across the condenser C_2. The resistance of the telephone is so high that it does not interfere with the oscillations of the condenser system across which it is placed.

499. The electromagnetic theory of light. The study of electromagnetic radiations, like those discussed in the preceding paragraphs, has shown not only that they have the speed of light, but that they are reflected, refracted, and polarized; in fact, that they possess all the properties of light waves, the only apparent difference being in their greater wave length. Hence *modern physics regards light as an electromagnetic phenomenon;* that is, light waves are thought to be generated by the oscillations of the electrically charged parts of the atoms. It was as long ago as 1864 that Clerk-Maxwell, of Cambridge, England, one of the world's most brilliant physicists and mathematicians, showed that it ought to be possible to create ether waves by means of electrical disturbances. But the experimental confirmation of his theory did not come until the time of Hertz's experiments (1888). Maxwell and Hertz together, therefore, share the honor of establishing the modern electromagnetic theory of light (p. 54).

Cathode and Röntgen Rays

500. The electric spark in partial vacua. Let *a* and *b* (Fig. 457) be the terminals of an induction coil or static machine, *e* and *f* electrodes sealed into a glass tube 60 or 80 centimeters long, *g* a rubber tube leading to an air pump by which the tube may be exhausted. Let the coil be started before the exhaustion is begun. A spark will pass between *a* and *b*, since *ab* is a very much shorter path than *ef*. Then let the tube be rapidly exhausted. When the pressure has been reduced to a few centimeters of mercury the discharge will be seen to choose the long path *ef* in preference to the short path *ab*, thus showing that *an electrical discharge takes place more readily through a partial vacuum than through air at ordinary pressures.*

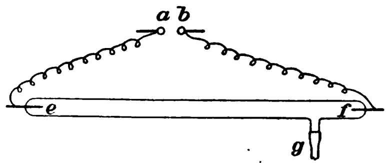


Fig. 457. Discharge in partial vacua

When the spark first begins to pass between *e* and *f*, it will have the appearance of a long ribbon of crimson light. As the pumping is continued this ribbon will spread out until the crimson glow fills the whole tube. Ordinary so-called Geissler tubes are tubes precisely like the above, except that they are usually twisted into fantastic shapes, and are sometimes surrounded with jackets containing colored liquids, which produce pretty color effects.

501. Cathode rays. When a tube like the above is exhausted to a very high degree, say, to a pressure of about .001 millimeter of mercury, the character of the discharge changes completely. The glow almost entirely disappears from the residual gas in the tube, and certain invisible radiations called *cathode rays* are found to be emitted by the cathode (the terminal of the tube which is connected to the negative terminal of the coil or static machine). These rays manifest themselves first by the brilliant fluorescent effects which they produce in the glass walls of the tube, or in other substances within the tube upon which they fall; second, by powerful heating effects; and third, by the sharp shadows which they cast.

†

Thus, if the negative electrode is concave, as in Fig. 458, and a piece of platinum foil is placed at the center of the sphere of which the cathode is a portion, the rays will come to a focus upon a small part of the foil and will heat it white-hot, thus showing that the rays, whatever they are, travel out in straight lines at right angles to the surface of the cathode. This may also be shown nicely by an ordinary bulb of the shape shown in Fig. 460. If the electrode A is made the cathode and B the anode, a sharp shadow of the piece of platinum in the middle of the tube will be cast on the wall opposite to A, thus showing that the cathode rays, unlike the ordinary electric spark, do not pass between the terminals of the tube, but pass out in a straight line from the cathode surface.

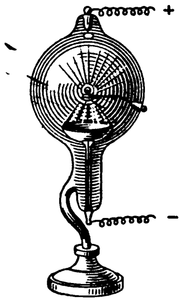


FIG. 458. Heating effect of cathode rays

502. Nature of the cathode rays. The nature of the cathode rays was a subject of much dispute between the years 1875, when they first began to be carefully studied, and 1898. Some thought them to be streams of negatively charged particles shot off with great speed from the surface of the cathode, while others thought they were waves in the ether — some sort of invisible light. The following experiment furnishes very convincing evidence that the first view is correct.

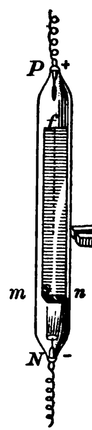


FIG. 459. Deflection of cathode rays by a magnet

NP (Fig. 459) is an exhausted tube within which has been placed a screen *sf* coated with some substance like zinc sulphide, which fluoresces brilliantly when the cathode rays fall upon it; *mn* is a mica strip containing a slit *s*. This mica strip absorbs all the cathode rays which strike it; but those which pass through the slit *s* travel the full length of the tube, and although they are themselves invisible, their path is completely traced out by the fluorescence which they excite upon *sf* as they graze along it. If a magnet *M* is held in the position shown, the cathode rays will be seen to be deflected, and in exactly the direction to be expected if they consisted of negatively charged particles. For we

learned in § 308, p. 240, that a moving charge constitutes an electric current, and in § 360, p. 287, that an electric current tends to move in an electric field in the direction given by the motor rule. On the other hand, a magnetic field is not known to exert any influence whatever on the direction of a beam of light or on any other form of ether waves.

When, in 1895, J. J. Thomson, of Cambridge, England, proved that the cathode rays were also deflected by electric charges, as was to be expected if they consist of negatively charged particles, and when Perrin in Paris had proved that they actually impart negative charges to bodies on which they fall, all opposition to the projected-particle theory was abandoned. The mass and speed of these particles are computed from their deflectibility in magnetic and electric fields.

Cathode rays are then to-day universally recognized as streams of electrons shot off from the surface of the cathode with speeds which may reach the stupendous value of 100,000 *miles per second.*

503. X rays. It was in 1895 that the German physicist, Röntgen, first discovered that wherever the cathode rays impinge upon the walls of a tube, or upon any obstacles placed inside the tube, they give rise to another type of invisible radiation which is now known under the name of X rays, or Röntgen rays. In the ordinary X-ray tube (Fig. 460) a thick piece of platinum *P* is placed in the center to serve as a target for the cathode rays, which, being emitted at right angles to the concave surface of the cathode *C*, come to a focus at a point on the surface of this plate. This is the point at which the X rays are generated and from which they radiate in all directions.

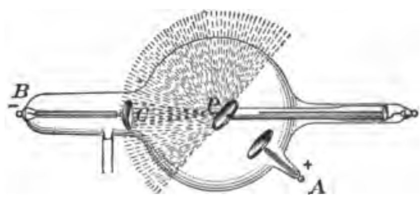


FIG. 460. An X-ray bulb

In order to convince oneself of the truth of this statement, it is only necessary to observe an X-ray tube in action. It will be seen to be

divided into two hemispheres by the plane which contains the platinum plate (see Fig. 460). The hemisphere which is facing the source of the X rays will be aglow with a greenish fluorescent light, while the other hemisphere, being screened from the rays, is darker. By moving a fluoroscope (a zinc sulphide screen) about the tube it will be made evident that the rays which render the bones visible (Fig. 461) come from P.

504. Nature of X rays. While X rays are like cathode rays in producing fluorescence, and also in that neither of them can be reflected, refracted, or polarized, as is light, they nevertheless differ from cathode rays in several important respects. First, X rays penetrate many substances which are quite impervious to cathode rays; for example, they pass through the walls of the glass tube, while cathode rays ordinarily do not. Again, X rays are not deflected either by a magnet or by an electrostatic charge, nor do they carry electrical charges of any sort. Hence it is certain that they do not consist, like cathode rays, of streams of electrically charged particles. Their real nature is still unknown, but they are at present generally regarded as irregular pulses in the ether, set up by the sudden stopping of the cathode-ray particles when they strike an obstruction.

505. X rays render gases conducting. One of the notable properties which X rays possess in common with cathode rays is the property of causing any electrified body on which they fall to slowly lose its charge.

To demonstrate the existence of this property, let any X-ray bulb be set in operation within 5 or 10 feet of a charged gold-leaf electroscope. The leaves at once begin to fall together.

The reason for this is that the X rays shake loose electrons from the atoms of the gas and thus fill it with positively and negatively charged particles, each negative particle being at the instant of separation an electron, and each positive particle an atom from which an electron has been detached. Any charged body in the gas therefore draws toward itself charges of sign opposite to its own, and thus becomes discharged.

JOSEPH JOHN THOMSON (1856–   )

Most conspicuous figure in the development of the "physics of the electron"; born in Manchester, England; educated at Cambridge University; Cavendish professor of experimental physics in Cambridge since 1884; author of a number of books, the most important of which is the "Conduction of Electricity through Gases," 1903; author or inspirer of much of the recent work, both experimental and theoretical, which has thrown light upon the connection between electricity and matter; worthy representative of twentieth-century physics

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506. X-ray pictures. The most striking property of X rays is their ability to pass through many substances which are wholly opaque to light, such, for example, as cardboard, wood, leather, flesh, etc. Thus, if the hand is held close to a photographic plate and then exposed to X rays, a shadow picture of the denser portions of the hand, that is, the bones, is formed upon the plate. Fig. 461 shows a copy of such a picture.

FIG. 461. An X-ray picture of a living hand

RADIOACTIVITY

507. Discovery of radioactivity. In 1896 Henri Becquerel, in Paris, performed the following experiment. He wrapped a photographic plate in a piece of perfectly opaque black paper, laid a coin on top of the paper, and suspended above the coin a small quantity of the mineral uranium. He then set the whole away in a dark room and let it stand for several days. When he developed the photographic plate he found upon it a shadow picture of the coin similar to an X-ray picture. He concluded, therefore, that *uranium possesses the property of spontaneously emitting rays of some sort which have the power of penetrating opaque objects and of affecting photographic plates, just as X rays do.* He also found that these rays, which he called *uranium rays*, are like X rays in that they discharge electrically charged bodies on which they fall. He found also that the rays are emitted by all uranium compounds.

508. Radium. It was but a few months after Becquerel's discovery that Madame Curie, in Paris, began an investigation of all the known elements, to find whether any of the rest of them possessed the remarkable property which had been found to be possessed by uranium. She found that one of the remaining known elements, namely thorium, the chief constitue

of Welsbach mantles, is capable, together with its compounds, of producing the same effect. After this discovery the rays from all this class of substances began to be called *Becquerel rays*, and all substances which emitted such rays were called *radioactive* substances.

But in connection with this investigation Madame Curie noticed that pitchblende, the crude ore from which uranium is extracted, and which consists largely of uranium oxide, would discharge her electroscope about four times as fast as pure uranium. She inferred, therefore, that the radioactivity of pitchblende could not be due solely to the uranium contained in it, and that pitchblende must therefore contain some hitherto unknown element which has the property of emitting Becquerel rays more powerfully than uranium or thorium. After a long and difficult search she succeeded in separating from several tons of pitchblende a few hundredths of a gram of a new element which was capable of discharging an electroscope more than a million times as rapidly as either uranium or thorium. She named this new element *radium*.

509. Nature of Becquerel rays. That these rays which are spontaneously emitted by radioactive substances are not X rays, in spite of their similarity in affecting a photographic plate, in causing fluorescence, and in discharging electrified bodies, is proved by the fact that they are found to be deflected by both magnetic and electric fields, and by the further fact that they impart electric charges to bodies upon which they fall. These properties constitute strong evidence that *radioactive substances project from themselves electrically charged particles.*

But an experiment performed in 1899 by Rutherford, then of McGill University, Montreal, showed that Becquerel rays are complex, consisting of three different types of radiation, which he named the *alpha*, the *beta*, and the *gamma* rays. The beta rays are found to be identical in all respects with cathode rays, that is, *they are streams of electrons* projected with

velocities varying from 60,000 to 180,000 miles per second. The alpha rays are distinguished from these by their very much smaller penetrating power, by their very much greater power of rendering gases conductors, by their very much smaller deflectability in magnetic and electric fields, and by the fact that *the direction of the deflection is opposite to that of the beta rays.* From this last fact, discovered by Rutherford in 1903, the conclusion is drawn that the alpha rays consist of *positively* charged particles; and from the amount of their deflectability their mass has been calculated to be about four times that of the hydrogen atom, that is, about 7000 times the mass of the electron, and their velocity to be about 20,000 miles per second. Rutherford and Boltwood have collected the alpha particles in sufficient amount to identify them definitely as *positively charged atoms of helium.*

The difference in the sizes of the alpha and beta particles explains why the latter are so much more penetrating than the former, and why the former are so much more efficient than the latter in knocking electrons out of the molecules of a gas and rendering it conducting. A sheet of aluminium foil .005 centimeter thick cuts off completely the alpha rays, but offers practically no obstruction to the passage of the beta and gamma rays.

The gamma rays are very much more penetrating even than the beta rays, and are not at all deflected by magnetic or electric fields. They are commonly supposed to be X rays produced by the impact of the beta particles on surrounding matter.

510. Crookes's spinthariscope. In 1903 Sir William Crookes devised a little instrument, called the spinthariscope, which furnishes very direct and striking evidence that particles are being continuously shot off from radium with enormous velocities. In the spinthariscope a tiny speck of radium R (Fig. 462) is placed about a millimeter above a zinc sulphide screen S, and the latter is then viewed through a lens L, which gives from ten to twenty diameters magnification. The continuous soft glow of the screen, which is all one sees with the naked eye, is resolved by the microscope into hundreds of tiny flashes of light. The appearance

as though the screen were being fiercely bombarded by an incessant rain of projectiles, each impact being marked by a flash of light, just as sparks fly from a flint when struck with steel. The experiment is a very beautiful one, and it furnishes very direct and convincing evidence that radium is continually projecting particles from itself at stupendous speeds. The flashes are due to the impacts of the *alpha*, not the *beta*, particles against the zinc sulphide screen.

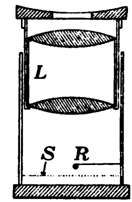


FIG. 462. Crookes's spinthariscope

511. Photographing the tracks of alpha and beta rays. In 1912 C. T. R. Wilson, of Cambridge, England, succeeded in actually photographing, with the aid of the electric spark (see § 495), the tracks of alpha and beta particles as they shoot through air. Some of his photographs are reproduced in the frontispiece. The white streaks there shown are directly due to water vapor condensed upon ions formed along the paths of the rays. In the case of the alpha particles the little water drops are so close together that the photograph shows a continuous white streak. This is on account of the tremendous ionizing power of the alpha particle. In the case of the beta rays the ionization is relatively feeble, and the paths are not so straight, both of which results are to be expected from the smallness of the electron (beta particle) in comparison with the helium atom (alpha particle). The photograph obtained when an X-ray beam was passed through the gas shows that the effect of the X ray is to eject electrons from the molecules of the gas. The path of each of these ejected electrons may be easily traced in the figure.

512. The disintegration of radioactive substances. Whatever be the cause of this ceaseless emission of particles exhibited by radioactive substances, it is certainly not due to any ordinary chemical reactions; for Madame Curie showed, when she discovered the activity of thorium, that the activity of all the radioactive substances is simply proportional to the amount of the active element present, and has nothing whatever to do

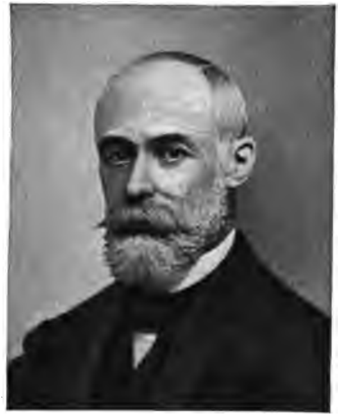
with the nature of the chemical compound in which the element is found. Thus thorium may be changed from a nitrate to a chloride or a sulphide, or it may undergo any sort of chemical reaction, without any change whatever being noticeable in its activity. Furthermore, radioactivity has been found to be independent of all *physical* as well as chemical conditions. The lowest cold or greatest heat does not appear to affect it in the least. Radioactivity, therefore, seems to be as unalterable a property of the atoms of radioactive substances as is weight itself. For this reason Rutherford has advanced the theory that the atoms of radioactive substances are slowly disintegrating into simpler atoms. Uranium and thorium have the heaviest atoms of all the elements. For some unknown reason they seem not infrequently to become unstable and project off a part of their mass. This projected mass is the alpha particle. What is left of the atom after the explosion is a new substance with chemical properties different from those of the original atom. This new atom is, in general, also unstable and breaks down into something else. This process is repeated over and over again until some stable form of atom is reached. Somewhere in the course of this atomic catastrophe some electrons leave the mass; these are beta rays.

According to this point of view, which is now generally accepted, radium is simply one of the stages in the disintegration of the uranium atom. The atomic weight of uranium is 238.5, that of radium about 226, that of helium 3.994. Radium would then be uranium after it has lost 3 helium atoms. The further disintegration of radium through four additional transformations has been traced. It has been conjectured that the fifth and final one is lead. If we subtract 8×3.994 from 238.5, we obtain 206.5, which is very close to the accepted value for lead, namely 207. In a similar way six successive stages in the disintegration of the thorium atom (atomic weight 232) have been found, but the final product is unknown.

513. Energy stored up in the atoms of the elements. In 1903 the two Frenchmen, Curie and Labord, made an epoch-making discovery. It was that radium is continually evolving heat at the rate of about one hundred calories per hour per gram. More recent measurements have given one hundred eighteen calories. This result was to have been anticipated from the fact that the particles which are continually flying off from the disintegrating radium atoms subject the whole mass to an incessant internal bombardment which would be expected to raise its temperature. This measurement of the exact amount of heat evolved per hour enables us to estimate how much heat energy is evolved in the disintegration of one gram of radium. It is about two thousand million calories — fully three hundred thousand times as much as is evolved in the combustion of one gram of coal. Furthermore, it is not impossible that similar enormous quantities of energy are locked up in the atoms of *all* substances, existing there perhaps in the form of the kinetic energy of rotation of the electrons. The most vitally interesting question which the physics of the future has to face is, Is it possible for man to gain control of any such store of subatomic energy and to use it for his own ends? Such a result does not now seem likely or even possible; and yet the transformations which the study of physics has wrought in the world within a hundred years were once just as incredible as this. In view of what physics has done, is doing, and can yet do for the progress of the world, can any one be insensible either to its value or to its fascination?

WILLIAM CONRAD RÖNTGEN, MUNICH

Discoverer of X rays



ANTOINE HENRI BECQUEREL, PARIS

Discoverer of radioactivity

MADAME CURIE, UNIVERSITY OF PARIS

Discoverer of radium

E. RUTHERFORD, UNIVERSITY OF MANCHESTER (ENGLAND)

Discoverer of radioactive transformations

A GROUP OF MODERN PHYSICISTS

APPENDIX

REVIEW QUESTIONS AND PROBLEMS

1. A cubical box 20 cm. on a side is filled with equal parts of mercury and water. What is the entire force on the inner surface of the box?

2. Suppose a tube 5 mm. square and 200 cm. long is inserted into the top of the box mentioned in the previous problem and filled with water, what will the entire force be?

3. A floating dock is shown in Fig. 463. When the chambers *c* are filled with water the dock sinks until the water line is at *A*. The vessel is then floated into the dock. As soon as it is in place, the water is pumped from the chambers until the water line is as low as *B*. Workmen can then get at all parts of the bottom. If each of the chambers is 10 ft. high and 10 ft. wide, what must be the length of the dock if it is to be available for the *Imperator* (Hamburg-American Line), of 50,000 tons' weight?

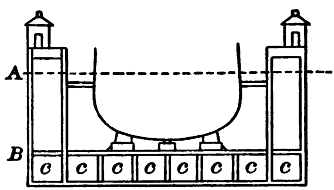


FIG. 463. Floating dock

4. The density of stone is about 2.5. If a boy can lift 120 lb., how heavy a stone can he lift to the surface of a pond?

5. How many cubic centimeters of a liquid of specific gravity 1.5 must be mixed with 1 l. of a liquid of specific gravity .8 to make a mixture of specific gravity 1.3?

6. A diver with his diving suit weighs 100 kg. It requires 15 kg. of lead to sink him. If the density of lead is 11.3, what is the volume of the diver and his suit?

7. A body loses 25 g. in water, 23 g. in oil, and 20 g. in alcohol. Find the density of the oil and of the alcohol.

8. A platinum ball weighs 330 g. in air, 315 g. in water, and 303 g. in sulphuric acid. Find the density of the platinum, the density of the acid, and the volume of the ball.

9. What fraction of the total volume of an irregular block of wood of density .6 will float above the surface of alcohol of density .8?

10. What must be the specific gravity of a liquid in which a body having a specific gravity of 6.8 will float with half its volume submerged?

11. How large a balloon filled with hydrogen is needed to raise a weight of 300 lb., including the balloon? Explain.

12. What is Boyle's law? A mass of air 3 cc. in volume is introduced into the space above a barometer column which originally stands at 760 mm. The column sinks until it is only 570 mm. high. Find the volume now occupied by the air.

13. The diameters of the piston and cylinder of a hydrostatic press are respectively 3 in. and 30 in. The piston rod is attached 2 ft. from the fulcrum of a lever 12 ft. long (Fig. 12, p. 17). What force must be applied at the end of the lever to make the press exert a force of 5000 lb.?

14. How high will a lift pump raise water if it is located upon the side of a mountain where the barometer reading is 71 cm.?

15. If the cylinder of an air pump is $\frac{1}{3}$ the size of the receiver, what fractional part of the original air will be left after 5 strokes?

16. A gas at constant pressure expands $\frac{1}{273}$ of its volume at 0° C. for every degree it is raised above 0° C. How much will it expand for every degree F. above 32° F.?

17. A water tank 8 ft. deep, standing some distance above the ground, closed everywhere except at the top, is to be emptied. The only means of emptying it is a flexible tube.

(a) What is the most convenient way of using the tube, and how could it be set into operation?

(b) How long must the tube be to empty the tank completely?

18. If when the barometric height is 76 cm. and the temperature is 30° C. some water is introduced into an air-tight vessel, what will a barometer in the vessel read?

19. A ball shot straight upward near a pond was seen to strike the water in 10 sec. How high did it rise? What was its initial speed?

20. With what velocity must a ball be shot upward to rise to the height of the Washington Monument (555 ft.)? How long before it will return?

21. A pull of a dyne acts for 3 sec. on a mass of 1 g. What velocity does it impart?

22. How long must a force of 100 dynes act on a mass of 20 g. to impart to it a velocity of 40 cm. per second?

23. A force of 1 dyne acts on 1 g. for 1 sec. How far has the gram been moved at the end of the second?

24. A steamboat weighing 20,000 metric tons is being pulled by a tug which exerts a pull of 2 metric tons. (A metric ton is equal to 1000 kg.) If the friction of the water were negligible, what velocity would the boat acquire in 4 min.? (Reduce mass to grams, force to dynes, and remember that $F = mv/t$.)

25. If a train of cars weighs 200 metric tons, and the engine in pulling 5 sec. imparts to it a velocity of 2 m. per second, what is the pull of the engine in metric tons?

26. A steel ball dropped into a pail of moist clay from a height of a meter sinks to a depth of 2 cm. How far will it sink if dropped 4 m.?

27. Neglecting friction, find how much force a boy would have to exert to pull a 100-lb. wagon up an incline which rises 5 ft. for every 100 ft. of length traversed on the incline. Give not merely the numerical solution of the problem, but state why you solve it as you do, and how you know that your solution is correct.

28. Describe fully how you would proceed to find the density of an irregular solid heavier than water, showing *why* in every case you proceed as you do.

29. A rifle weighing 5 lb. discharges a 4-oz. bullet with a velocity of 100 ft. per sec. What will be the velocity of the rifle in the opposite direction?

30. A bullet weighing 2 oz. is shot into a body weighing 30 lb. hanging freely suspended. If the velocity of the bullet is 1500 ft. per second, what will be the vertical height to which the body will be raised?

31. How many times as much weight will a wire which is twice as thick as another of similar material support?

32. A force of 3 lb. stretches 1 mm. a wire that is 1 m. long and .1 mm. in diameter. How much force will it take to stretch 5 mm. a wire of the same material 4 m. long and .15 mm. in diameter?

33. Why do some liquids rise while others are depressed in capillary tubes?

34. A metal rod 230 cm. long expanded 2.75 mm. in being raised from 0° C. to 100° C. Find its coefficient of linear expansion.

35. If iron rails are 30 ft. long, and if the variation of temperature throughout the year is 50° C., what space must be left between their ends?

36. If the total length of the iron rods *b*, *d*, *e*, and *i* in a compensated pendulum (Fig. 129) is 2 m., what must be the total length of the copper rods *c* if the period of the pendulum is independent of temperature?

37. Decide from the table of expansion coefficients given on page 128 why the wires which lead the current through the walls of incandescent electric-light bulbs are always made of platinum; that is, why it is impossible to seal any other metal into glass.

38. If a diver descends to a depth of 100 ft., what is the pressure to which he is subjected? What is the density of the air in his suit, the density at the surface where the pressure is 75 cm. being .00123? (Assume the temperature to remain unchanged.)

39. A bubble escapes from a diver's suit at a depth of 100 ft. where the temperature is 4° C. To how many times its original volume has the bubble grown by the time it reaches the surface, where the temperature is 30° C. and the barometric height 75 cm.?

40. Find the density of the air in a furnace whose temperature is 1000° C., the density at 0° C. being .001293.

41. The air within a half-inflated balloon occupies a volume of 100,000 l. The temperature is 15° C. and the barometric height 75 cm. What will be its volume after the balloon has risen to the height of Mt. Blanc, where the pressure is 37 cm. and the temperature − 10° C.?

42. If the volume of a quantity of air at 30° C. is 200 cc., at what temperature will its volume be 300 cc., the pressure remaining the same?

43. When the barometric height is 76 cm. and the temperature 0° C., the density of air is .001293. Find the density of air when the temperature is 38° C. and the barometric height is 73 cm. Find the density of air when the temperature is − 40° C. and the barometric height 74 cm.

44. A lever is 3 ft. long. Where must the fulcrum be placed so that a weight of 300 lb. at one end shall be balanced by 50 lb. at the other?

45. Where must a load of 100 lb. be placed on a stick 10 ft. long, if the man who holds one end is to support 30 lb., while the man at the other end supports 70 lb.?

46. Two horses of unequal strength must be hitched as a team. The one is to pull 360 lb., while the other pulls 288 lb. In the doubletree 50 in. long, where must the pin be placed to permit an even pull?

47. How many gallons of water (8 lb. each) could a 10 H.P. engine raise in one hour to a height of 60 ft.?

48. Find graphically the resultant of 40 lb. N.E. and 70 lb. W.

49. In the course of a stream is a waterfall 22 ft. high. It is shown by measurement that 450 cu. ft. of water per second pour over it. How many foot pounds of energy could be obtained from it? What horse power? What becomes of this energy if not used in driving machinery?

50. A car weighing 60,000 kilos slides down a grade which is 2 m. lower at the bottom than at the top, and is brought to rest at the bottom by the brakes. How many calories of heat are developed by the friction?

51. A body weighing 10 kilos is pushed 10 m. along a level plane. If the coefficient of friction between the block and the plane is .125, how many gram centimeters of work have been done? How many ergs? How many calories of heat have been developed?

52. Meteorites are small cold bodies moving about in space. Why do they become luminous when they enter the earth's atmosphere?

53. A piece of platinum weighing 10 g. is taken from a furnace and plunged instantly into 40 g. of water at 10° C. The temperature of the water rises to 24° C. What was the temperature of the furnace?

54. A body is projected along a horizontal plane with a velocity of 100 ft. per second, the coefficient of friction being $\frac{1}{10}$. How far will it go before coming to rest?

55. With what velocity must a body be moving in order, before coming to rest, to pass over 20 m. on a horizontal plane, the coefficient of friction of which is $\frac{1}{8}$?

56. The efficiency of a good condensing engine is about 16%. How much coal is consumed per hour by a 40,000 H.P. condensing engine, each gram of coal being assumed to produce 6000 calories?

57. The average locomotive has an efficiency of about 6%. What horse power does it develop when it is consuming 1 ton of coal per hour? (See Problem 56.)

58. What pull does a 1000 H.P. locomotive exert when it is running at 25 mi. per hour and exerting its full horse power?

59. Equal weights of hot water and ice are mixed and the result is water at 0° C. What was the temperature of the hot water?

60. From what height must a gram of ice at 0° C. fall in order to melt itself by the heat generated in the impact?

61. What temperature will result from mixing 10 g. of ice at 0° C. with 200 g. of water at 25° C.?

62. One hundred grams of water at 80° C. are thoroughly mixed with 500 g. of mercury at 0° C. What is the temperature of the mixture?

63. Just what will occur if 1000 calories be applied to 20 g. of ice at 0° C.?

64. If the specific heat of lead is .031 and the mechanical equivalent of a calorie 427 g. m., through how many degrees centigrade will a 1000-g. lead ball be raised if it falls from a height of 100 m., provided all of the heat developed by the impact goes into the lead?

65. How many grams of ice must be put into 200 g. of water at 40° C. to lower the temperature 10° C.?

66. Ten grams of steam at 100° C. are cooled to 41° F. How much heat is given out?

67. A magnetic pole of 80 units' strength is 20 cm. distant from a similar pole of 30 units' strength. Find the force between them.

68. Two small spheres are charged with + 16 and − 4 units of electricity. With what force will they attract each other when at a distance of 4 cm.?

69. If the two spheres of the previous problem are made to touch and are then returned to their former positions, with what force will they act on each other? Will this force be attraction or repulsion?

70. If an electrified rod is brought near to a pith ball suspended by a silk thread, the ball is first attracted to the rod and then repelled from it. Explain this.

71. The diameter of No. 20 wire is 31.96 mils (1 mil = .001 in.) and that of No. 30 wire 10.025 mils. Compare the resistances of equal lengths of No. 20 and No. 30 German-silver wires.

72. What length of No. 30 copper wire will have the same resistance as 20 ft. of No. 20 copper wire?

73. What length of No. 20 German-silver wire will have the same resistance as 100 ft. of No. 30 copper wire?

74. If a certain Daniell cell has an internal resistance of 2 ohms and an E.M.F. of 1.08 volts, what current will it send through an ammeter whose resistance is negligible? What current will it send through a copper wire of 2 ohms' resistance? through a German-silver wire of 100 ohms resistance?

75. A Daniell cell indicates a certain current when connected to a galvanometer of negligible resistance. When a piece of No. 20 German-silver wire is inserted into the circuit, it is found to require a length of 5 ft. to reduce the current to one half its former value. Find the resistance of the cell in ohms, No. 20 German-silver wire having a resistance of 190.2 ohms per 1000 ft.

76. A coil of unknown resistance is inserted in series with a considerable length of No. 30 German-silver wire and joined to a Daniell cell. When the terminals of a high-resistance galvanometer are touched to the wire at points 10 ft. apart, the deflection is found to be the same as when they are touched across the terminals of the unknown resistance. What is the resistance of the unknown coil? (See § 317, p. 252.)

77. Find the joint resistance of 10 ft. of No. 30 copper wire and 1 ft. of No. 20 German-silver wire connected in series; in parallel.

78. Three wires, each having a resistance of 15 ohms, were joined abreast and a current of 3 amperes sent through them. How much was the E.M.F. of the current?

79. The E.M.F. of a certain battery is 10 volts and the strength of the current obtained through an external resistance of 4 ohms is 1.25 amperes. What is the internal resistance of the battery?

80. How many cells, each of E.M.F. 1.5 volts and internal resistance 2 ohms, will be needed to send a current of at least 1 ampere through an external resistance of 40 ohms?

81. How many lamps, each of resistance 20 ohms, and requiring a current of .8 ampere, can be lighted by a dynamo that has an output of 4000 watts?

82. How many calories will be developed in 10 sec. by a current of 20 amperes flowing through a resistance of 100 ohms?

83. Draw a diagram of a two-station telegraph line, showing receiving and sending instruments at each station and a relay at one station.

84. Draw a diagram of an induction coil and explain its action.

85. (a) Describe and illustrate resonance.

(b) Find the number of vibrations per second of a fork which produces resonance in a pipe 1 ft. long. (Take the speed of sound as 1120 ft. per second.)

86. Build up a diatonic scale on $C = 264$.

87. If a vibrating string is found to produce the note C when stretched by a force of 10 lb., what must be the force exerted to cause it to produce (a) the note E? (b) the note G?

88. What is meant by the phenomenon of beats in sound? How may it be produced, and what is its cause?

89. Show what relation exists between the wave length of a note and the lengths of the shortest closed and open pipes which will respond to this note.

90. (a) How can you show that the wave lengths of red and green lights are different, and how can you determine which one is the longer?

(b) Explain as well as you can how a telescope forms the image which you see when you look into it.

91. A clapper strikes a bell once every two seconds. How far from the bell must a man be in order that the clapper may appear to hit the bell at the exact instant at which each stroke is heard?

92. The note from a piano string which makes 300 vibrations per second passes from indoors, where the temperature is 20° C., to outdoors, where it is 5° C. What is the difference in centimeters between the wave lengths indoors and outdoors?

93. A man riding on an express train moving at the rate of 1 mi. per minute hears a bell ringing in a tower in front of him. If the bell makes 300 vibrations per second, how many pulses will strike his ear per second, the velocity of sound being 1130 ft. per second? (The number of extra impulses received per second by the ear is equal to the number of wave lengths contained in the distance traveled per second by the train.)

94. How many candles will be required to produce the same intensity of illumination at 2 m. distance that is produced by 1 candle at 30 cm. distance?

95. In which medium, water or air, does light travel the faster? Give reasons for your answer.

†

96. Draw diagrams to show in what way a beam of light is bent (a) in passing through a prism; (b) in passing obliquely through a plate-glass window.

97. Distinguish between a real image and a virtual image, and state the conditions for the formation of each by a convex lens; by a concave mirror.

98. Show by a diagram and explanation what is meant by critical angle.

99. Does blue light travel slower or faster in glass than red light? How do you know?

100. Draw a figure to show how a spectrum is formed by a prism, and indicate the relative positions of the red, the yellow, the green, and the blue in this spectrum.

101. Draw a diagram of a slit, a prism, and a lens, so placed as to form a pure spectrum.

102. Why is the order of the colors in the secondary rainbow the reverse of the order in the primary bow?

103. An object 5 cm. long is 50 cm. from a concave mirror of focal length 30 cm. Where is the image, and what is its size?

104. An object is 20 cm. from a lens of focal length 30. Where is the image?

105. Why is it necessary to use a rectifying crystal in series with a telephone receiver to detect electric waves?

106. Explain why an electroscope is discharged when a bit of radium is brought near it.

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